Resonances in Chiral perturbation theory

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Und wenn du lange in einen Abgrund blickst,
blickt der Abgrund auch in dich hinein.

Fridrich Nietzsche

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Abstrakt: V předložené práci studujeme roli resonancí v chirální poruchové teorii (χPT), konkrétně vektorových resonancí 1^{--}. V první části je naznačena cesta, která vede k zavedení resonanční chirální teorii (RχT), jež je aproximací ke QCD v limitě velkého počtu barev. V další části práce na příkladě výpočtu konkrétních korelátorů studujeme vysokoenergetické podmínky plynoucí z OPE a vztah RχPT a χPT při nízkých energií. V poslední kapitole se také krátce dotkneme otázky renormalizace v RχT a naznačíme případné problémy, které se zde mohou objevit.
Klíčová slova: chirální poruchová teorie, vektorové resonance, QCD, resonanční chirální teorie

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Abstract: In the present work we study the role of resonances in Chiral Perturbation Theory (χPT), concretely the vector resonances 1^{--}. In the first part there is presented the way which leads to the introduction of the Resonance Chiral Theory (RχT). This is an approximation to QCD with infinite number of colors. Then we do the calculations of various correlators and we study the high energy constraints dictated by OPE and the relationship between RχT and χPT at low energies. The last chapter briefly mentions the problem of renormalization in RχT.
Keywords: Chiral Perturbation Theory, vector resonances, QCD, Resonance Chiral Theory

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I declare that I wrote the thesis by myself and listed all used sources. I agree with making this thesis publicly available.

Prague, 10.8.2007 Jaroslav Trnka

ARTICLES

The thesis is based on the results presented in these articles:

[A] K. Kampf, J. Novotny and J. Trnka, *On different Lagrangian formalisms for vector resonances within chiral perturbation theory*, Eur. Phys. J. C **50**, 385 (2007), `arXiv:hep-ph/0608051`

[B] K. Kampf, J. Novotný and J. Trnka, *Tensor and vector formulations of resonance chiral theory*, Presented by K.K. at the Final Euridice Meeting, 24-27 August 2006, Kazimierz, Poland, `arXiv:hep-ph/0701041.`

[C] K. Kampf, J. Novotný and J. Trnka, *First order formalism for spin one fields*, Presented by J.T. at the Petrov summer school and conference, 30 June 2007, Kazan, Russia

[D] K. Kampf, J. Novotný and J. Trnka, *Ghosts in theories with spin one fields*, in preparation

[E] K. Kampf, J. Novotný and J. Trnka, *Compton-like scattering in Resonance Chiral Theory*, in preparation

Contents

Introduction

Chiral Perturbation Theory (χPT) [1, 2, 3, 4, 5] is an effective theory for strong interactions and describes the dynamics of the lightest hadrons and their interactions at low energies. Underlying theory, Quantum chromodynamics, is formulated in terms of quarks and gluons as its degrees of freedom. The process of spontaneous symmetry breaking gives rise to the octet of the Goldstone bosons. In χPT we identify these Goldstone bosons (or pseudogoldstone bosons when the quark masses are taken into account) with the octet of the lightest hadrons, i.e. with the octet of the pseudoscalar mesons ($\pi^0, \pi^\pm, K^\pm, \dots$). In the low energy region (under some scale Λ that is typically $\Lambda \approx 1\,\mathrm{GeV}$, the approximate mass of nongoldstone particles) these degrees of freedom dominate and they can be assumed as the only effective hadronic degrees of freedom.

χPT is formulated as a perturbative theory in terms of the small external momentum $p/\Lambda \ll \Lambda$[1]. This is a well-founded approach because Goldstone bosons interact weakly at small energies and therefore, the Lagrangian of χPT can be then written in the form: $\mathcal{L}_\chi = \mathcal{L}_2 + \mathcal{L}_4 + \dots$ where $\mathcal{L}_n = \mathcal{O}(p^n)$. Weinberg formula [1] provides us with the consistent power counting, i.e. the rule which operators should be used when calculating concrete tree level or loop diagrams up to a given order.

The Lagrangian of χPT contains a set of coupling constants (called LEC - low energy constants)[2] that describe not only the interactions of the lightest hadrons but also effectively include the contributions of the heavy degrees of freedom (resonances). For energies $p \approx \Lambda$, χPT looses its convergence and it is necessary to introduce the phenomenological Lagrangians based on the large N_C QCD that describe the direct interactions of resonances. Of course, when integrating out these heavier states and coming back to low energies we reestablish the original χPT Lagrangian. This can help us to learn how the χPT coupling constants are saturated by the interactions of resonances. Restricting ourselves only to the lightest resonances in each channel we introduce the Resonance chiral theory (RχT)[6, 7]. Matching with experiments can give us

[1] In the massive case we do the expansion also in the quark masses which are of the second order, $m_q = \mathcal{O}(p^2)$

[2] For $\mathcal{O}(p^2)$ we have 2 constants, for $\mathcal{O}(p^4)$ 14 constants and for $\mathcal{O}(p^6)$ approximately 100 constants

the predictions of the values of LEC [8]. RχT has not been yet formulated as a closed theory, despite a considerable progress has already been done [9, 10, 11, 12].

This thesis makes a simplification of RχT to the case of one type of resonances, vector resonances 1^{--}, but the general case does not principally differ from it. Our contributions to the study of RχT can be divide in several areas that will be discussed in the following chapters.

1. The different Lagrangian formalisms for description of (vector) resonances are not fully equivalent. The contributions to the effective chiral Lagrangian start at different orders in vector and antisymmetric tensor formalisms. This problem is briefly discussed in chapter 2. The detailed version can be found in article [A] and some fragments in [B] and [C].

2. There is no complete study of high energy constraints and their applications to various correlators. The discussion of two and three point Green functions is proposed in chapter 3. Moreover, we have also studied the more difficult example of four point correlator $\langle VVPP \rangle$ and Compton-like scattering. The results can be found in chapter 4 and in appendix C. The detailed study will appear in [E].

3. Quantum loops in RχPT were briefly studied in [13, 14] but with the simplest Lagrangian terms only. In chapter 5 can be found the systematic study of the renormalization of resonance propagators and its interesting consequences. Complete version will be published in article [D].

Formally, this thesis is segmented in the following way. In chapter 1 it is briefly described the Chiral Perturbation Theory and its connection with QCD. Chapter 2 is focused on the basis of Resonance Chiral Theory, the way how to describe resonances in the framework of effective theories for QCD. Next chapters provide with explicit calculations of some processes together with the interpretation of the results that can help us to study the formal properties of the Resonance Chiral Theory. In chapter 3 we study the two point and the three point Green functions, in chapter 4 there are proposed the calculations of Compton-like scattering and in chapter 5 the one loop corrections to resonance propagators. Some technical tools and complementary results can be found in appendices.

CHAPTER 1

Introduction to Chiral Perturbation Theory

In the first chapter we want to describe briefly the motivation that leads to the construction of Chiral Perturbation Theory, the effective theory for QCD at low energies.

First, we discuss the realization of symmetries in quantum field theory and then we formulate Goldstone theorem that connects the spontaneously symmetry breaking and the presence of Goldstone bosos in the spectrum. Next, we concentrate on the case of QCD, the gauge theory for strong interactions, and we mention some of its formal properties. We write $SU(3)_C$ invariant Lagrangian and we focus only on the light quark sector with massless quarks u, d, s. The Lagrangian then possesses the additional flavor symmetry $U(3)_L \times U(3)_R$ which is broken on the quantum level to $SU(3)_L \times SU(3)_R \times U(1)_V$ (the axial symmetry $U(1)$ is not present).

We also comment some features of chiral Ward identities as the relations between various Green functions that represent the symmetry properties of the Lagrangian on the quantum level. In order to incorporate all Ward identities we introduce the external sources (currents and densities) into the Lagrangian. The Lagrangian $\mathcal{L} = \mathcal{L}^0_{QCD} + \mathcal{L}_{ext}$ is then invariant under the local chiral symmetry group $SU(3)_L \times SU(3)_R \times U(1)_V$. The external sources are coupled on the interpolating fields and they are often used to introduce other interactions (e.g. electroweak,...) or the quark mass matrix into the massless QCD.

Furthermore, we discuss the effect of symmetry breaking in QCD. The explicit symmetry breaking is provided by the introduction of the quark mass matrix. Consequently, the flavor symmetry is then completely destroyed in the general case. In QCD, there exists an order parameter and therefore, the chiral symmetry is also spontaneously broken to the subgroup $SU(3)_V$. According to Goldstone theorem, this phenomenon leads to the existence of 8 massless Goldstone bosons in the spectrum of QCD.

In Chiral Perturbation Theory, these particles are identified with an octet of pseudoscalar mesons which are the lightest degrees of freedom in the hadronic spectrum. We formulate χPT as a perturbation theory based on the symmetry properties of QCD with an external momentum p as an expansion parameter. The connection between χPT and QCD can also be expressed as the equality of generating functionals after both are expanded in terms of p. Theoretically, we should express the coupling constants in ChPT Lagrangian in terms of QCD parameters but practically, this is impossible to do.

1.1 Symmetries in QFT

Let us assume that the Lagrangian of the system is invariant under the symmetry group G with the conserved currents $J^a_\mu(x)$. There are two possible ways how to realize this symmetry: Wigner-Weyl realization and Nambu-Goldstone realization.

Wigner-Weyl realization

This situation occurs when not only the Lagrangian but also the vacuum is invariant under the action of symmetry group G. Its elements can be represented by means of unitary operators

$$U = \exp\left(-i\alpha^a Q^a\right), \qquad a = 1, \ldots n \tag{1.1.1}$$

where n is the dimension of the group generated by the charges Q^a,

$$Q^a = \int d^3x J^a_0(x) \tag{1.1.2}$$

which commute with the Hamiltonian, $[H, Q^a]$=0. For the vacuum we have

$$U|0\rangle = |0\rangle \qquad \rightarrow \qquad Q^a|0\rangle = 0. \tag{1.1.3}$$

Because the generators of symmetry group commute with Hamiltonian, the energy eigenstates are degenerate and they form the multiplets with the same energies. The number of states then relates to the dimension of a representation of the group G.

Nambu-Goldstone realization

If the vacuum is not invariant under the action of elements of G, the situation is different. We can then divide the generators into two parts $Q^a = (H^i, X^j)$ where

$$H^i|0\rangle = 0, \qquad X^j|0\rangle \neq 0 \tag{1.1.4}$$

The generators H^i form the subgroup H (little group) of the symmetry group G and the realization is of Wigner-Weyl type. This is no longer possible for the generators X^j. Let us denote the energy of the vacuum E_0, $H|0\rangle = E_0|0\rangle$. Then the states $X^j|0\rangle$ have the same energy,

$$H(X^j|0\rangle) = X^j H|0\rangle = E_0(X^j|0\rangle \tag{1.1.5}$$

and therefore, the vacuum is degenerate. But the cluster decomposition theorem (see for example [18]) indicates that the vacuum must be non-degenerate. Moreover, the states $X^j|E\rangle$ are not well defined on the Hilbert space and the corresponding multiplets are missing in the physical spectrum.

Goldstone theorem

The spontaneously broken symmetry relates very closely to the spectrum of the theory. Goldstone theorem claims:

If the Lagrangian is invariant under the symmetry of the continuous group G and the vacuum is invariant only under the symmetry of continuous group $H \subset G$, then there appear n massless scalar particles in the spectrum, where $n = \dim G - \mathrm{dimH}$.

Another formulation of Goldstone theorem says that for every generator of the symmetry group Q^a for which there exists an operator $\mathcal{O}$ such that

$$\langle 0|[Q^a, \mathcal{O}]|0\rangle \neq 0, \tag{1.1.6}$$

there appears in the spectrum one independent massless state $|\phi^a\rangle$ with

$$\langle 0|J_0^a(0)|\phi^a\rangle\langle\phi^a|\mathcal{O}|0\rangle \neq 0 \tag{1.1.7}$$

where $J_0^a(0)$ is the zero component of the conserved current. We call the quantity $\delta^a\mathcal{O} = [Q^a, \mathcal{O}]$ the *order parameter*. Its non-vanishing vacuum expectation value leads to spontaneously symmetry breaking and (according to Goldstone theorem) to the existence of the Goldstone bosons.

1.2 Quantum chromodynamics

Quantum chromodynamics (QCD) is the quantum field theory of strong interactions. It is based on $SU(3)_C$ color gauge symmetry and describes quarks and gluons as its fundamental degrees of freedom. However, quarks have been never observed as free asymptotic states, only their composite particles are in the physical spectrum. Regardless, we believe that QCD is the fundamental theory and it is principally possible to use it for description of the behavior of all strongly interacting hadrons.

QCD Lagrangian

As was said QCD describes quarks and gluons as their fundamental degrees of freedom. We introduce the quark colour triplet as the basic building block

$$q_f = \begin{pmatrix} q_f^r \\ q_f^g \\ q_f^b \end{pmatrix} \tag{1.2.1}$$

where f stands for flavour of the quark triplet and upper index is the color one (r - red, g - green, b - blue). This triplet transforms as

$$q_f \to U(x)q_f \tag{1.2.2}$$

where $U(x)$ stands for an element of $SU(3)_C$ color group. This means that each flavor triplet transforms separately in the same way as others. The $SU(3)_C$ invariant quark Lagrangian can be written in the form

$$\mathcal{L}_q = \sum_f \overline{q}_f(i\gamma^\mu D_\mu - m_f)q_f \tag{1.2.3}$$

where D^μ is the covariant derivative such that $D^\mu q_f$ transforms as the triplet too.

$$D_\mu q_f = \partial_\mu q_f - ig\mathcal{A}_\mu(x)q_f, \tag{1.2.4}$$

with $\mathcal{A}_\mu(x)$, the octet of $SU(3)_C$ gauge fields

$$\mathcal{A}_\mu(x) = \sum_{a=1}^{8} \frac{\lambda^a}{2}\mathcal{A}^a_\mu(x). \tag{1.2.5}$$

that transform as

$$\mathcal{A}_\mu(x) \mapsto U(x)\mathcal{A}_\mu(x)U^\dagger(x) - \frac{i}{g}\partial_\mu U(x)U^\dagger(x) \tag{1.2.6}$$

The gauge particles for QCD, gluons, mediate the interactions between quarks. The construction of an invariant object made of gluon fields leads to the introduction of the nonabelian stress tensor

$$\mathcal{G}^a_{\mu\nu} = \partial_\mu\mathcal{A}^a_\nu - \partial_\nu\mathcal{A}^a_\mu + gf^{abc}\mathcal{A}^b_\mu\mathcal{A}^c_\nu \tag{1.2.7}$$

with the transformation property

$$\mathcal{G}_{\mu\nu} \to U(x)\mathcal{G}_{\mu\nu}U^\dagger(x) \tag{1.2.8}$$

The only nontrivial scalar (dim ≤ 4), which can be made from given objects, is the contraction of two stress tensors. The complete QCD Lagrangian is then

$$\mathcal{L}_{QCD} = \sum_f \overline{q}_f(i\gamma^\mu D_\mu - m_f)q_f - \frac{1}{4}\sum_{a=1}^{8}\mathcal{G}^a_{\mu\nu}\mathcal{G}^{a,\mu\nu}. \tag{1.2.9}$$

In contradiction to the abelian case of QED, the nonabelian gluon Lagrangian involves not only kinetic term but also the self-interaction vertices with three and four gluons. Moreover, the invariance under $SU(3)_C$ allows us to add one another term, the so called θ-term,

$$\mathcal{L}_\theta = \frac{g^2\overline{\theta}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{a=1}^{8}\mathcal{G}^a_{\mu\nu}G^a_{\rho\sigma} \tag{1.2.10}$$

This term implies explicit P and CP violation of strong interactions and it is the origin of the nonzero electric dipole moment of the neutron. Regardless, due to empirical information θ term is small and is often omitted.

Global symmetries of QCD Lagrangian

Let us now concentrate on the flavor sector. There are six quark flavors in the spectrum, which are often divided into two parts - light quarks u, d, s and heavy quarks c, b, t. For quark masses we have the relation

$$m_u, m_d, m_s \ll 1\,\mathrm{GeV} < m_c, m_b, m_t \tag{1.2.11}$$

where the scale $1\,\mathrm{GeV}$ called Λ_H (hadron scale) is the natural value which is associated with the masses of hadrons containing the lightest quarks, therefore, in the low-energy region only light quarks can be taken into account. The approximation with massless quarks is called *chiral limit*. In this limit the massless QCD Lagrangian

$$\mathcal{L}^0_{QCD} = \sum_{f=u,d,s} \overline{q}_f i\gamma^\mu D_\mu q_f - \frac{1}{4}\sum_{a=1}^{8} \mathcal{G}^a_{\mu\nu} G^{a,\mu\nu} \tag{1.2.12}$$

is invariant not only under $SU(3)_C$ group but also possesses $U(3)$ flavor symmetry. Now we can introduce the projection operators P_L and P_R[1]

$$P_L = \frac{1}{2}(1+\gamma_5), \qquad P_R = \frac{1}{2}(1-\gamma_5) \tag{1.2.13}$$

with the expected properties

$$P_R^2 = P_R, \qquad P_L^2 = P_L, \qquad P_R P_L = P_L P_R = 0, \qquad P_L + P_R = 1. \tag{1.2.14}$$

Acting by these operators on the quark field we get its left-handed and right-handed chiral components

$$q_R = P_R q, \qquad q_L = P_L q \tag{1.2.15}$$

with the relation $q = q_L + q_R$. The properties of gamma matrices allow us to rewrite the massless QCD Lagrangian in terms of chiral components q_R, q_L

$$\mathcal{L}^0_{QCD} = \sum_{f=u,d,s} \left(\overline{q}_{R,f} i\gamma^\mu D_\mu q_{R,f} + \overline{q}_{L,f} i\gamma^\mu D_\mu q_{L,f}\right) - \frac{1}{4}\sum_{a=1}^{8} \mathcal{G}^a_{\mu\nu} G^{a,\mu\nu} \tag{1.2.16}$$

It is easy to see that this Lagrangian is invariant not only under the global flavor $U(3)$ transformation of quark fields q, but also under the independent transformation of their chiral components q_R and q_L.

$$q_L \to U_L q_L, \qquad q_R \to U_R q_R \tag{1.2.17}$$

with 3×3 unitary matrices U_L and U_R. $\mathcal{L}^0_{QCD}$ is said to have the classical $U(3)_L \times U(3)_R$ symmetry. The element of the group $U(3)$ can be divided into $SU(3)$ component and the phase $U(1)$ part. According to Noerther's theorem there are 18 conserved currents associated with the transformations of left-handed and right-handed quarks. The octets of $SU(3)$ currents are then

$$L^{a,\mu} = \overline{q}_L \gamma^\mu \frac{\lambda^a}{2} q_L, \qquad R^{a,\mu} = \overline{q}_R \gamma^\mu \frac{\lambda^a}{2} q_R \tag{1.2.18}$$

[1] The indices L and R correspond to left and right.

where λ^a are Gellmann matrices[2] and $\lambda^0 = \sqrt{2/3}\mathbf{1}$. The conservation laws corresponding to these currents are $\partial_\mu L^{a,\mu} = \partial_\mu R^{a,\mu} = 0$. Instead of these chiral currents it is suitable to use their linear combinations,

$$V^{a,\mu} = R^{a,\mu} + L^{a,\mu} = \overline{q}\gamma^\mu \frac{\lambda^a}{2} q, \tag{1.2.19}$$

$$A^{a,\mu} = R^{a,\mu} - L^{a,\mu} = \overline{q}\gamma^\mu \gamma_5 \frac{\lambda^a}{2} q \tag{1.2.20}$$

where $a = 1, \dots 8$ which transform as vector and axial vector under parity transformations

$$V^{a\mu}(\mathbf{x}, t) \to V_\mu^a(-\mathbf{x}, t), \qquad A^{a\mu}(\mathbf{x}, t) \to -A_\mu^a(-\mathbf{x}, t). \tag{1.2.21}$$

and the singlet $U(1)$ currents ($a = 0$) can also be associated with the vector and axial vector currents

$$V^\mu = \overline{q}\gamma^\mu q, \qquad A^\mu = \overline{q}\gamma^\mu\gamma_5 q \tag{1.2.22}$$

Both currents are conserved on the classical level but after the quantization the axial current is not conserved anymore. The symmetry is not preserved due to the anomaly,

$$\partial_\mu A^\mu = \frac{3g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \sum_{a=1}^{8} \mathcal{G}^{a,\mu\nu} G^{a,\rho\sigma} \tag{1.2.23}$$

Consequently, on the quantum level the Lagrangian $\mathcal{L}^0_{QCD}$ is invariant under the chiral group $SU(3)_L \times SU(3)_R \times U(1)_V$.

In addition to the vector and axial vector currents it is convenient to define scalar and pseudoscalar densities of the form

$$S^a = \overline{q}\frac{\lambda^a}{2} q, \qquad P^a = i\overline{q}\gamma_5 \frac{\lambda^a}{2} q \tag{1.2.24}$$

where the octet $a = 1, \dots 8$ forms the $SU(3)$ part and for index $a = 0$ we have the $U(1)$ part which is useful to write separately as

$$S = \overline{q}q, \qquad P = i\overline{q}\gamma_5 q \tag{1.2.25}$$

The parity transformation for these densities reads

$$S_a(\mathbf{x}, t) \to S_a(-\mathbf{x}, t), \qquad P_a(\mathbf{x}, t) \to -P_a(-\mathbf{x}, t) \tag{1.2.26}$$

and same for singlets $S(x)$ and $P(x)$.

Chiral Ward identities

The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields. The Green functions are connected through very important relations - Ward identities that reflect the symmetry properties

[2] They are described detailed in appendix A

of a given theory on the quantum level. Their knowledge helps us to determine the structure of Green functions and their important features.

The correlator of chiral currents and densities is defined as

$$G(x_1, x_2, \dots, x_n) = \langle 0|T[A_1(x_1)A_2(x_2)\dots A_n(x_n)|0\rangle \tag{1.2.27}$$

with $A_i = V, A, S, P$ where the Lorentz and group indices were suppressed. The *chiral Green functions* are then the time, ordered vacuum expectation values of the currents and densities where at least one factor of $V^{a,\mu}$ or $A^{a,\mu}$ is present. The divergences of chiral Green functions correspond to the linear combinations of other Green functions. These relations we call *chiral Ward identities*, explicitly

$$\begin{aligned}\partial^x_\mu \langle 0|T[J^\mu(x)A_1(x_1)\dots A_n(x_n)]|0\rangle &= \langle 0|T[(\partial^x_\mu J^\mu(x))A_1(x_1)\dots A_n(x_n)]0\rangle \\ &+ \sum_{i=1}^{n}\delta(x^0 - x^i)\langle 0|T[A_1(x_1)\dots[J_0(x), A_i(x_i)]\dots A_n(x_n)]|0\rangle\end{aligned} \tag{1.2.28}$$

where $J^\mu(x)$ stands for any of the Noether currents and $A_i(x_i)$ are arbitrary chiral currents or densities (again the indices are suppressed). For evaluating the concrete chiral Ward identity we have to know the equal-time commutation relations among V, A, S and P. Omitting the Schwinger terms we can write

$$[V^a_0(\mathbf{x},t), V^b_\mu(\mathbf{y},t)] = [A^a_0(\mathbf{x},t), A^b_\mu(\mathbf{y},t)] = \delta^3(\mathbf{x}-\mathbf{y})if^{abc}V^c_\mu(\mathbf{x},t), \tag{1.2.29}$$

$$[V^a_0(\mathbf{x},t), A^b_\mu(\mathbf{y},t)] = [A^a_0(\mathbf{x},t), V^b_\mu(\mathbf{y},t)] = \delta^3(\mathbf{x}-\mathbf{y})if^{abc}A^c_\mu(\mathbf{x},t), \tag{1.2.30}$$

$$[V^a_0(\mathbf{x},t), S^b(\mathbf{y},t)] = \delta^3(\mathbf{x}-\mathbf{y})if^{abc}S^c(\mathbf{x},t), \tag{1.2.31}$$

$$[V^a_0(\mathbf{x},t), P^b(\mathbf{y},t)] = \delta^3(\mathbf{x}-\mathbf{y})if^{abc}P^c(\mathbf{x},t), \tag{1.2.32}$$

$$[A^a_0(\mathbf{x},t), P^b(\mathbf{y},t)] = -\delta^3(\mathbf{x}-\mathbf{y})i\left(d^{abc}S^c(\mathbf{x},t) + \frac{2}{3}\delta^{ab}S(\mathbf{x},t)\right), \tag{1.2.33}$$

$$[A^a_0(\mathbf{x},t), S^b(\mathbf{y},t)] = \delta^3(\mathbf{x}-\mathbf{y})i\left(d^{abc}P^c(\mathbf{x},t) + \frac{2}{3}\delta^{ab}P(\mathbf{x},t)\right), \tag{1.2.34}$$

$$[V^a_0(\mathbf{x},t), V^\mu(\mathbf{y},t)] = [V^a_0(\mathbf{x},t), S(\mathbf{y},t)], \tag{1.2.35}$$

$$[V^a_0(\mathbf{x},t), P(\mathbf{y},t)] = [A^a_0(\mathbf{x},t), V^\mu(\mathbf{y},t)] = 0, \tag{1.2.36}$$

$$[A^a_0(\mathbf{x},t), S(\mathbf{y},t)] = \delta^3(\mathbf{x}-\mathbf{y})iP^a(\mathbf{x},t), \tag{1.2.37}$$

$$[A^a_0(\mathbf{x},t), P(\mathbf{y},t)] = -\delta^3(\mathbf{x}-\mathbf{y})iS^a(\mathbf{x},t) \tag{1.2.38}$$

In χPT $P^a(\mathbf{x},t)$ and $A^a(\mathbf{x},t)$ are interpolating fields for Goldstone bosons, $V^{a\mu}(\mathbf{x},t)$ correspond to electroweak currents and $S^a(\mathbf{x},t)$ is related to the quark mass matrix.

Generating functional

It is useful to introduce the generating functional of currents and densities in QCD. Varying it with respect to the external sources one obtains all chiral Green functions. To construct the generating functional we have to couple the nine vector currents and eight axial-vector currents as well as the scalar and pseudoscalar quark densities to the external c-number fields $v^\mu(x)$, $v^\mu_{(s)}(x)$, $a^\mu(x)$, $s(x)$ and $p(x)$,

$$\mathcal{L} = \mathcal{L}^0_{QCD} + \mathcal{L}_{ext} = \mathcal{L}^0_{QCD} + \overline{q}\gamma_\mu(v^\mu + \frac{1}{3}v^\mu_{(s)} + \gamma_5 a^\mu)q - \overline{q}(s - i\gamma_5 p)q \tag{1.2.39}$$

The external fields are color-neutral, they transform as the singlets under color $SU(3)_C$ group. In the flavor sector they are represented by Hermitian 3×3 matrices, where the matrix character is

$$v^\mu = \sum_{a=1}^{8} \frac{\lambda^a}{2} v^\mu_a, \quad a^\mu = \sum_{a=1}^{8} \frac{\lambda^a}{2} a^\mu_a, \quad s = \sum_{a=0}^{8} \frac{\lambda_a}{2} s_a, \quad p = \sum_{a=0}^{8} \frac{\lambda_a}{2} p_a \tag{1.2.40}$$

The ordinary three flavor QCD Lagrangian is recovered by setting $v^\mu = v^\mu_{(s)} = a^\mu = p = 0$ and $s = \mathcal{M}$ in (1.2.39). The generating functional is defined as

$$\exp\left(iZ[v,a,s,p]\right) = \langle 0|T \exp\left[i\int d^4x\, \mathcal{L}_{ext}(x)\right]|0\rangle. \tag{1.2.41}$$

The n-point Green functions can be obtained by variation with respect to corresponding external sources. For example,

$$\langle 0|T[V^\mu_a(x)V^b_\nu(0)|0\rangle = (-i)^2 \frac{\delta^2}{\delta v^{a,\mu}(x)\,\delta v^{b,\nu}(0)} \exp\left(iZ[v,a,s,p]\right)\Big|_{v=a=p=0,\, s=\mathcal{M}} \tag{1.2.42}$$

More tricky task is to derive the correlator of quark fields. They appear non-linearly in the currents and densities, for $\langle \overline{u}u\rangle$ we have

$$\langle 0|\overline{u}(x)u(x)|0\rangle_0 = \frac{i}{2}\left[\sqrt{\frac{2}{3}}\frac{\delta}{\delta s_0(x)} + \frac{\delta}{\delta s_3(x)} + \frac{1}{\sqrt{3}}\frac{\delta}{\delta s_8(x)}\right] \exp\left(iZ[v,a,s,p]\right)\Big|_{v=a=p=s=0} \tag{1.2.43}$$

The Lagrangian (1.2.39) can be written in terms of left-handed and right-handed quark fields q_L, q_R. Defining the vector and axial-vector currents

$$v^\mu = \frac{1}{2}(r^\mu + l^\mu), \qquad a^\mu = \frac{1}{2}(r^\mu - l^\mu). \tag{1.2.44}$$

we obtain

$$\begin{aligned}\mathcal{L} = \mathcal{L}^0_{QCD} + \overline{q}_L\gamma^\mu\left(l_\mu + \frac{1}{3}v^{(s)}\right)q_L + \overline{q}_R\gamma^\mu\left(r_\mu + \frac{1}{3}v^{(s)}_\mu\right)q_R \\ - \overline{q}_R(s+ip)q_L - \overline{q}_L(s-ip)q_R\end{aligned} \tag{1.2.45}$$

This Lagrangian is manifestly invariant under the local $SU(3)_L \times SU(3)_R \times U(1)_V$ group with

$$q_R \quad \to \quad \exp\left(-i\frac{\Theta(x)}{3}\right)V_R(x)q_R, \tag{1.2.46}$$

$$q_L \quad \to \quad \exp\left(-i\frac{\Theta(x)}{3}\right)V_L(x)q_L \tag{1.2.47}$$

where $V_R(x)$ and $V_L(x)$ are $SU(3)$ matrices. The following transformation properties of the external sources are

$$r_\mu \rightarrow V_R r_\mu V_R^\dagger + iV_R \partial_\mu V_R^\dagger, \tag{1.2.48}$$
$$l_\mu \rightarrow V_L l_\mu V_L^\dagger + iV_L \partial_\mu V_L^\dagger, \tag{1.2.49}$$
$$v_\mu^{(s)} \rightarrow v_\mu^{(s)} - \partial_\mu \Theta, \tag{1.2.50}$$
$$s + ip \rightarrow V_R(s + ip_) V_L^\dagger, \tag{1.2.51}$$
$$s - ip \rightarrow V_L(s - ip) V_R^\dagger. \tag{1.2.52}$$

The part of Lagrangian $\mathcal{L}_{ext}$ represents the interaction of quarks with the external fields. For example, we can restore the electroweak interaction Lagrangian by setting r_μ and l_μ dependent on the gauge fields $\mathcal{Z}_\mu$, $\mathcal{W}_\mu^\pm$. The result is then the usual electroweak quark Lagrangian.

Although the Lagrangian (1.2.39) is invariant under the local transformations (1.2.48)-(1.2.52), it is no longer true for the generating functional $Z[v, a, s, p]$. The anomalies of fermionic determinant leads to the breaking of the chiral symmetry at the quantum level. If we assume the infinitesimal chiral transformations

$$V_L(x) = 1 + i\alpha(x) - i\beta(x), \qquad V_R(x) = 1 + i\alpha(x) + i\beta(x), \tag{1.2.53}$$

the change of the generating functional under (1.2.48)-(1.2.52) is given by

$$\delta Z[v, a, s, p] = -\frac{N_C}{16\pi^2} \int d^4x \langle \beta(x) \Omega(x) \rangle \tag{1.2.54}$$

where

$$\Omega(x) = \epsilon^{\mu\nu\rho\sigma} \left\{ v_{\mu\nu} v_{\rho\sigma} + \frac{4}{3} \nabla_\mu a_\nu \nabla_\sigma a_\rho + \frac{2}{3} i \{v_{\mu\nu}, a_\sigma a_\rho\} + \frac{8}{3} i a_\sigma v_{\mu\nu} a_\rho + \frac{4}{3} a_\mu a_\nu a_\sigma a_\rho \right\} \tag{1.2.55}$$

with

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu], \qquad \nabla_\mu v_\nu = \partial_\mu a_\nu - i[v_\mu, a_\nu]. \tag{1.2.56}$$

This anomalous variation of Z is an $\mathcal{O}(p^4)$ effect, in chiral power counting. The source for this change of functional was found by Wess and Zumino [15] and reformulated in a geometrical way by Witten [17].

Explicit symmetry breaking

So far we have not considered the quark masses. If we take them into account the flavor symmetry is explicitly broken due to the presence of the mass term in the Lagrangian. So let us consider the quark-mass matrix of the three light quarks

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{1.2.57}$$

The mass term then mixes between left-handed and right-handed quarks

$$\mathcal{L}_M = -\overline{q}\mathcal{M}q = -(\overline{q}_R\mathcal{M}q_L + \overline{q}_L\mathcal{M}q_R). \tag{1.2.58}$$

The divergences of the constructed currents are

$$\partial_\mu V^{a,\mu} = i\overline{q}\left[\mathcal{M}, \frac{\lambda^a}{2}\right] q, \tag{1.2.59}$$

$$\partial_\mu A^{a,\mu} = i\overline{q}\left\{\mathcal{M}, \frac{\lambda^a}{2}\right\} q, \tag{1.2.60}$$

$$\partial_\mu V^{\mu} = 0, \tag{1.2.61}$$

$$\partial_\mu A^{\mu} = 2i\overline{q}\mathcal{M}\gamma_5 q + \frac{3g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathcal{G}^{a,\mu\nu}\mathcal{G}^{a,\rho\sigma}. \tag{1.2.62}$$

Let us analyze the results according to the form of the quark-mass matrix. For the general values of m_u, m_d and m_s we have no flavor symmetry (except for $U(1)_V$ which is present all the time and represents the conservation of baryon number). In the special cases are

1. $m_u = m_d = m_s = 0$ - The octet vector and axial vector currents are conserved. The symmetry group is $SU(3)_L \times SU(3)_R \times U(1)_V$.

2. $m_u = m_d = m_s \neq 0$ - Vector current is conserved and the Lagrangian is invariant under $SU(3)_V \times U(1)_V$.

3. $m_u = m_d = 0$ - The model with two massless quarks implies the $SU(2)_L \times SU(2)_R \times U(1)_{SV} \times U(1)_V$ invariance where $U(1)_{SV}$ symmetry stands for the conservation of the strangeness.

4. $m_u = m_d \neq 0$ - The chiral limit of lightest quarks indicates the symmetry $SU(2)_V \times U(1)_{SV} \times U(1)_V$.

Spontenous Symmetry breaking in QCD

As it is well known, the symmetry group of the massless QCD $SU(3)_L \times SU(3)_R \times U(1)_V$ is spontaneously broken to $SU(3)_V \times U(1)_V$ due to the presence of an order parameter in QCD. According to the Goldstone theorem, to each generator, which does not anihilate the vacuum state, there corresponds one massless Goldstone boson. Therefore, an octet of these particles appears in the spectrum of QCD.

Our goal is now to find the order parameter for QCD which is responsible for the spontaneous symmetry breaking. The generators of $SU(3)_V$ symmetry are defined as

$$Q_V^a(t) = \int d^3x V_0^a(x,t) = \int q^\dagger(x,t)\frac{\lambda^a}{2}q(x,t). \tag{1.2.63}$$

The equal time commutation relations with the $SU(3)_V$ octet of scalar densities are

$$[Q_V^a(t), S^b(y)] = if^{abc}S^c(y). \tag{1.2.64}$$

We can reverse this relation,

$$S^a(y) = -\frac{i}{3} f^{abc}[Q_V^b(t), S^c(y)] \tag{1.2.65}$$

The vacuum is invariant under $SU(3)_V$, so we can write

$$\langle 0|S^a(y)|0\rangle = 0. \tag{1.2.66}$$

Taking $a = 3$ and $a = 8$ we get

$$\langle \overline{u}u\rangle - \langle \overline{d}d\rangle = 0, \quad \langle \overline{u}u\rangle + \langle \overline{d}d\rangle - 2\langle \overline{s}s\rangle = 0 \quad \Rightarrow \quad \langle \overline{u}u\rangle = \langle \overline{d}d\rangle = \langle \overline{s}s\rangle. \tag{1.2.67}$$

Now assuming the non-vanishing singlet scalar density and using previous results we find

$$\langle 0|S|0\rangle = \langle \overline{q}q\rangle = 3\langle \overline{u}u\rangle \neq 0 \tag{1.2.68}$$

For the equal-time commutation relation

$$i[Q_A^a(t), P^a(y)] = d^{aac} S^c(x,t) + \frac{2}{3} S(x,t) \tag{1.2.69}$$

we calculate the vacuum expectation value

$$\langle 0|i[Q_A^a(t), P^a(y)]|0\rangle = \frac{2}{3}\langle \overline{q}q\rangle \neq 0 \tag{1.2.70}$$

So we have found the order parameter for QCD, $\delta\mathcal{O} = \langle \overline{q}q\rangle$. Consequently, the octet of Goldstone bosons $\phi^a(x)$ appear in the spectrum. Moreover, Goldstone theorem and Lorentz covariance permit us to write

$$\langle 0|A_\mu^a(0)|\phi^b(p)\rangle = ip_\mu F_0 \delta^{ab} \tag{1.2.71}$$

where F_0 denotes the decay constant. Because $\mathcal{O} = P^a$ the Goldstone bosons have the quantum numbers of pseudoscalar particles.

1.3 Chiral perturbation theory

The effective theory is the way how to construct the general S matrix for low energy degrees of freedom that satisfies all necessary conditions (analycity, unitarity, crossing symmetry). Moreover, the effective theories are based only on the symmetry properties of the fundamental theory when all other aspects are forgotten. Finally, the particle contents of such a theory should agree with the real physical spectrum.

We have seen in the last chapter that the spontaneous symmetry breaking in QCD leads, according to Goldstone theorem, to the presence of Goldstone bosons. Identifying them with the octet of psedoscalar mesons, which are the lightest particles in hadronic spectrum, we can construct the low energy effective theory for QCD called Chiral Perturbation Theory (χPT).

Pseudoscalar mesons

In the chiral limit (when all quark masses are set to zero), the Lagrangian of χPT must be invariant under the symmetry group of massless QCD - $G = SU(3)_L \times SU(3)_R \times U(1)_V$. The eight pseudoscalar mesons then transform as an octet under the subgroup $H = SU(3)_V$. Let us now define the essential building block of χPT

$$u(\phi) = \exp\left(i\frac{\phi}{\sqrt{2}F_0}\right) \tag{1.3.1}$$

where $\phi = \phi^a T^a$ with $T^a = \lambda^a/\sqrt{2}$ and

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\overline{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} \tag{1.3.2}$$

is the matrix describing the pseudoscalar mesons fields. The Goldstone bosons are parametrized by the elements $u(\phi)$ of the coset space $SU(3)_L \times SU(3)_R/SU(3)_V$, transforming as

$$u(\phi) \mapsto V_R u(\phi) h(g,\phi)^{-1} = h(g,\phi) u(\phi) V_R \tag{1.3.3}$$

under a general chiral rotation $g = (V_L, V_R) \subset G$ in terms of the $SU(3)_V$ compensator field $h(g,\phi)$.

It is also useful to introduce the classical sources s, p, v^μ and a^μ (with transformation properties (1.2.48)-(1.2.52)) that couple on the scalar density S^a, pseudoscalar density P^a, vector currents $V^{a\mu}$ and axial currents $A^{a\mu}$. These are the interpolating fields for the external particles entering the process, coupled to quark mass matrix and so on. For instance the process $\pi^0 \to 2\gamma$ corresponds to 3-point Green function composed from two vector currents and one pseudoscalar density $\langle VVP\rangle$.

Construction of Lagrangian

As in all effective theories also in χPT the Lagrangian can be expanded in powers of small physical quantity. Here it is the external momenta p which should be much smaller than an energy scale $\Lambda \approx 1\,\mathrm{GeV}$. It is related to the typical (nongoldstone) hadron masses. Another small quantities are the quark masses (in quark mass matrix) and correspond to the second order in momenta[3], $\mathcal{M} \sim \mathcal{O}(p^2)$.

Expansion of the Lagrangian in terms of p has the following form (according to the symmetry conditions only even terms can contribute)

$$\mathcal{L}_\chi = \mathcal{L}_\chi^{(2)} + \mathcal{L}_\chi^{(4)} + \mathcal{L}_\chi^{(6)} + ... \tag{1.3.4}$$

[3] There is another approach, based on an assumption $\mathcal{M} \sim \mathcal{O}(p)$, which is called Generalized Chiral Perturbation Theory.

where $\mathcal{L}^{(n)}$ stands for a part of the Lagrangian which is of the n-th order in p, ie. $\mathcal{L}^{(n)} = \mathcal{O}(p^n)$. The Lagrangian must possess the same symmetry as the underlying theory, i.e. the local $SU(3)_L \times SU(3)_R$ symmetry. The lowest order Lagrangian reads

$$\mathcal{L}_\chi^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[u_\mu u^\mu + \chi_+] \tag{1.3.5}$$

where

$$u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger], \tag{1.3.6}$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \quad \chi = 2B_0(s+ip) \tag{1.3.7}$$

are the chiral building blocks. The left and right sources l_μ,r_μ are related to the vector and axial vector sources as

$$v_\mu = \frac{1}{2}(l_\mu + r_\mu), \qquad a_\mu = \frac{1}{2}(l_\mu - r_\mu). \tag{1.3.8}$$

There exist more chiral building blocks in higher order Lagrangians. For our next calculation we need

$$f_\pm^{\mu\nu} = uf_L^{\mu\nu}u^\dagger \pm u^\dagger f_R^{\mu\nu}u \tag{1.3.9}$$

where

$$f_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], \tag{1.3.10}$$

$$f_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]. \tag{1.3.11}$$

Moreover, we can define the covariant derivative of a field X respecting the symmetry properties

$$D_\mu X = \partial_\mu X + [\Gamma_\mu, X] \tag{1.3.12}$$

with

$$\Gamma_\mu = \frac{1}{2}\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\}. \tag{1.3.13}$$

We see that the second order Lagrangian contains only two unknown constants F_0 and B_0 (in chiral limit). But it is not true for higher orders. In the next-to-leading order (order $\mathcal{O}(p^4)$)[2, 3] the Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\chi PT}^{(4)} =& L_1\langle u_\mu u^\mu\rangle^2 + L_2\langle u_\mu u^\nu\rangle\langle u^\mu u_\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle + L_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle \\ &+ L_5\langle u_\mu u^\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 + L_8/2\langle\chi_-^2 + \chi_+^2\rangle \\ &- iL_9\langle f_+^{\mu\nu}u_\mu u_\nu\rangle + L_{10}/4\langle f_{+\mu\nu}f_+^{\mu\nu} - f_{-\mu\nu}f_-^{\mu\nu}\rangle \\ &+ iL_{11}\langle\chi_-(D_\mu u^\mu + i/2\chi_-)\rangle - L_{12}\langle(D_\mu u^\mu + i/2\chi_-)^2\rangle \\ &+ H_1/2\langle f_{+\mu\nu}f_+^{\mu\nu} + f_{-\mu\nu}f_-^{\mu\nu}\rangle + H_2/4\langle\chi_+^2 - \chi_-^2\rangle\end{aligned} \tag{1.3.14}$$

The number of coupling constants grows rapidly, $\mathcal{L}_{\chi,6}$ has already about 100 constants [20].

Generating functional

The generating functional for χPT is defined as

$$Z_{\chi PT}[s,p,v,a] = \int \mathcal{D}u \exp\left\{ i \int d^4x \mathcal{L}_\chi \right\} \tag{1.3.15}$$

Because χPT is the effective theory of QCD we must demand the equality of corresponding generating functionals after expanding in terms of small momenta

$$\begin{aligned} Z_{\chi PT}[s,p,v,a] &= Z_{QCD}[s,p,v,a] \\ &= \int \mathcal{D}q\mathcal{D}\overline{q}\mathcal{D}G \exp\left\{ i \int d^4x \left[\mathcal{L}_{QCD} + \overline{q}\gamma_\mu(v^\mu + \gamma_5 a^\mu)q - \overline{q}(s - i\gamma_5 p)q \right] \right\} \end{aligned} \tag{1.3.16}$$

Unfortunately, we don't know this functional from the first principles, so the constants in χPT Lagrangian cannot be computed directly from QCD.

Weinberg power counting formula

Weinberg power counting scheme describes a behavior of Feynman diagram under a linear rescaling of an external momenta, $p \mapsto \lambda p$. [4] If we define the chiral dimension D of a given diagram, the amplitude of such a diagram satisfies

$$\mathcal{M}(\lambda p, \lambda^2 m_q) = \lambda^D \mathcal{M}(p, m_q). \tag{1.3.17}$$

Power counting formula gives the expression for the chiral dimension

$$D = 2 + 2L + \sum_{n=0}^{\infty} (2n-2) N_{2n} \tag{1.3.18}$$

where L is number of loops and N_{2n} denotes the number of vertices from $\mathcal{L}_{2n}$. Because the number of possible counterterms with the chiral dimension $D \leq D_{max}$ is finite, the theory is then renormalizable if we take into account only diagrams up to a given chiral order [1].

[4] In the same way, we rescale quadratically the masses of light quarks, $m_q \mapsto \lambda^2 m_q$. Therefore, the masses of Goldstone bosons (outside the chiral limit) are rescaled $M^2 \mapsto \lambda^2 M^2$.

CHAPTER 2

Resonance chiral theory

We have seen that χPT describing pseudoscalar mesons as the only degrees of freedom can be used as an effective theory for QCD at low energies. If we go to energies $E \geq 1\,\text{GeV}$ χPT looses its convergence and cannot be used anymore because the higher mass states become active in dynamics of hadrons. We use the tool of effective theory to describe these degrees of freedom (resonances) using phenomenological Lagrangians based on symmetries of QCD.

It was shown [16] that QCD in the limit of infinite number of colors can be formulated as a perturbative expansion in $1/N_C$. Its spectrum contains the infinite tower of resonances [17] and provides us with an exact theory for resonances based on QCD.

In the intermediate energy region $1\,\text{GeV} \leq E \leq 2\,\text{GeV}$ it is justified to take into account only one type of resonance in each channel. The final theory based partially on χPT and large N_C QCD (we do the matching at low and high energies), Resonance Chiral Theory (RχT), is the topic of this chapter. For simplification, after general discussion we restrict ourselves to one type of resonances - vector resonances 1^{--} which are the most interesting in the spectrum of resonances. The results in complete RχT are longer but principally similar to our results.

2.1 Phenomenological Lagrangians

In contrast to χPT where the expansion parameter is the external momentum p, in the Resonance Chiral Theory the standard chiral power counting breaks because the momenta and the masses of resonances are not neglected in comparison with the typical scale in the intermediate energy region. This reason together with the absence of an energy gap in the spectrum of hadrons make difficult to build the resonance theory as an effective theory of resonances for QCD. Fortunately, the short distance constraints, OPE results and large N_C behavior can help us with construction of RχT Lagrangian.

$1/N_C$ expansion

The basic proposal is to investigate the properties of QCD in the large N_C limit when the symmetry group is enlarged from $SU(3)_C$ to $SU(N_C)$. Despite this seems to be quite strange because the case $N_C = 3$ is far from the infinite value $N_C \to \infty$, this generalization of QCD suggested by Gerard t'Hooft has many simple properties that are partially shared by real QCD. Witten showed [17] that the Green functions calculated in large N_C QCD in the leading order contain the exchange of infinite tower of resonances.

Taking g_s to be of order $\mathcal{O}(1/\sqrt{N_C})$ and letting $N_C \to \infty$ while $\alpha_s N_C$ fixed we obtain important results

- Mesons are free, stable and non-interacting and the number of meson states is infinite.
- Elasting scattering amplitudes are of order $\mathcal{O}(N^{1-k/2})$ where k is the number of mesons in the process.
- The dynamics of mesons in the leading order in $1/N_C$ is dominated by the tree level diagrams, the loops are of higher orders in $1/N_C$.
- The flavor group of the theory is $U(N_f)_L \times U(N_f)_R$ because there is no axial anomaly in large N_C limit. This symmetry is spontaneously broken to $U(N_f)_V$.

Despite the expansion in $1/N_C$ has a beautiful theoretical sense based directly on the properties of the underlying theory (large N_C QCD), its use in hierarchy of Lagrangian terms in RχT is problematic. In practice, power counting in $1/N_C$ is an expansion in number of mesons and the terms with many derivatives are not suppressed.

Relation between χPT and RχT

As was said, in $N_C \to \infty$ we can construct the effective Lagrangian (RχT) for QCD for intermediate energy region that satisfies all symmetry properties dictated by the underlying theory. Unfortunately, the Lagrangian of this theory is not known from first principles. However, its coupling constants can be related to the phenomenology of the resonance sector. Up to the order $O(p^6)$ it has the general form

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{GB} + \mathcal{L}_{res} = \mathcal{L}_{GB}^{(2)} + \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{res}^{(4)} + \mathcal{L}_{res}^{(6)} \tag{2.1.1}$$

where $\mathcal{L}_{GB} = \mathcal{L}_{GB}^{(2)} + \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{GB}^{(6)}$ contains only the (pseudo)Goldstone bosons and $\mathcal{L}_{GB}^{(2n)}$ has the same form as the $O(p^{2n})$ χPT Lagrangian $\mathcal{L}_{\chi}^{(2n)}$. The corresponding LECs are, however, different. Actually, in concrete resonance saturation calculation, these LECs are treated as negligible at the resonance scale. $\mathcal{L}_{res}^{(4)}$ and $\mathcal{L}_{res}^{(6)}$ are the resonance Lagrangians of the chiral order

$O(p^4)$ and $O(p^6)$ respectively[1]. Integrating out the resonance fields from the RχT Lagrangian and expanding to the given chiral order yields an effective χPT Lagrangian $\mathcal{L}_\chi$. The LECs in this Lagrangian are now expressed in terms of the resonance parameters and the LECs from $\mathcal{L}_{GB}$. Schematically, up to the order $O(p^6)$

$$\mathcal{L}_{\chi PT} = \mathcal{L}_{GB} + \mathcal{L}_{\chi,eff} = \mathcal{L}_{GB}^{(2)} + \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{\chi,eff}^{(4)} + \mathcal{L}_{\chi,eff}^{(6)}. \tag{2.1.2}$$

So, we can write

$$\mathcal{L}_{\chi PT}^{(2)} = \mathcal{L}_{GB}^{(2)}, \tag{2.1.3}$$
$$\mathcal{L}_{\chi PT}^{(4)} = \mathcal{L}_{GB}^{(4)} + \mathcal{L}_{\chi,eff}^{(4)}, \tag{2.1.4}$$
$$\mathcal{L}_{\chi PT}^{(6)} = \mathcal{L}_{GB}^{(6)} + \mathcal{L}_{\chi,eff}^{(6)}. \tag{2.1.5}$$

Here $\mathcal{L}_{\chi,res}^{(2n)}$ has the same form as $\mathcal{L}_{GB}^{(2n)}$ with LECs depending on the resonance masses and couplings of $\mathcal{L}_{res}$.

It was shown in [6] that the hypothesis of the successful saturation of $\mathcal{O}(p^4)$ LECs by the finite number of resonances is legitimate. Since then this idea has been often used in particular cases in order to estimate also the contribution of the $O(p^6)$ LECs to various quantities calculated within the $O(p^6)$ χPT. Quite recently, the first steps towards a systematic and consistent estimate of the $O(p^6)$ LECs via resonance saturation have been made in [12, 8] and confirms the validity of RχT results.

2.2 Spin one particles

In this short section we describe the basic properties of two essential ways how to describe massive spin one particles in the framework of quantum field theory. We can either use the formalism of vector fields or antisymmetric tensor fields. In the first case we can write the free field Lagrangian in the form

$$\mathcal{L}_V = -\frac{1}{4}\hat{V}_{\mu\nu}\hat{V}^{\mu\nu} + \frac{1}{2}M^2 V_\mu V^\mu, \tag{2.2.1}$$

where $\hat{V}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. Classical equation of motion gives

$$\partial^2 V_\mu + M^2 V_\mu - \partial_\mu(\partial \cdot V) = 0. \tag{2.2.2}$$

Taking the divergence we get

$$\partial \cdot V = 0 \tag{2.2.3}$$

and hence

$$\left(\partial^2 + M^2\right) V_\mu = 0. \tag{2.2.4}$$

[1]The chiral order of the resonance fields depend on the formalism used and will be clarified in what follows.

A real vector field satisfying these two equations can be expressed as Fourier transform

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \left\{B_\mu(\mathbf{p},\sigma)e^{ip\cdot x} + B^*_\mu(\mathbf{p},\sigma)e^{-ip\cdot x}\right\} \tag{2.2.5}$$

where $E = \sqrt{\mathbf{p}^2 + m^2}$. In the quantization procedure we substitute operators for functions, i.e. $B_\mu \to \widehat{B}_\mu$. Separating the tensor structure we can write

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \left\{\varepsilon_\mu(\mathbf{p},\sigma)a(\mathbf{p},\sigma)e^{ip\cdot x} + \varepsilon_{\mu*}(\mathbf{p},\sigma)a^\dagger(\mathbf{p},\sigma)e^{-ip\cdot x}\right\}, \tag{2.2.6}$$

where $\varepsilon^\mu(\mathbf{p},\sigma)$ are three independent polarization vectors satisfying

$$\begin{aligned}
\sum_{\sigma=-1}^{\sigma=1} \varepsilon^\mu(\mathbf{p},\sigma)\varepsilon^{\nu*}(\mathbf{p},\sigma) &= -g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}, \\
\varepsilon_\mu(\mathbf{p},\sigma)\varepsilon^\mu(\mathbf{p},\sigma') &= -\delta_{\sigma\sigma'}, \\
p_\mu\varepsilon^\mu(\mathbf{p},\sigma) &= 0.
\end{aligned}$$

and $a(\mathbf{p},\sigma)$, $a^\dagger(\mathbf{p},\sigma)$ are anihilation and creation operators that satisfy commutation relations

$$\begin{aligned}
\left[a(\mathbf{p},\sigma), a^\dagger(\mathbf{p}',\sigma')\right] &= \delta^3(\mathbf{p}'-\mathbf{p})\delta_{\sigma,\sigma'}, \\
\left[a(\mathbf{p},\sigma), a(\mathbf{p}',\sigma')\right] &= \left[a^\dagger(\mathbf{p},\sigma), a^\dagger(\mathbf{p}',\sigma')\right] = 0.
\end{aligned}$$

Then fields $V_\mu(x)$ transform in $(1/2, 1/2)$ representation of Lorentz group. The 2-point correlator of these fields (called propagators) is defined as

$$i\Delta_F^V(x-y)_{\mu\nu} \equiv \langle 0|T[V_\mu(x)V_\nu(y)]|0\rangle. \tag{2.2.7}$$

where the covariant part of the result has the form

$$i\Delta_F^V(x-y)_{\mu\nu} = \int \frac{d^4p}{(2\pi)^4} i\Delta_F(p)_{\mu\nu}e^{-ip\cdot(x-y)} \tag{2.2.8}$$

and

$$i\Delta_F^V(p)_{\mu\nu} = \frac{-i}{p^2 - m^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}\right) \tag{2.2.9}$$

is the propagator in momentum representation.

For the description of vector resonances using antisymmetric tensor formalism we use the free field Lagrangian

$$\mathcal{L}_T = -\frac{1}{2}W_\mu W^\mu + \frac{1}{4}M^2 R_{\mu\nu}R^{\mu\nu}. \tag{2.2.10}$$

where $W_\mu = \partial^\alpha R_{\alpha\mu}$. Classical equation of motion has the form

$$\partial_\mu\partial^\alpha R_{\alpha\nu} - \partial_\nu\partial^\alpha R_{\alpha\mu} + m^2 R_{\mu\nu} = 0. \tag{2.2.11}$$

Applying the derivative ∂^ν we obtain for $\partial^\alpha R_{\alpha\mu}$ (multiplied by $1/m$ because of proper dimension of the field)

$$(\partial^2 + m^2)\left(\frac{1}{m}\partial^\alpha R_{\alpha\mu}\right) = 0. \tag{2.2.12}$$

The condition of transversality is satisfied identically due to the antisymmetry of $R_{\mu\nu}$. So we obtain again Proca field equation and it is possible to write for $\partial^\alpha R_{\alpha\mu}$ the same expression as in previous case (using the same creation and anihilation operators!). Guessing the general form of the expansion for $R_{\mu\nu}$ we get[2]

$$R_{\mu\nu}(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \left\{ A_{\mu\nu}(\mathbf{p},\sigma)a(\mathbf{p},\sigma)e^{ip\cdot x} + B_{\mu\nu}(\mathbf{p},\sigma)a^\dagger(\mathbf{p},\sigma)e^{-ip\cdot x} \right\}. \tag{2.2.13}$$

Applying the derivative in the momentum space we obtain

$$\begin{aligned} ip^\mu A_{\mu\nu} &= m\varepsilon^\mu(\mathbf{p},\sigma), \\ -ip^\mu B_{\mu\nu} &= m\varepsilon^{\mu *}(\mathbf{p},\sigma). \end{aligned}$$

Easy calculation using the relation $p_\mu \varepsilon^\mu(\mathbf{p},\sigma) = 0$ gives the result

$$\begin{aligned} R_{\mu\nu}(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-1}^{\sigma=1} \frac{d^3\mathbf{p}}{\sqrt{2E}} \frac{i}{m} \Big\{ & \left(p_\nu \varepsilon_\mu(\mathbf{p},\sigma) - p_\mu \varepsilon_\nu(\mathbf{p},\sigma)\right) a(\mathbf{p},\sigma)e^{ip\cdot x} + \\ & \left(p_\nu \varepsilon^*_\mu(\mathbf{p},\sigma) - p_\mu \varepsilon^*_\nu(\mathbf{p},\sigma)\right) a^\dagger(\mathbf{p},\sigma)e^{-ip\cdot x} \Big\}. \end{aligned}$$

The covariant propagator of the field is then

$$i\Delta_F^T(x-y)_{\alpha\beta\mu\nu} \equiv \langle 0|T[R_{\alpha\beta}(x)R_{\mu\nu}(y)]|0\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} i\Delta_F(p)_{\alpha\beta\mu\nu} e^{-ip\cdot(x-y)} \tag{2.2.14}$$

where

$$\begin{aligned} & i\Delta_F^T(p)_{\alpha\beta\mu\nu} = \\ & \quad \frac{-i}{p^2 - m^2 + i\epsilon} \frac{1}{m^2} \left((m^2 - p^2) g_{\alpha\mu} g_{\beta\nu} + g_{\alpha\mu} p_\beta p_\nu - g_{\alpha\nu} p_\beta p_\mu - (\mu \leftrightarrow \nu) \right). \end{aligned} \tag{2.2.15}$$

In the following sections we propose the study of both mentioned ways of description of vector resonances in RχT up to $\mathcal{O}(p^6)$ together with study of their equivalence and the introduction of alternative formulation.

2.3 Construction of Lagrangian

The Resonance Chiral Theory enlarges the number of degrees of freedom of standard χPT (which contains only pseudo Goldstone bosons) by including also massive multiplets of resonances -

[2] Actually, this is not a guess. Antisymmetric tensor field transforms under $(1,0) + (0,1)$ representation of Lorentz group which guaranties the possibility of the expansion of the field in this form [18], [19].

vector 1^{--}, axial vector 1^{++}, scalar 0^{++} and pseudoscalar 0^{-+}. The same procedure as in χPT can be done when constructing resonance Lagrangian, only the symmetry group is now $U(3)_L \times U(3)_R$.

Let us now restrict ourselves to the octet of vector resonances 1^{--} which is the subject of our interest. The resonance field in the antisymmetric tensor formalism (the form in vector formalism is analogous and both formalisms will be discussed later) can be written as

$$R_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_3 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu} \tag{2.3.1}$$

The resonance fields transform in the nonlinear realization of the $U(3)_L \times U(3)_R$. These massive states transform as octets R_8 or singlets R_0 under $SU(3)_L \times SU(3)_R$

$$R_8 \mapsto h(g,\phi)R_8 h(g,\phi)^{-1}, \qquad R_0 \mapsto R_0. \tag{2.3.2}$$

where $R_8 = \sum_i R_i T_i$. In the large N_C limit with massless quarks (chiral limit) we can collect these states into a nonet state (with the same mass)

$$R = \sum_{i=1}^{8} T_i R_i + T_0 R_0 = \sum_{j=1}^{9} T_j R_j \tag{2.3.3}$$

where $T_0 = \sqrt{1/3}\mathbf{1}$. Moreover, the Lagrangian must be invariant under P and C transformations and hermitian self-conjugate. The resonance fields $R_{\mu\nu}$ transform under these symmetries as

$$\begin{aligned} P : R^{\mu\nu} &\mapsto R^{\mu\nu} \\ C : R^{\mu\nu} &\mapsto -R^T_{\mu\nu} \\ h.c. : R^{\mu\nu} &\mapsto R_{\mu\nu} \end{aligned}$$

Then in the leading order (the complete lists of Lagrangian terms are provided in the following sections) in $1/N_C$ (terms with one resonance) we can construct the interaction resonance Lagrangian that is invariant under all these symmetries

$$\mathcal{L}_R^{int} = \frac{F_V}{2\sqrt{2}}\langle R_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle R_{\mu\nu}[u^\mu, u^\nu]\rangle. \tag{2.3.4}$$

where the resonance fields are coupled on the $\mathcal{O}(p^2)$ chiral building blocks. The complete resonance Lagrangian is then

$$\mathcal{L}_R = \mathcal{L}_R^0 + \mathcal{L}_R^{int} \tag{2.3.5}$$

where $\mathcal{L}_R^0$ represents the kinetic and mass terms (that will be discussed in following). Integrating out the resonance fields we obtain the effective chiral Lagrangian

$$\int \mathcal{D}R \exp\left(i\int d^4x(\mathcal{L}_R)\right) = \exp\left(i\int d^4x L_{\chi,eff}\right) \tag{2.3.6}$$

After Gaussian integration we obtain the result

$$\mathcal{L}_{\chi,eff} = \frac{G_V^2}{2M^2}\langle u_\mu u^\mu\rangle^2 + \frac{G_V^2}{4M^2}\langle u_\mu u^\nu\rangle\langle u^\mu u_\nu\rangle - \frac{3G_V^2}{4M^2}\langle u_\mu u^\mu u_\nu u^\nu\rangle - \frac{F_V^2}{8M^2}\langle f_{+\mu\nu} f_+^{\mu\nu}\rangle \tag{2.3.7}$$

We can easily find that this result can be decomposed into the χPT Lagrangian. The form of the terms in $\mathcal{L}_{\chi,eff}$ indicates that the constants F_V and G_V contribute into $\mathcal{O}(p^4)$ coupling constants. Doing the precise matching we obtain

$$L_1' = \frac{G_V^2}{2M^2}, \quad L_2' = \frac{G_V^2}{4M^2}, \quad L_3' = -\frac{3G_V^2}{4M^2}, \quad L_{10}' = -\frac{F_V^2}{4M^2}, \quad H_1' = -\frac{F_V^2}{8M^2} \tag{2.3.8}$$

where the prime denotes that this is a contribution of vector resonances only. We have observed that the constants in $\mathcal{O}(p^4)$ chiral Lagrangian are saturated by the constants from $\mathcal{L}_R^{int}$, so we can assign the chiral order to the resonance fields $R_{\mu\nu} = \mathcal{O}(p^2)$. The situation for vector fields is similar, we can just replace $R \to V$, the simplest interaction term (analogous to previous one)

$$\mathcal{L}_V^{int} = \frac{F_V}{2\sqrt{2}}\langle (D_\mu V_\nu - D_\nu V_\mu) f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle R(D_\mu V_\nu - D_\nu V_\mu)[u^\mu, u^\nu]\rangle. \tag{2.3.9}$$

We can easily find that the corresponding effective chiral Lagrangian cannot be decomposed into the terms of $\mathcal{O}(p^4)$ χPT Lagrangian form. $\mathcal{L}_V^{(2)}$ contributes only to the order $\mathcal{O}(p^6)$ which indicates the chiral order, $V = \mathcal{O}(p^3)$. The fact there is no $\mathcal{O}(p^4)$ contribution to saturation of LECs indicates the future problems with the equivalence of descriptions.

2.4 Vector field formalism

The most natural way how to describe vector resonances is the vector field formalism (called often Proca field formalism).

The short hand notation used in this and following sections is explained in appendix A.

General properties

The resonance Lagrangian can be written in the form

$$\mathcal{L}_V = \mathcal{L}_V^{kin} + \mathcal{L}_V^{int} \tag{2.4.1}$$

where the kinetic and mass terms (covariant derivative include interaction part, of course) are

$$\mathcal{L}_V^{kin} = -\frac{1}{4}(\hat{V} : \hat{V}) + \frac{1}{2}M^2(V \cdot V) \tag{2.4.2}$$

and the interaction part can be expand in terms of chiral order

$$\mathcal{L}_V^{int} = \mathcal{L}_V^{(4)} + \mathcal{L}_V^{(6)} + \mathcal{L}_V^{(8)} + \ldots \tag{2.4.3}$$

In the beginning of the chapter we have seen that integrating out the resonances no effective chiral Lagrangian up to $\mathcal{O}(p^4)$ is generated, so $\mathcal{L}_V^{(4)} = 0$. Moreover, we have found that the chiral order of the vector fields is $V = \mathcal{O}(p^3)$. In the following discussion, it is also useful to introduce the alternative expansion in terms of the number of resonance fields

$$\begin{aligned}\mathcal{L}_V^{int} &= (J_1 \cdot V) + (J_2 : \hat{V}) + \frac{1}{2}(V \cdot K \cdot K) + (V \cdot J_3 : \hat{V}) \\ &\quad + \text{terms trilinear and higher in resonance fields}\end{aligned} \tag{2.4.4}$$

External sources J_i are built from the usual chiral building blocks that determine the chiral orders of the sources

$$J_1 = \mathcal{O}(p^3), \tag{2.4.5}$$

$$J_2 = \mathcal{O}(p^2), \tag{2.4.6}$$

$$J_3 = \mathcal{O}(p), \tag{2.4.7}$$

$$K = \mathcal{O}(p^2). \tag{2.4.8}$$

For example, we have

$$T^a J^a_{2\mu\nu} = -\frac{f_V}{2\sqrt{2}} f_{+\mu\nu} + \frac{ig_V}{2\sqrt{2}} g_V [u_\mu, u_\nu]. \tag{2.4.9}$$

Effective chiral Lagrangian

Dividing the resonance Lagrangian into $\mathcal{O}(p^6)$ and $\mathcal{O}(p^8)$ parts we can write

$$\mathcal{L}_V^{(6)} = \frac{1}{2} M^2 (V \cdot V) + (J_1 \cdot V) + (J_2 \cdot \hat{V}), \tag{2.4.10}$$

$$\mathcal{L}_V^{(8)} = -\frac{1}{4}(\hat{V} : \hat{V}) + \frac{1}{2}(V \cdot K \cdot V) + (V \cdot J_3 : \hat{V}) \tag{2.4.11}$$

Integrating out the resonance fields we get the corresponding effective chiral Lagrangian up to the given order. The integration is Gaussian, what effectively means the insertion of the solution of the classical equation of motion into the original Lagrangian. To the lowest non-trivial order $\mathcal{O}(p^3)$ we obtain

$$V^{(3)} = \frac{1}{M^2}(J_1 - 2D \cdot J_2). \tag{2.4.12}$$

The result up to the order $\mathcal{O}(p^6)$ is then

$$\begin{aligned}\mathcal{L}^{(6)}_{\chi,V} &= -\frac{1}{2M^2}((J_1 - 2D \cdot J_2) \cdot (J_1 - 2D \cdot J_2)) \\ &= -\frac{1}{2M^2}(J_1 \cdot J_1) + \frac{2}{M^2}(D \cdot J_2 \cdot J_1) + \frac{2}{M^2}\left(D \cdot J_2 \cdot J_2 \cdot \overleftarrow{D}\right)\end{aligned} \tag{2.4.13}$$

This Lagrangian can be rewritten in the standard $\mathcal{O}(p^6)$ basis [20] and we can find the saturation of $\mathcal{O}(p^6)$ LECs by the resonance couplings. As pointed out in [7], the contributions to the $O(p^4)$ LEC are not generated, unless extra contact terms are added to the Lagrangian. On the other hand, the interaction terms contained in the $\mathcal{L}_V^{(8)}$ give contributions only to the $O(p^8)$ chiral Lagrangian and could be therefore ignored (there is no study of χPT Lagrangian up of this order). Note also that, in principle, higher derivative terms as well as terms cubic or higher in the resonance fields can be added to the Lagrangian, but the chiral order are higher and these terms are irrelevant with respect to the possible contribution to $\mathcal{O}(p^6)$ LECs. Despite it, both types of additional terms mentioned above could be useful to satisfy high energy constraints of Green functions dictated by OPE, [7], [9], or as the counterterms to kill the infinities in one loop calculations.

Complete basis of terms

The Lagrangian terms can be divided into two parts representing odd and even intrinsic parity sector. The basis of $\mathcal{O}(p^6)$ terms that has been already studied in [9] and also in [A] has the form

	$\mathcal{O}(p^6)$ even parity	coupling
1	$\mathrm{i}\langle V^\mu [u^\nu, f_{-\mu\nu}]\rangle$	α_V
2	$\langle V^\mu [u_\mu, \chi_-]\rangle$	β_V
3	$\langle \hat{V}^{\mu\nu} f_{+\mu\nu}\rangle$	$-\frac{1}{2\sqrt{2}} f_V$
4	$\mathrm{i}\langle \hat{V}^{\mu\nu} [u_\mu, u_\nu]\rangle$	$-\frac{1}{2\sqrt{2}} g_V$

	$\mathcal{O}(p^6)$ odd parity	coupling
5	$\mathrm{i}\varepsilon_{\mu\nu\alpha\beta}\langle V^\mu u^\nu u^\alpha u^\beta\rangle$	θ_V
6	$\varepsilon_{\mu\nu\alpha\beta}\langle V^\mu \{u^\nu, f_+^{\alpha\beta}\}\rangle$	h_V

Moreover, we mention one of the $\mathcal{O}(p^8)$ terms that will have the analogue in the first order formalism:

	$\mathcal{O}(p^8)$ odd parity with VV	coupling
8	$\varepsilon_{\alpha\beta\mu\nu}\langle \{V^\alpha, \hat{V}^{\mu\nu}\} u^\beta\rangle$	$\frac{1}{2}\sigma_V$

2.5 Antisymmetric tensor formalism

The alternative description of vector resonances uses the antisymmetric tensor fields $R^a_{\mu\nu}$.

General properties

The resonance Lagrangian in the antisymmetric tensor formalism has the same form as in the Proca field case

$$\mathcal{L}_R = \mathcal{L}_R^{kin} + \mathcal{L}_R^{int} \tag{2.5.1}$$

The kinetic and mass terms are then

$$\mathcal{L}_R^{kin} = -\frac{1}{2}(W \cdot W) + \frac{1}{4}M^2(R : R) \tag{2.5.2}$$

Similarly as in the vector formalism we can use the chiral expansion of the interaction part of Lagrangian

$$\mathcal{L}_R^{int} = \mathcal{L}_R^{(4)} + \mathcal{L}_R^{(6)} + \mathcal{L}_R^{(8)} + \dots \tag{2.5.3}$$

We have seen that the leading interaction term is of the order $\mathcal{O}(p^4)$ and the chiral order of resonance field is $R = \mathcal{O}(p^2)$. The expansion in terms of resonance fields and external sources has the form

$$\begin{aligned}\mathcal{L}_R^{int} =& (J_1 \cdot W) + (J_2 : R) + (W \cdot J_3 : R) + (R : J_4 : R) \\ &+ (R : J_5 \cdot D : R) + (R : J_6 :: RR) + \text{terms higher in resonance fields}\end{aligned} \tag{2.5.4}$$

On the contrary to the vector formalism (where analogous term would be of the order at least $\mathcal{O}(p^{10})$) the trilinear term is present here. The leading chiral orders of the external sources J_i are

$$J_1 = \mathcal{O}(p^3), \tag{2.5.5}$$
$$J_2^{(2)} = \mathcal{O}(p^2), \tag{2.5.6}$$
$$J_2^{(4)} = \mathcal{O}(p^4), \tag{2.5.7}$$
$$J_3 = \mathcal{O}(p), \tag{2.5.8}$$
$$J_4 = \mathcal{O}(p^2), \tag{2.5.9}$$
$$J_5 = \mathcal{O}(p), \tag{2.5.10}$$
$$J_6 = \mathcal{O}(p^0). \tag{2.5.11}$$

where we divide the J_2 source into $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ parts.

Effective chiral Lagrangian

The $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ parts of resonance Lagrangian are

$$\mathcal{L}_R^{(4)} = \frac{1}{4} M^2 (R : R) + \left(J_2^{(2)} : R \right), \tag{2.5.12}$$

$$\begin{aligned}\mathcal{L}_R^{(6)} =& -\frac{1}{2}(W \cdot W) + \left(J_2^{(4)} : R \right) + (J_1 \cdot W) + (W \cdot J_3 : R) + (R : J_4 : R) \\ &+ (R : J_5 \cdot D : R) + (R : J_6 :: RR)\end{aligned} \tag{2.5.13}$$

There is, of course, additional $\mathcal{O}(p^8)$ contribution but it is not relevant in the following. Equation of motion to the lowest order is then

$$R^{(2)} = -\frac{2}{M^2} J_2^{(2)}. \tag{2.5.14}$$

Integrating out resonance fields in the original Lagrangian by the inserting this solution of EOM we obtain the effective chiral Lagrangian with $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ contributions

$$\mathcal{L}^{(4)}_{\chi,eff} = -\frac{1}{M^2}\left(J_2^{(2)} : J_2^{(2)}\right), \tag{2.5.15}$$

$$\begin{aligned}\mathcal{L}^{(6)}_{\chi,eff} = & -\frac{2}{M^2}\left(J_2^{(2)} : J_2^{(4)}\right) + \frac{2}{M^4}\left(D\cdot J_2^{(2)}\cdot J_2^{(2)}\cdot \overleftarrow{D}\right) - \frac{2}{M^2}\left(D\cdot J_2^{(2)}\cdot J_1\right) \\ & + \frac{4}{M^4}\left(D\cdot J_2^{(2)}\cdot J_3 : J_2^{(2)}\right) + \frac{4}{M^4}\left(J_2^{(2)} : J_4 : J_2^{(2)}\right) + \frac{4}{M^4}\left(J_2^{(2)} : J_5\cdot D : J_2^{(2)}\right) \\ & - \frac{8}{M^6}(J_2^{(2)} : J_6 :: J_2^{(2)} J_2^{(2)}).\end{aligned} \tag{2.5.16}$$

Complete basis of terms

The $\mathcal{O}(p^4)$ basis reads

	$\mathcal{O}(p^4)$ terms	coupling
1	$\langle R^{\mu\nu} f_{+\mu\nu}\rangle$	$\frac{1}{2\sqrt{2}}F_V$
2	$\mathrm{i}\langle R^{\mu\nu}[u_\mu, u_\nu]\rangle$	$\frac{1}{2\sqrt{2}}G_V$

The complete basis of $\mathcal{O}(p^6)$ in antisymmetric tensor formalism has not been constructed yet. In [12] is provided the complete list of the even intrinsic parity sector terms, in [11] we can find the odd intrinsic parity terms that contribute to all correlators that we will compute in the following.

The list of even parity terms with two resonances is

	$\mathcal{O}(p^6)$ even parity with RR	coupling
1	$\langle R_{\mu\nu}R^{\mu\nu}u^\alpha u_\alpha\rangle$	λ_1^{VV}
2	$\langle R_{\mu\nu}u^\alpha R^{\mu\nu}u_\alpha\rangle$	λ_2^{VV}
3	$\langle R_{\mu\alpha}R^{\nu\alpha}u^\mu u_\nu\rangle$	λ_3^{VV}
4	$\langle R_{\mu\alpha}R^{\nu\alpha}u^\mu u_\nu\rangle$	λ_4^{VV}
5	$\langle R_{\mu\alpha}(u^\alpha R^{\mu\beta}u_\beta + u_\beta R^{\mu\beta}u^\alpha)\rangle$	λ_5^{VV}
6	$\langle R_{\mu\nu}R^{\mu\nu}\chi_+\rangle$	λ_6^{VV}
7	$\mathrm{i}\, g^{\beta\mu}\langle R_{\mu\alpha}R^{\alpha\nu}f_{+\beta\nu}\rangle$	λ_7^{VV}

	$\mathcal{O}(p^6)$ even parity with R	
1	$\mathrm{i}\langle R_{\mu\nu}u^\mu u_\alpha u^\alpha u^\nu\rangle$	λ_1^V
2	$\mathrm{i}\langle R_{\mu\nu}u^\alpha u^\mu u^\nu u_\alpha\rangle$	λ_2^V
3	$\mathrm{i}\langle R_{\mu\nu}\{u^\alpha, u^\mu u_\alpha u^\nu\}\rangle$	λ_3^V
4	$\mathrm{i}\langle R_{\mu\nu}\{u^\mu u^\nu, u^\alpha u_\alpha\}\rangle$	λ_4^V
5	$\mathrm{i}g_{\alpha\beta}\langle R_{\mu\nu}f_-^{\mu\alpha}f_-^{\nu\beta}\rangle$	λ_5^V
6	$\langle R_{\mu\nu}\{f_+^{\mu\nu}, \chi_+\}\rangle$	λ_6^V
7	$\mathrm{i}g_{\alpha\beta}\langle R_{\mu\nu}f_+^{\mu\alpha}f_+^{\nu\beta}\rangle$	λ_7^V
8	$\mathrm{i}\langle R_{\mu\nu}\{\chi_+, u^\mu u^\nu\}\rangle$	λ_8^V
9	$\mathrm{i}\langle R_{\mu\nu}u^\mu\chi_+u^\nu\rangle$	λ_9^V
10	$\mathrm{i}\langle R_{\mu\nu}[u^\mu, D^\nu\chi_-]\rangle$	λ_{10}^V
11	$\mathrm{i}\langle R_{\mu\nu}\{f_+^{\mu\nu}, u^\alpha u_\alpha\}\rangle$	λ_{11}^V

	$\mathcal{O}(p^6)$ even parity with R	
12	$\langle R_{\mu\nu}u_\alpha f_+^{\mu\nu}u^\alpha\rangle$	λ_{12}^V
13	$\langle R_{\mu\nu}(u^\mu f_+^{\nu\alpha}u_\alpha + u_\alpha f_+^{\nu\alpha}u^\mu)\rangle$	λ_{13}^V
14	$\langle R_{\mu\nu}(u^\mu u_\alpha f_+^{\alpha\nu} + f_+^{\alpha\nu}u_\alpha u^\mu)\rangle$	λ_{14}^V
15	$\langle R_{\mu\nu}(u_\alpha u^\mu f_+^{\alpha\nu} + f_+^{\alpha\nu}u^\mu u_\alpha)\rangle$	λ_{15}^V
16	$\mathrm{i}\langle R_{\mu\nu}[D^\mu f_-^{\nu\alpha}, u_\alpha]\rangle$	λ_{16}^V
17	$\mathrm{i}\langle R_{\mu\nu}[D_\alpha f_-^{\mu\nu}, u^\alpha]\rangle$	λ_{17}^V
18	$\mathrm{i}\langle R_{\mu\nu}[D_\alpha f_-^{\alpha\mu}, u^\nu]\rangle$	λ_{18}^V
19	$\mathrm{i}\langle R_{\mu\nu}[f_-^{\mu\alpha}, h_\alpha^\nu]\rangle$	λ_{19}^V
20	$\langle R_{\mu\nu}[f_-^{\mu\nu}, \chi_-]\rangle$	λ_{20}^V
21	$\mathrm{i}\langle R_{\mu\nu}D_\alpha D^\alpha(u^\mu u^\nu)\rangle$	λ_{21}^V
22	$\langle R_{\mu\nu}D_\alpha D^\alpha f_+^{\mu\nu}\rangle$	λ_{22}^V

This is quite a new classification. In the older papers where the basis is incomplete alternative representation of these terms is used.

	alternative $\mathcal{O}(p^6)$ even parity	coupling
1	$\langle D^\mu R_{\mu\nu}[\chi_-, u^\nu]\rangle$	$-f_\chi/M$
2	$\mathrm{i}\langle R^{\mu\nu}\{[u_\mu, u_\nu], \chi_+\}\rangle$	$\frac{1}{2}g_{V1}^m/M$
3	$\mathrm{i}\langle R^{\mu\nu}(u^\mu\chi_+u_\nu - u_\nu\chi_+u_\mu)\rangle$	$\frac{1}{2}g_{V1}^m/M$
4	$\langle R^{\mu\nu}\{f_{+\mu\nu}, \chi_+\}\rangle$	f_{V1}^m/M
5	$\langle R^{\mu\nu}[f_{-\mu\nu}, \chi_-]\rangle$	f_{V2}^m/M
6	$\langle\chi_+\{R^{\mu\nu}, R_{\mu\nu}\}\rangle$	$\frac{1}{4}e_V^m$

The correspondence between these two sets of terms yields

$$\lambda_6^V \leftrightarrow f_{V1}^m, \qquad \lambda_8^V, \lambda_9^V, \lambda_{10}^V \leftrightarrow g_{V1}^m, g_{V2}^m, f_\chi,$$
$$\lambda_{20}^V \leftrightarrow f_{V2}^m, \qquad \lambda_6^{VV} \leftrightarrow \frac{1}{4}e_V^m$$

The direct calculation leads to the following relations between some constants

$$\lambda_6^V = \frac{f_{V1}^m}{M}, \qquad \lambda_{20}^V = \frac{f_{V1}^m}{M}, \qquad \lambda_6^{VV} = \frac{e_V^m}{2} \tag{2.5.17}$$

The odd intrinsic parity sector has not been classified yet. We have just an incomplete list of contributing terms

	$\mathcal{O}(p^6)$ odd parity with R	coupling
1	$\varepsilon_{\mu\nu\rho\sigma}\langle R^{\mu\nu}\{f_+^{\rho\alpha}, D_\alpha u^\sigma\}\rangle$	c_1/M
2	$\varepsilon_{\mu\kappa\rho\sigma}\langle R^{\mu\nu}\{f_+^{\rho\sigma}, D_\nu u^\kappa\}\rangle$	c_2/M
3	$\mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\langle R^{\mu\nu}\{f_+^{\rho\sigma}, \chi_-\}\rangle$	c_3/M
4	$\mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\langle R^{\mu\nu}[f_-^{\rho\sigma}, \chi_+]\rangle$	c_4/M
5	$\varepsilon_{\mu\nu\rho\sigma}\langle D_\lambda R^{\mu\nu}\{f_+^{\rho\lambda}, u^\sigma\}\rangle$	c_5/M
6	$\varepsilon_{\mu\kappa\rho\sigma}\langle D_\nu R^{\mu\nu}\{f_+^{\rho\sigma}, u^\kappa\}\rangle$	c_6/M
7	$\varepsilon_{\mu\nu\rho\sigma}\langle D^\sigma R^{\mu\nu}\{f_+^{\rho\lambda}, u_\lambda\}\rangle$	c_7/M

	$\mathcal{O}(p^6)$ odd parity with RR	coupling
1	$\varepsilon_{\mu\nu\alpha\sigma}\langle\{R^{\mu\nu},R^{\alpha\beta}\}D_\beta u^\sigma\rangle$	d_1
2	$\varepsilon_{\mu\nu\alpha\beta}\langle\{R^{\mu\nu},R^{\alpha\beta}\}\chi_-\rangle$	d_2
3	$\varepsilon_{\rho\sigma\mu\lambda}\langle\{D_\nu R^{\mu\nu},R^{\rho\sigma}\}u^\lambda\rangle$	d_3
4	$\varepsilon_{\rho\sigma\mu\alpha}\langle\{D^\alpha R^{\mu\nu},R^{\rho\sigma}\}u_\nu\rangle$	d_4

Up to $\mathcal{O}(p^6)$ we have to take into account also a term which is trilinear in the resonance fields.

	$\mathcal{O}(p^6)$ with RRR	coupling
1	$i\langle R_{\mu\nu}R^{\mu\rho}R^{\nu\sigma}\rangle g_{\rho\sigma}$	λ^{VVV}

2.6 Equivalence of both approaches

In this section we will study the correspondence between vector and antisymmetric tensor formalisms. We have already mentioned the problems connected with the contribution to the effective chiral Lagrangians and now we show this feature directly on the Lagrangian level.

As it was recognized in [7] the naive correspondence connecting free vector and antisymmetric tensor fields

$$\begin{aligned} R &\leftrightarrow \frac{1}{M}\widehat{V}, \\ V &\leftrightarrow -\frac{1}{M}W \end{aligned} \tag{2.6.1}$$

does not relate the Lagrangians properly. Let us now start with the simple antisymmetric tensor Lagrangian

$$\mathcal{L}_T = \frac{1}{4}M^2(R:R) - \frac{1}{2}(W\cdot W) + (J_2:R). \tag{2.6.2}$$

From the naive correspondence we obtain

$$\mathcal{L}_R \to \mathcal{L}_V = -\frac{1}{4}(\widehat{V}:\widehat{V}) + \frac{1}{2}m^2(V\cdot V) + \frac{1}{m}(J_2:\widehat{V}). \tag{2.6.3}$$

However, the contributions to the effective chiral Lagrangians up to $\mathcal{O}(p^6)$ are not identical (as can be shown from last sections). For instance to restore equality up to $O(p^4)$ we have to add the contact term

$$\mathcal{L}_T \to \mathcal{L}_V - \frac{1}{m^2}(J_2^{(2)}:J_2^{(2)}). \tag{2.6.4}$$

Therefore the naive substitution into the interaction terms with the sources J_i does not ensure the equivalence of both formulations.

The correspondence of these two formulations was studied in the past (cf. references [6], [7], [21], [22], [23], [24], [10]).

Vector → tensor correspondence

In this subsection we start with the vector field Lagrangian $\mathcal{L}_V$ and try to construct the antisymmetric tensor field Lagrangian $\mathcal{L}_R^{eff}$ which is equivalent to $\mathcal{L}_V$. Let us consider the Goldstone boson effective action $\Gamma_V[J_i, K]$ defined as

$$Z_V[J_i,K] = \exp\left(i\Gamma_V[J_i,K]\right) = \int \mathcal{D}V \exp\left(i\int d^4x\mathcal{L}_V\right). \tag{2.6.5}$$

The equivalence of $\mathcal{L}_V$ and $\mathcal{L}_T^{eff}$ means the equivalence of the contributions to the effective action $\Gamma_V[J_i, K]$

$$Z_V[J_i,K] = \exp\left(i\Gamma_V[J_i,K]\right) = \int \mathcal{D}R \exp\left(i\int d^4x\mathcal{L}_T^{eff}\right). \tag{2.6.6}$$

Introducing an auxiliary antisymmetric tensor field R we can write

$$Z_V[J_i,K] = \int \mathcal{D}V \exp\left(i\int d^4x\mathcal{L}_V\right) \tag{2.6.7}$$
$$= \frac{\int \mathcal{D}V\mathcal{D}R \exp\left(i\int d^4x\left(\frac{1}{4}m^2(R:R)+\mathcal{L}_V\right)\right)}{\int \mathcal{D}R\exp\left(i\int d^4x\frac{1}{4}m^2(R:R)\right)} \approx \int \mathcal{D}R\exp\left(i\int d^4x\mathcal{L}_R^{eff}\right).$$

The auxiliary field R is merely an integration variable, it can be therefore freely redefined. In the following we try to integrate out the vector field and get the expression for the effective Lagrangian $\mathcal{L}_R^{eff}$ which is completely equivalent to $\mathcal{L}_V$. The detailed calculation is done in [A]. The result is an infinite series in powers of p and can be found in the same article. The antisymmetric tensor field Lagrangian[3] $\mathcal{L}_R^{eff(\leq 6)}$ is not completely equivalent to the original $\mathcal{L}_V$ but is equivalent up to $\mathcal{O}(p^6)$ and gives the same $\mathcal{O}(p^6)$ chiral Lagrangian. The result for $\mathcal{L}_R^{eff(\leq 6)}$ can be written in the same form as the Lagrangian $\mathcal{L}_R$

$$\begin{aligned}\mathcal{L}_R^{eff(\leq 6)} &= \frac{1}{4}M^2(R:R) - \frac{1}{2}(W\cdot W) + \left(J_1^{eff}\cdot W\right) + (J_2^{eff}:R) \\ &+ (W\cdot J_3^{eff}:R) + (R:J_4^{eff}:R) + (R:J_5^{eff}\cdot D:R) + \mathcal{L}_T^{eff,(\leq 6)contact}.\end{aligned} \tag{2.6.8}$$

where

$$J_1^{eff} = -\frac{1}{m}J_1, \tag{2.6.9}$$
$$J_2^{eff} = mJ_2 - \frac{2}{m}J_2:J_3\cdot J_3 - \frac{1}{m}J_1\cdot J_3, \tag{2.6.10}$$
$$J_3^{eff} = -J_3, \tag{2.6.11}$$
$$J_4^{eff} = -\frac{1}{2}J_3\cdot J_3, \tag{2.6.12}$$
$$J_5^{eff} = 0, \tag{2.6.13}$$

and the contact term

$$\mathcal{L}_R^{eff(\leq 6),contact} = (J_2:J_2) - \frac{1}{2M^2}(J_1\cdot J_1) - \frac{2}{M^2}(J_2:J_3\cdot J_1) - \frac{2}{M^2}(J_2:J_3\cdot J_3:J_2). \tag{2.6.14}$$

[3]We have denoted $\mathcal{L}_R^{eff(\leq 6)}$ the effective Lagrangian $\mathcal{L}_R^{eff}$ where only terms up to $\mathcal{O}(p^6)$ are taken into account.

The sources J_i are taken, of course, from the vector formalism. The equivalence between the vector and the effective antisymmetric tensor Lagrangian up to the order $\mathcal{O}(p^6)$ cannot be complete unless $K = J_3 = 0$ in the original model. Then we have explicitly[4] $\mathcal{L}_R^{eff(\geq 8)} = 0$ and the infinite series reduces to $\mathcal{L}_R^{eff(\leq 6)}$. This condition is satisfied in the vector field formulation so the equivalence between $\mathcal{L}_R$ and $\mathcal{L}_R^{eff(\leq 6)}$ is guaranteed.

Tensor → vector correspondence

Analogously, we want to find the effective vector Lagrangian $\mathcal{L}_V^{eff}$ which is completely equivalent to the antisymmetric tensor Lagrangian $\mathcal{L}_R$. Detailed calculation is done again in [A] and we again obtain the infinite series of terms. The result up to $\mathcal{O}(p^6)$ is then

$$\mathcal{L}_V^{eff(\leq 6)} = -\frac{1}{4}(\widehat{V}:\widehat{V}) + \frac{1}{2}m^2(V\cdot V) + (J_1^{eff}\cdot V) + (J_2^{eff}:\widehat{V}) \tag{2.6.15}$$

$$+\frac{1}{2}(V\cdot K^{eff}\cdot V) + (V\cdot J_3^{eff}:\widehat{V}) + \mathcal{L}_V^{eff(\leq 6),contact} \tag{2.6.16}$$

where

$$\begin{aligned} J_1^{eff} &= mJ_1, \\ J_2^{eff} &= -\frac{1}{m}J_2^{(2)}, \\ K^{eff} &= J_3^{eff} = 0. \end{aligned}$$

and the contact term

$$\begin{aligned} \mathcal{L}_V^{eff(\leq 6),contact} = \; & \frac{1}{2}(J_1\cdot J_1) - \frac{1}{m^2}(J_2^{(2)}:J_2^{(2)}) - \frac{2}{m^2}(J_2^{(2)}:J_2^{(4)}) + \\ & \frac{4}{m^2}(J_2^{(2)}:J_4:J_2^{(2)}) + \frac{4}{m^4}(J_2^{(2)}:D\cdot J_5:J_2^{(2)}). \end{aligned}$$

The equivalence between the antisymmetric tensor and the effective vector Lagrangian up to the order $\mathcal{O}(p^6)$ cannot be complete unless $J_3 = J_4 = J_5 = 0$ in the original model. But the concrete forms of the sources J_i in the antisymmetric tensor field formulation up to $\mathcal{O}(p^6)$ do not satisfy these conditions so the infinite series does not generally reduce to the finite number of terms.

2.7 First order formalism

General properties

In last two subsections we have tried to prove the equivalence between the vector and the antisymmetric tensor formulation up to $\mathcal{O}(p^6)$. We have seen that the equivalence is not obtained in the general case. It can be observed already on the level of the effective chiral Lagrangians

[4] $\mathcal{L}_R^{eff(\geq 8)}$ are the terms from $\mathcal{L}_R^{eff}$ that are of order $\mathcal{O}(p^8)$ or higher

which neither start at the same order nor all terms are analogous. The antisymmetric tensor formulation seems to be better (and it really is) but as it was mentioned in [8] and [12] it does not create the contact term

$$\mathcal{L}_{\chi}^{J_1} = -\frac{1}{2}(J_1 \cdot J_1) \tag{2.7.1}$$

in the effective chiral $\mathcal{O}(p^6)$ Lagrangian after integrating out the resonances. So it must be added by hand as in [8]. All these problems lead us to find another formulation from which both previous cases can be derived and which will be more general than the traditional descriptions. Let us now start with the following simple first order Lagrangian

$$\mathcal{L}_{VT} = \frac{1}{4}M^2(R:R) + \frac{1}{2}m^2(V\cdot V) - \frac{1}{2}M(R:\widehat{V}) + (J_1\cdot V) + (J_2 : R) \tag{2.7.2}$$

Using the derivation presented in the appendix of [A] we can write for the fields V^{μ} and $R_{\mu\nu}$ in the momentum representation

$$\begin{aligned} \widetilde{V}(p) &= -\Delta_F^V(p)\cdot\left(\widetilde{J}_1(p) + \frac{2\mathrm{i}}{m}p\cdot\widetilde{J}_2(p)\right), \\ \widetilde{R}(p) &= -\Delta_F^R(p) : \left(\widetilde{J}_2(p) - \frac{\mathrm{i}}{2m}\widehat{p\widetilde{J}_1(p)}\right) \end{aligned} \tag{2.7.3}$$

and thus

$$\begin{pmatrix} \widetilde{V}(p) \\ \widetilde{R}(p) \end{pmatrix} = -\begin{pmatrix} \Delta_F^V(p) & -\frac{\mathrm{i}}{m}\widehat{\Delta_F^V(p)p} \\ \frac{\mathrm{i}}{m}\Delta_F^R(p)\cdot p & \Delta_F^R(p) \end{pmatrix}\begin{pmatrix} \widetilde{J}_1(p) \\ \widetilde{J}_2(p) \end{pmatrix}, \tag{2.7.4}$$

where $\mathrm{i}\Delta_F^V(p)_{\mu\nu}$ and $\mathrm{i}\Delta_F^R(p)_{\mu\nu\ \rho\sigma}$ are the covariant parts of the propagators of vector and anti-symmetric tensor fields (it means that they are reconstructed in the first order formalism). We use the notation of two-point Green functions as in [A]

$$= \langle T\widetilde{V}_{\mu}(p)V_{\nu}(0)\rangle = \mathrm{i}\Delta_F^V(p)_{\mu\nu},$$

$$= \langle T\widetilde{R}_{\mu\nu}(p)R_{\rho\sigma}(0)\rangle = \mathrm{i}\Delta_F^R(p)_{\mu\nu\ \rho\sigma},$$

$$= \langle T\widetilde{V}_{\sigma}(p)R_{\mu\nu}(0)\rangle = \mathrm{i}\Delta_F^{RV}(p)_{\sigma\ \mu\nu},$$

$$= \langle T\widetilde{R}_{\mu\nu}(p)V_{\sigma}(0)\rangle = \mathrm{i}\Delta_F^{RV}(-p)_{\sigma\ \mu\nu} = -\mathrm{i}\Delta_F^{RV}(p)_{\sigma\ \mu\nu}. \tag{2.7.5}$$

It is not difficult to prove, that the off-diagonal mixed propagator reads

$$\Delta_F^{RV}(p) = -\frac{\mathrm{i}}{m}\widehat{\Delta_F^V(p)p} = -\frac{\mathrm{i}}{m}\Delta_F^R(p)\cdot p = \frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}0}\frac{\mathrm{i}}{m}(g_{\sigma\mu}p_{\nu} - g_{\sigma\nu}p_{\mu}). \tag{2.7.6}$$

This approach was first introduced in [A], we call it *the first order formalism* as it is clear from the construction.

Effective chiral Lagrangian

Let us now discuss the general case of first order Lagrangian up to $\mathcal{O}(p^6)$ maximally bilinear in resonances,

$$\begin{aligned} \mathcal{L}_{VT} =& \frac{1}{4}m^2(R:R) + \frac{1}{2}m^2(V\cdot V) - \frac{1}{2}m(R:\widehat{V}) + \frac{1}{2}(V\cdot K\cdot V) + (J_1\cdot V) \\ &+ (J_2 : R) + (V\cdot J_3 : R) + (R : J_4 : R) + (R : J_5\cdot D : R). \end{aligned} \tag{2.7.7}$$

The solutions of the equations of motion to the lowest order are

$$\begin{aligned} R &= \frac{2}{m^2} J_2^{(2)}, \\ V &= -\frac{1}{m^2}\left(J_1 - \frac{2}{m^3} J_3 : J_2^{(2)} - \frac{2}{m} D \cdot J_2^{(2)}\right) \end{aligned}$$

that indicate the chiral counting $R = \mathcal{O}(p^2)$ and $V = \mathcal{O}(p^3)$ (as it is usual). We can then organize the Lagrangian as follows

$$\mathcal{L}_{VT} = \mathcal{L}_{VT}^{(4)} + \mathcal{L}_{VT}^{(6)} + \mathcal{L}_{VT}^{(8)} \tag{2.7.8}$$

where

$$\begin{aligned} \mathcal{L}_{VT}^{(4)} &= \frac{1}{4} m^2 (R : R) + (J_2^{(2)} : R), & (2.7.9) \\ \mathcal{L}_{VT}^{(6)} &= \frac{1}{2} m^2 (V \cdot V) - \frac{1}{2} m (R : \widehat{V}) + (J_1 \cdot V) + (J_2^{(4)} : R) + (V \cdot J_3 : R) \\ &\quad + (R : J_4 : R) + (R : J_5 \cdot D : R), & (2.7.10) \\ \mathcal{L}_{VT}^{(8)} &= \frac{1}{2} (V \cdot K \cdot V). & (2.7.11) \end{aligned}$$

The corresponding effective chiral Lagrangian up to $\mathcal{O}(p^6)$ is

$$\mathcal{L}_{\chi,VT} = \mathcal{L}_{\chi,VT}^{(4)} + \mathcal{L}_{\chi,VT}^{(6)} \tag{2.7.12}$$

where

$$\begin{aligned} \mathcal{L}_{\chi,VT}^{(4)} &= -\frac{1}{m^2} (J_2^{(2)} : J_2^{(2)}), \\ \mathcal{L}_{\chi,VT}^{(6)} &= -\frac{1}{2m^2} (J_1 \cdot J_1) - \frac{2}{m^2} (J_2^{(2)} : J_2^{(4)}) + \frac{2}{m^4} (D \cdot J_2^{(2)} \cdot J_2^{(2)} \cdot \overleftarrow{D}) \\ &\quad + \frac{2}{m^3} (D \cdot J_2^{(2)} \cdot J_1) - \frac{4}{m^5} (D \cdot J_2^{(2)} \cdot J_3 : J_2^{(2)}) + \frac{4}{m^4} (J_2^{(2)} : J_4 : J_2^{(2)}) \\ &\quad + \frac{4}{m^4} (J_2^{(2)} : J_5 \cdot D : J_2^{(2)}) - \frac{2}{m^6} (J_2^{(2)} : J_3 \cdot J_3 : J_2^{(2)}) + \frac{2}{m^4} (J_1 \cdot J_3 : J_2^{(2)}). \end{aligned}$$

As it was shown in [A] it is possible to integrate out the vector or the antisymmetric tensor fields and to derive the corresponding effective vector or effective antisymmetric tensor Lagrangians up to $\mathcal{O}(p^6)$ that are completely equivalent to the original vector, resp. antisymmetric tensor Lagrangians. So, the first order formalism can be assumed as a new way how to describe vector resonances in RχT or at least as a consistent method how to find the contact terms that must be added to vector or antisymmetric tensor Lagrangians when generating complete effective chiral Lagrangians.

Complete basis of terms

The complete basis of terms is identical with the sum of terms from the vector and antisymmetric tensor formalisms. Moreover, we have one mixing term coming from the source J_3 coupled to the resonance fields.

	$\mathcal{O}(p^6)$ mixing term	coupling
1	$\epsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha, R^{\mu\nu}\}u^\beta\rangle$	$\frac{1}{2}M\sigma_V$

2.8 Summary of the chapter

In this chapter, we have investigated the general properties of Resonance Chiral Theory (RχT). It is motivated by large N_C QCD which contains the infinite tower of resonances and fully describes the spectrum of hadrons. In RχT we restrict ourselves just to the lightest resonances in each channel. Satisfying all symmetry properties dictated by QCD we can construct the phenomenological Lagrangians for resonances and find the connection of their coupling constants with LEC from χPT. In the following we have discussed one type of resonances - vector resonances 1^{--}, the discussion and the calculations with other types of resonances would be analogous.

The vector resonances can be described in two ways - using vector or antisymmetric tensor fields. It is shown in this chapter and in [A] that these formalisms are not fully equivalent and when integrating out the resonances they give different effective chiral Lagrangians (effective contribution of resonances in χPT) so it is necessary to add some contact terms in both resonance Lagrangians. Therefore, we have introduced the alternative formulation - the first order formalism, that is in some sense a generalization of both traditional descriptions. The effective chiral Lagrangian then contains all possible terms and there is not necessary to add any terms by hand. Finally, we have also presented the complete basis of interaction terms in all three formalisms that will be useful in the concrete calculations in the following chapters.

CHAPTER 3

Green functions at tree level

Our first goal is to investigate the behavior of two and three point Green function in the framework of Resonance Chiral Theory. In the leading order in $1/N_C$ tree diagrams dominate and the contributions from loops could be neglected.

First, we mention the general properties of correlators and then we do the explicit calculations in all three formalisms. Finally, some relations between coupling constants are found in order to satisfy high energy constraints.

3.1 General properties

Correlators

In quantum mechanics the Green function $G(x_1, x_2, t_1, t_2)$ describes the propagation of a particle from one point to another. In QFT we work with quantum fields and analogously as in quantum mechanics, we can define the two point Green function as

$$\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta]|0\rangle. \tag{3.1.1}$$

where α, β represent both Lorentz and group indices. Because of the translation invariance of the theory we can use the relation

$$\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(y)]|0\rangle = \langle 0|T[O_\alpha(x-y)\mathcal{O}_\beta(0)]|0\rangle. \tag{3.1.2}$$

In is often useful to define this object (often referred to a correlator of quantum fields) also in the momentum representation

$$\Pi_{\alpha\beta}(p) \equiv \langle 0|T[\tilde{\mathcal{O}}_\alpha(p)\mathcal{O}_\beta(0)]|0\rangle = \int d^4x e^{ipx}\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)]|0\rangle. \tag{3.1.3}$$

The asymptotic behavior of two point Green functions can provide us with information about the spectrum of one particle states $|p,\lambda\rangle$. They correspond to the poles in $\Pi_{\alpha\beta}(p)$

$$\langle 0|T[\tilde{\mathcal{O}}_\alpha(p)\mathcal{O}_\beta(0)]|0\rangle \approx_{p^2\to m^2} \frac{i}{p^2-m^2+i\epsilon}\sum_\lambda \langle 0|\mathcal{O}_\alpha|p,\lambda\rangle\langle p,\lambda|\mathcal{O}^\dagger_\beta|0\rangle + \text{regular terms} \tag{3.1.4}$$

where we denote λ all internal indices. So, if we know the Green function we can reconstruct the matrix element $\langle 0|\mathcal{O}_\alpha|p,\lambda\rangle \neq 0$ of one particle state $|p,\lambda\rangle$.

We can also define the n-point Green functions as

$$\langle 0|T[\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\mathcal{O}_n(x_n)]|0\rangle. \tag{3.1.5}$$

where all Lorentz and internal symmetry indices are suppressed. In the momentum representation we have

$$\begin{aligned}&\langle 0|T[\tilde{\mathcal{O}}_1(p_1)\tilde{\mathcal{O}}_2(p_2)\ldots\mathcal{O}_n(0)]|0\rangle \\ &= \int d^4x_1 d^4x_2\ldots d^4x_{n-1}e^{i(p_1x_1+p_2x_2+\ldots p_{n-1}x_{n-1})}\langle 0|T[\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\mathcal{O}_n(0)]|0\rangle\end{aligned} \tag{3.1.6}$$

For example, in the future we will discuss $\langle VVP\rangle$ correlator which can be obtained from the general case setting $\mathcal{O}_1 = V^a_\mu$, $\mathcal{O}_2 = V^b_\nu$ and $\mathcal{O}_3 = P^c$.

Operator product expansion

Following the arguments in [25] we construct the expansion of the time order product of operators called OPE (operator product expansion) for the limit $x \to 0$

$$T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0))] = \sum_{n=0} C^{\alpha\beta}_n(x)\mathcal{A}_n(0). \tag{3.1.7}$$

where $C^{\alpha\beta}_n(x)$ is are c-numbers. It provides us with information about short distance (high energy) behavior of a given correlator. The vacuum expectation value of (3.1.7) can be written in the form

$$\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)]|0\rangle = \sum_{n=0} C^{\alpha\beta}_n(x)a_n \tag{3.1.8}$$

with $a_n = \langle 0|\mathcal{A}_n(0)|0\rangle$. This result can be expanded in terms of x.

$$\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)]|0\rangle = \sum_{n=-\infty}^{\infty} c^{\alpha\beta}_n x^n \tag{3.1.9}$$

Let us now investigate the short distance behavior of the correlator, i.e. $x \to 0$. In what follows we are interested only in leading order and the result can be written in the simplified form

$$\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)]|0\rangle = \frac{c^{\alpha\beta}_n}{x^n} + \mathcal{O}\left(\frac{1}{x^{n-1}}\right) \tag{3.1.10}$$

If $\mathcal{O}_\alpha$ are bosonic fields then only the terms proportional to even powers of x can survive and we can write

$$\langle 0|T[\mathcal{O}_\alpha(x)\mathcal{O}_\beta(0)]|0\rangle = \frac{c_n^{\alpha\beta}}{x^{2n}} + \mathcal{O}\left(\frac{1}{x^{2(n-1)}}\right) \tag{3.1.11}$$

In the general case we can find the OPE for the more point Green function. Considering that all x_i are of the same order ($\sim \epsilon$) we can then write the expansion in terms of this small quantity.

If we write OPE in momentum representation and take all non-exceptional momenta to infinity (it is the analog of $x_i \to 0$), $p_i \to \lambda p_i$ where $\lambda \to \infty$ we obtain the expansion in terms of λ in the deep euclidean region that gives us the constraints which we compare with the results calculated in RχT. In order to satisfy these constraints we can obtain the set of relations between coupling constants. These relations will be then applied on the result in the low energy limit and compared with the χPT prediction. Finally, we find the set of relations for saturation of LECs valid in the leading order in $1/N_C$ expansion.

3.2 Simple Green functions

In this section we focus on the properties of simple Green functions, concretely two-point correlators $\langle VV\rangle$, $\langle PP\rangle$ and vector formfactor.

$\langle PP\rangle$ correlator

Two point $\langle PP\rangle$ correlator is defined as

$$(\Pi_{PP})^{ab}_{\mu\nu}(p) = \int d^4x e^{ip\cdot x}\langle 0|T[P^a(x)P^b(0)]|0\rangle. \tag{3.2.1}$$

There is no tensor structure so the general form of the correlator can be written in the form

$$(\Pi_{PP})^{ab}_{\mu\nu}(p) = i\delta^{ab}\Pi_{PP}(p^2). \tag{3.2.2}$$

The OPE expansion can be found in appendix A, the high energy behavior then reads

$$\Pi_{PP}(\lambda^2 p^2) = \frac{3p^2}{16\pi^2}\lambda^2\ln\lambda^2 + \mathcal{O}(\lambda^0, \alpha_s) \tag{3.2.3}$$

The $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ Lagrangians contributing to this correlator are

$$\begin{aligned}\mathcal{L}^{(4)}_\chi =& \frac{L_8}{2}\langle\chi_-^2 + \chi_+^2\rangle + iL_{11}\langle\chi_-(D_\mu u^\mu + i/2\chi_-)\rangle \\ & - L_{12}\langle(D_\mu u^\mu + i/2\chi_-)^2\rangle + \frac{H_2}{4}\langle\chi_+^2 - \chi_-^2\rangle, \end{aligned}\tag{3.2.4}$$

$$\mathcal{L}^{(6)}_\chi = c_{91}\langle D_\mu\chi D^\mu\chi^\dagger\rangle. \tag{3.2.5}$$

There is no 1^{--} resonance contribution at tree level. We have only pure χPT result.

$$\begin{aligned}\Pi_{PP}(p^2) &= -\frac{F^2B_0^2}{p^2}\left(1 - \frac{4(L_{11}-L_{12})p^2}{F^2}\right)^2 - 4B_0^2(2L_{11} + H_2 - L_{12} - 2L_8) - 4B_0^2p^2c_{91} \\ &= -\frac{FB_0^2}{p^2} - 4B_0^2(L_{12} + H_2 - 2L_8) - 4B_0^2\left(c_{91} + \frac{4(L_{11}-L_{12})^2}{F^2}\right)p^2 \end{aligned}\tag{3.2.6}$$

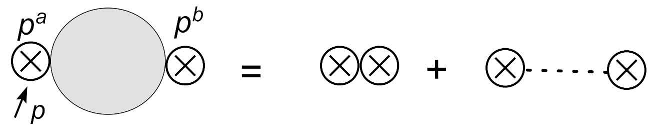


Figure 3.1: Diagrams contributing to PP correlator.

In order to satisfy the high energy constraints we do not have to impose any relation. In the low energy limit we just restore the result in Chiral perturbation theory, when the LEC with tilde are substituted by the LEC from χPT Lagrangian. As a result, there is no saturation of LEC from vector resonances (of course other types of resonances could contribute).

$\langle VV \rangle$ correlator

Two point $\langle VV \rangle$ correlator is defined as

$$(\Pi_{VV})^{ab}_{\mu\nu}(p) = \int d^4x e^{ip\cdot x} \langle 0|T[V^a_\mu(x)V^b_\nu(0)]|0\rangle. \tag{3.2.7}$$

Using the Ward identities, $p^\mu(\Pi_{VV})^{ab}_{\mu\nu} = 0$ we get

$$(\Pi_{VV})^{ab}_{\mu\nu}(p) = i\delta^{ab}\Pi_{VV}(p^2)(p^2 g_{\mu\nu} - p_\mu p_\nu) \tag{3.2.8}$$

The OPE constraints up to leading order have the form

$$\Pi_{VV}(\lambda^2 p^2) = -\frac{1}{8\pi^2}\ln\lambda^2 + \mathcal{O}\left(\frac{1}{\lambda^2}, \alpha_s\right) \tag{3.2.9}$$

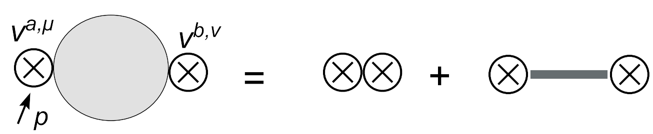


Figure 3.2: Diagrams contributing to VV correlator.

The low energy result calculated in χPT is determined by the contribution of the terms

$$\mathcal{L}^{(4)}_\chi = \frac{L_{10}}{4}\langle f_{+\mu\nu}f^{\mu\nu}_+ - f_{-\mu\nu}f^{\mu\nu}_-\rangle + \frac{H_1}{2}\langle f_{+\mu\nu}f^{\mu\nu}_+ + f_{-\mu\nu}f^{\mu\nu}_-\rangle, \tag{3.2.10}$$

$$\mathcal{L}^{(6)}_\chi = c_{93}\langle D_\rho F_{L\mu\nu}D^\rho F^{\mu\nu}_L\rangle + L \to R. \tag{3.2.11}$$

and the result for the formfactor has the form

$$\Pi^\chi_{VV}(p^2) = 2L_{10} + 4H_1 + 4c_{93}p^2. \tag{3.2.12}$$

Vector formalism

There is only one simple contribution coming from interaction term

$$\mathcal{L}_V = -\frac{f_V}{2\sqrt{2}} \langle \hat{V}^{\mu\nu} f_{+\mu\nu} \rangle, \tag{3.2.13}$$

and the formfactor in vector formalism reads

$$\Pi_{VV}(p^2) = \frac{f_V^2 p^2}{p^2 - M^2} + 2\tilde{L}_{10} + 4\tilde{H}_1 + 4\tilde{c}_{93} p^2. \tag{3.2.14}$$

In high energy limit for $\lambda \to \infty$ we have

$$\Pi_{VV}(\lambda^2 p^2) = 4\tilde{c}_{93}\lambda^2 p^2 + 2\tilde{L}_{10} + 4\tilde{H}_1 + f_V^2 + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \tag{3.2.15}$$

Compatibility with high energy constraints requires

$$\tilde{c}_{93} = 0. \tag{3.2.16}$$

For $p \to 0$ we can write

$$\Pi_{VV}(p^2) = 2\tilde{L}_{10} + 4\tilde{H}_1 - \left(\frac{f_V^2}{M^2} + 4\tilde{c}_{93}\right) p^2 + \mathcal{O}(p^4) \tag{3.2.17}$$

Applying the high energy relation we find the relations

$$L_{10} + 2H_1 = \tilde{L}_{10} + 2\tilde{H}_1, \tag{3.2.18}$$

$$c_{93} = -\frac{f_V^2}{4M^2}. \tag{3.2.19}$$

The first relation says that there is now resonance saturation in the combination of these $\mathcal{O}(p^4)$ low energy constants. For the $\mathcal{O}(p^6)$ constant c_{93} we have found the exact prediction in terms of resonance couplings.

Antisymmetric tensor formalism

Analogous interaction Lagrangian term in antisymmetric tensor formalism has not only $\mathcal{O}(p^6)$ but also $\mathcal{O}(p^4)$ contribution

$$\mathcal{L}_R = \frac{F_V}{2\sqrt{2}} \langle R^{\mu\nu} f_{+\mu\nu} \rangle + \lambda_{22}^V \langle R_{\mu\nu} D^\alpha D_\alpha f_+^{\mu\nu} \rangle. \tag{3.2.20}$$

The result can be then written in the form

$$\Pi_{VV}(p^2) = \frac{1}{p^2 - M^2}\left(F_V^2 - 4\sqrt{2}F_V\lambda_{22}^V p^2 + 8(\lambda_{22}^V)^2 p^4\right) + 2\tilde{L}_{10} + 4\tilde{H}_1 + 4\tilde{c}_{93}p^2. \tag{3.2.21}$$

In high energy limit for $\lambda \to \infty$ we have

$$\Pi_{VV}(\lambda^2 p^2) = \left(4\tilde{c}_{93} + 8(\lambda_{22}^V)^2\right)\lambda^2 p^2 - 4\sqrt{2}F_V\lambda_{22}^V + 8(\lambda_{22}^V)^2 M^2 + 2\tilde{L}_{10} + 4\tilde{H}_1 + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \tag{3.2.22}$$

Compatibility with high energy constraints requires

$$\tilde{c}_{93} = -2(\lambda_{22}^V)^2. \tag{3.2.23}$$

For $p \to 0$ we can write

$$\Pi_{VV}(p^2) = 2\tilde{L}_{10} + 4\tilde{H}_1 - \frac{F_V^2}{M^2} - \left(\frac{F_V^2}{M^4} - \frac{4\sqrt{2}F_V\lambda_{22}^V}{M^2} - 4\tilde{c}_{93} \right) p^2 + \mathcal{O}(p^4) \tag{3.2.24}$$

Applying (3.2.23) we find again the saturation of the constant c_{93} in terms of couplings from the antisymmetric tensor Lagrangian and also the nontrivial contribution to the $\mathcal{O}(p^4)$ LECs

$$c_{93} = -\frac{1}{4M^4}(F_V - 2\sqrt{2}\lambda_{22}^V M^2)^2, \tag{3.2.25}$$

$$2L_{10} + 4H_1 = 2\tilde{L}_{10} + 4\tilde{H}_1 - \frac{F_V^2}{M^2}. \tag{3.2.26}$$

First order formalism

In first order formalism we have the terms of interaction Lagrangian which are the sum of those in vector and antisymmetric tensor formalisms.

$$\mathcal{L}_R = -\frac{f_V}{2\sqrt{2}}\langle \hat{V}^{\mu\nu} f_{+\mu\nu}\rangle + \frac{F_V}{2\sqrt{2}}\langle R^{\mu\nu} f_{+\mu\nu}\rangle + \lambda_{22}^V\langle R_{\mu\nu} D^\alpha D_\alpha f_+^{\mu\nu}\rangle. \tag{3.2.27}$$

The result is of the form

$$\begin{aligned}\Pi_{VV}(p^2) =& \frac{F_V^2}{p^2 - M^2} - \frac{p^2}{p^2 - M^2}\left[-f_V^2 + 4\sqrt{2}F_V\lambda_{22}^V + \frac{2f_V F_V}{M} \right] \\ &- \frac{p^4}{p^2 - M^2}\left[-8(\lambda_{22}^V)^2 - \frac{4\sqrt{2}f_V\lambda_{22}^V}{M} \right] + 2\tilde{L}_{10} + 4\tilde{H}_1 + 4\tilde{c}_{93}p^2\end{aligned} \tag{3.2.28}$$

In high energy limit for $\lambda \to \infty$ we have

$$\begin{aligned}\Pi_{VV}(\lambda^2 p^2) =& \left(4\tilde{c}_{93} - 8(\lambda_{22}^V)^2 - \frac{4\sqrt{2}f_V\lambda_{22}^V}{M} \right)\lambda^2 p^2 + 4\sqrt{2}F_V\lambda_{22}^V - 8(\lambda_{22}^V)^2 M^2 \\ &- 4\sqrt{2}f_V\lambda_{22}^V M - f_V^2 + \frac{2F_V f_V}{M} + 2\tilde{L}_{10} + 4\tilde{H}_1 + \mathcal{O}\left(\frac{1}{\lambda^2}\right)\end{aligned} \tag{3.2.29}$$

Analogously for $\tilde{c}_{93}$ we obtain

$$\tilde{c}_{93} = -2(\lambda_{22}^V)^2 - \frac{\sqrt{2}f_V\lambda_{22}^V}{M}. \tag{3.2.30}$$

The low energy result reads

$$\begin{aligned}\Pi_{VV}(p^2) =& 2\tilde{L}_{10} + 4\tilde{H}_1 - \frac{F_V^2}{M^2} \\ &- \left(\frac{F_V^2}{M^4} - \frac{4\sqrt{2}F_V\lambda_{22}^V}{M^2} + \frac{f_V^2}{M^2} - \frac{2F_V f_V}{M} - 4\tilde{c}_{93} \right) p^2 + \mathcal{O}(p^4)\end{aligned} \tag{3.2.31}$$

Applying (3.2.30) one finds

$$c_{93} = -\frac{1}{4M^4}(F_V - f_V M - 2\sqrt{2}\lambda_{22}^V M^2)^2, \tag{3.2.32}$$

$$L_{10} + 2H_1 = \tilde{L}_{10} + 2\tilde{H}_1 - \frac{F_V^2}{2M^2}. \tag{3.2.33}$$

Taking $f_V = 0$ we restore the result from the antisymmetric tensor formalism and for $F_V = \lambda_{22}^V = 0$ we get the relation known from the vector formalism.

Vector formfactor

The vector formfactor is defined as the matrix element

$$\mathcal{F}^{\mu,abc}(q^2) = i\langle\phi^b(p_1)\phi^c(p_2)|V^{\mu,a}|0\rangle \tag{3.2.34}$$

where $q^\mu = (p_1+p_2)^\mu$ and $p_1^2 = p_2^2 = 0$ (off-shell external pions). Symmetry properties determine the group and tensor structure uniquely.

$$\langle\phi^b(p_1)\phi^c(p_2)|V^{\mu,a}|0\rangle = i\mathcal{F}(q^2)f^{abc}(p_2 - p_1)^\mu \tag{3.2.35}$$

High energy constraints require vanishing of $\mathcal{F}(q^2)$ for $q^2 \to \infty$.

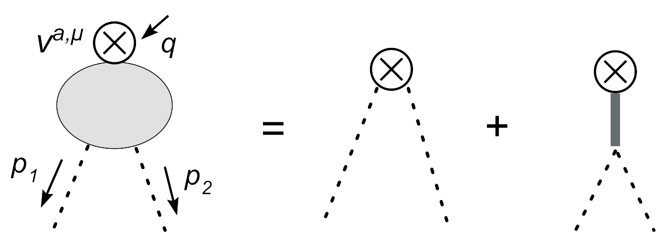


Figure 3.3: Diagrams contributing to vector form factor.

The Lagrangian terms of χPT contributing to the vector formfactor are (including $\mathcal{O}(p^2)$ term)

$$\mathcal{L}_\chi^{(4)} = -iL_9\langle f_+^{\mu\nu}u_\mu u_\nu\rangle - L_{12}\langle D^\mu u_\mu D^\nu u_\nu\rangle,$$
$$\mathcal{L}_\chi^{(6)} = ic_{88}\langle D^\rho f_+^{\mu\nu}[h_{\mu\rho}, u_\nu]\rangle + ic_{90}\langle D_\mu f_+^{\mu\nu}[h_{\nu\rho}, u^\rho]\rangle$$

with the result

$$\mathcal{F}_\chi(q^2) = 1 + \frac{2L_9 q^2}{F^2} + \frac{4(c_{90} - c_{88})q^4}{F^2}. \tag{3.2.36}$$

Vector formalism

The terms in interaction resonance Lagrangian that contribute to the formfactor are

$$\mathcal{L}_V = -\frac{f_V}{2\sqrt{2}}\langle\hat{V}^{\mu\nu}f_{+\mu\nu}\rangle - \frac{ig_V}{2\sqrt{2}}\langle\hat{V}^{\mu\nu}[u_\mu, u_\nu]\rangle \tag{3.2.37}$$

The formfactor then reads

$$\mathcal{F}(q^2) = 1 - \frac{f_V g_V}{F^2} \frac{q^4}{q^2 - M^2} + \frac{\tilde{L}_9 q^2}{2F^2} + \frac{4(\tilde{c}_{90} - \tilde{c}_{88}) q^4}{F^2}. \tag{3.2.38}$$

In high energy limit for $q^2 \to \infty$ we have

$$\mathcal{F}(q^2) = \frac{4(\tilde{c}_{90} - \tilde{c}_{88})}{F^2} q^4 - \left(\frac{f_V g_V}{F^2} - \frac{2\tilde{L}_9}{F^2} \right) q^2 + 1 - \frac{f_V g_V M^2}{F^2} + \mathcal{O}\left(\frac{1}{q^2} \right) \tag{3.2.39}$$

Compatibility with high energy constraints requires

$$\tilde{c}_{90} = \tilde{c}_{88}, \tag{3.2.40}$$
$$f_V g_V M^2 = F^2, \tag{3.2.41}$$
$$f_V g_V = 2\tilde{L}_9. \tag{3.2.42}$$

For low energies, $q^2 \to 0$, we can write

$$\mathcal{F}(q^2) = 1 + \frac{2\tilde{L}_9}{F^2} q^2 + \left(\frac{f_V g_V}{M^2 F^2} + \frac{4(\tilde{c}_{90} - \tilde{c}_{88})}{F^2} \right) q^4 + \mathcal{O}(q^6) \tag{3.2.43}$$

Applying the relations found in matching with OPE at large energies one finds

$$\mathcal{F}(q^2) = 1 + \frac{q^2}{M^2} + \frac{q^4}{M^4} + \mathcal{O}(q^6). \tag{3.2.44}$$

Comparing with the prediction of χPT we obtain the relations for LEC

$$L_9 = \frac{F^2}{2M^2}, \tag{3.2.45}$$
$$c_{90} - c_{88} = \frac{F^2}{4M^4}. \tag{3.2.46}$$

together with the relation (3.2.41).

Antisymmetric tensor formalism

In antisymmetric tensor formalism we have as usual $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ terms contributing to the formfactor

$$\begin{aligned} \mathcal{L}_R = & \frac{F_V}{2\sqrt{2}} \langle R^{\mu\nu} f_{+\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle R^{\mu\nu} [u_\mu, u_\nu] \rangle \\ & + i\lambda_{21}^V \langle R_{\mu\nu} D_\alpha D^\alpha (u^\mu u^\nu) \rangle + \lambda_{22}^V \langle R_{\mu\nu} D_\alpha D^\alpha f_+^{\mu\nu} \rangle \end{aligned} \tag{3.2.47}$$

The formfactor then reads

$$\begin{aligned} \mathcal{F}(q^2) = 1 - & \frac{1}{F^2} \frac{q^2}{q^2 - M^2} \left(F_V - 2\sqrt{2} q^2 \lambda_{22}^V \right) \left(G_V - \sqrt{2} q^2 \lambda_{21}^V \right) \\ & + \frac{2\tilde{L}_9 q^2}{F^2} + \frac{4(\tilde{c}_{90} - \tilde{c}_{88}) q^4}{F^2}. \end{aligned} \tag{3.2.48}$$

In high energy limit for $q^2 \to \infty$ we have

$$\mathcal{F}(q^2) = \frac{4q^4}{F^2}(\tilde{c}_{90} - \tilde{c}_{88} - \lambda_{21}^V \lambda_{22}^V) \tag{3.2.49}$$
$$+ \frac{q^2}{F^2}\left(\sqrt{2}F_V \lambda_{21}^V + 2\sqrt{2}G_V \lambda_{22}^V - 4M^2 \lambda_{21}^V \lambda_{22}^V + 2\tilde{L}_9\right)$$
$$+1 + \frac{1}{F^2}\left[2\sqrt{2}\lambda_{22}^V(G_V M^2 - \sqrt{2}M^4 \lambda_{21}^V) - F_V(G_V - \sqrt{2}M^2\lambda_{21}^V)\right] + \mathcal{O}\left(\frac{1}{q^2}\right)$$

The compatibility with high energy constraints requires

$$\tilde{c}_{90} - \tilde{c}_{88} = \lambda_{21}^V \lambda_{22}^V, \tag{3.2.50}$$
$$\tilde{L}_9 = -\frac{F_V \lambda_{21}^V}{\sqrt{2}} - \sqrt{2}G_V \lambda_{22}^V + 2M^2 \lambda_{21}^V \lambda_{22}^V, \tag{3.2.51}$$
$$F^2 = F_V(G_V - \sqrt{2}M^2 \lambda_{21}^V) - 2\sqrt{2}\lambda_{22}^V(G_V M^2 - \sqrt{2}M^4 \lambda_{21}^V). \tag{3.2.52}$$

For $q^2 \to 0$ we can write

$$\mathcal{F}(q^2) = 1 + \frac{q^2}{F^2}\left(\frac{F_V G_V}{M^2} + 2\tilde{L}_9\right) \tag{3.2.53}$$
$$+ \frac{q^4}{F^2}\left(4(c_{90} - c_{88}) - \frac{\sqrt{2}}{M^2}(F_V \lambda_{21}^V + 2G_V \lambda_{22}^V) + \frac{F_V G_V}{M^4}\right) q^4 + \mathcal{O}(q^6)$$

Taking into account the conditions (3.2.50)-(3.2.52) we obtain the same form of the formfactor (3.2.44) as in the vector formalism together with the same relations for LECs.

First order formalism

The contributing Lagrangian is here just the sum of $\mathcal{L}_V$ and $\mathcal{L}_R$, $\mathcal{L}_{RV} = \mathcal{L}_R + \mathcal{L}_V$. For the formfactor we obtain the more general result as in the previous cases

$$\mathcal{F}(q^2) = 1 - \frac{f_V g_V}{F^2}\frac{q^4}{q^2 - M^2} - \frac{1}{F^2}\frac{q^2}{q^2 - M^2}\left(F_V - 2\sqrt{2}q^2\lambda_{22}^V\right)\left(G_V - \sqrt{2}q^2\lambda_{21}^V\right)$$
$$+ \frac{g_V}{MF^2}\frac{q^4}{q^2 - M^2}\left(F_V - 2\sqrt{2}q^2\lambda_{22}^V\right) + \frac{f_V}{MF^2}\frac{q^4}{q^2 - M^2}\left(G_V - \sqrt{2}q^2\lambda_{21}^V\right)$$
$$+ \frac{2L_9 q^2}{F^2} + \frac{4(c_{90} - c_{88})q^4}{F^2} \tag{3.2.54}$$

In the high energy limit for $q^2 \to \infty$ we have

$$\begin{aligned}\mathcal{F}(q^2) =& \frac{4q^4}{F^2}\left(\tilde{c}_{90} - \tilde{c}_{88} - \lambda_{21}^V\lambda_{22}^V - \frac{f_V\lambda_{21}^V + 2g_V\lambda_{22}^V}{2\sqrt{2}M}\right) \\ &+ \frac{q^2}{F^2}\left\{\frac{(F_V - 2\sqrt{2}M^2\lambda_{22}^V)g_V}{M} - f_Vg_V + 2\tilde{L}_9 + \frac{f_V(G_V - \sqrt{2}M^2\lambda_{21}^V)}{M}\right. \\ &\left.+ \sqrt{2}F_V\lambda_{21}^V + 2\sqrt{2}\lambda_{22}^V(G_V - \sqrt{2}M^2\lambda_{21}^V)\right\} \\ &+ \left\{1 - \frac{f_Vg_VM^2}{F^2} + \frac{f_VM(G_V - \sqrt{2}M^2\lambda_{21}^V)}{F^2} + \frac{g_VM(F_V - 2\sqrt{2}M^2\lambda_{22}^V)}{F^2}\right. \\ &\left.+ \frac{2\sqrt{2}M^2\lambda_{22}^V(G_V - \sqrt{2}M^2\lambda_{21}^V)}{F^2} - \frac{F_VG_V}{F^2} + \frac{\sqrt{2}F_VM^2\lambda_{21}^V}{F^2}\right\} + \mathcal{O}\left(\frac{1}{q^2}\right)\end{aligned} \tag{3.2.55}$$

The compatibility with high energy constraints requires

$$\tilde{c}_{90} - \tilde{c}_{88} = \lambda_{21}^V\lambda_{22}^V + \frac{f_V\lambda_{21}^V + 2g_V\lambda_{22}^V}{2\sqrt{2}M}, \tag{3.2.56}$$

$$\begin{aligned}\tilde{L}_9 =& -\frac{F_V\lambda_{21}^V}{\sqrt{2}} - \sqrt{2}\lambda_{22}^V(G_V - \sqrt{2}M^2\lambda_{21}^V) + \frac{f_Vg_V}{2} \\ &+ \frac{(F_V - 2\sqrt{2}M^2\lambda_{22}^V)g_V}{2M} + \frac{f_V(G_V - \sqrt{2}M^2\lambda_{21}^V)}{2M},\end{aligned} \tag{3.2.57}$$

$$\begin{aligned}F^2 =& f_Vg_VM^2 - f_VM(G_V - \sqrt{2}M^2\lambda_{21}^V) - g_VM(F_V - 2\sqrt{2}M^2\lambda_{22}^V) \\ &- 2\sqrt{2}M^2\lambda_{22}^V(G_V - \sqrt{2}M^2\lambda_{21}^V) + F_VG_V - \sqrt{2}F_VM^2\lambda_{21}^V.\end{aligned} \tag{3.2.58}$$

For $q^2 \to 0$ we can write

$$\begin{aligned}\mathcal{F}(q^2) =& 1 + \frac{q^2}{F^2}\left(\frac{F_VG_V}{M^2} + 2\tilde{L}_9\right) + \frac{q^4}{F^2}\left\{\frac{f_Vg_V}{M^2} - \frac{(F_Vg_V + G_Vf_V)}{M^3}\right. \\ &\left.+ 4(\tilde{c}_{90} - \tilde{c}_{88}) + \frac{F_VG_V}{M^4} - \frac{\sqrt{2}F_V\lambda_{21}^V + 2\sqrt{2}G_V\lambda_{22}^V}{M^2}\right\} + \mathcal{O}(q^6)\end{aligned} \tag{3.2.59}$$

Again, we obtain the same relations for LECs as in the vector and the antisymmetric tensor formalisms.

We continue our discussion of Green functions in χT focusing on the more complicated case of the three point correlator.

3.3 $\langle VVP\rangle$ correlator

This correlator was already studied in [11, 9, 26]. We enlarge this study by taking account the complete $\mathcal{O}(p^6)$ Lagrangian in the antisymmetric tensor formalism and we also include the calculations in the first order formalism. The result can be then applied for many processes, ie. $\pi \to 2\gamma$.

General properties

The definition of the correlator in the momentum representation is

$$(\Pi_{VVP})^{abc}_{\mu}(p,q) = \int d^4x \int d^4y e^{i(p\cdot x+q\cdot y)}\langle 0|T[V^a_\mu(x)V^b_\nu(y)P^c(0)|0\rangle. \tag{3.3.1}$$

Denoting $r=-p-q$ we can write the Ward identities in the form

$$-ip^\mu(\Pi_{VVP})^{abc}_{\mu\nu} = if^{abd}\langle 0|T\widetilde{V}^d_\nu(p+q)P^c(0)|0\rangle + if^{acd}\langle 0|T\widetilde{V}^b_\nu(q)P^d(0)|0\rangle = 0 \tag{3.3.2}$$

where

$$(\Pi_{VP})^{cd}_\mu = \langle 0|T\tilde{V}^c_\mu(p)P^d(0)|0\rangle, \tag{3.3.3}$$

which is identically zero. The similar calculation can be done for the second independent momentum. As a result, we obtain

$$p^\mu(\Pi_{VVP})^{abc}_{\mu\nu} = q^\nu(\Pi_{VVP})^{abc}_{\mu\nu} = 0 \tag{3.3.4}$$

Together with the invariance under P and C transformation we get the structure,

$$(\Pi_{VVP})^{abc}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta d^{abc}\mathcal{F}_{VVP}(p^2,q^2,r^2) \tag{3.3.5}$$

with the four-vector $r=-p-q$ and formfactor $\mathcal{F}(p^2,q^2,r^2)$ is symmetric under $p\leftrightarrow q$. The OPE calculations can be found in the appendix, in the high energy limit we have

$$\lim_{\lambda\to\infty}\int d^4x\, d^4y\, e^{i\lambda px+i\lambda qy}\langle 0|T[V^{a,\mu}(x)V^{b,\nu}(y)P^c(0)]|0\rangle \tag{3.3.6}$$
$$= \frac{1}{\lambda^2}\frac{B_0F_0^2d^{abc}}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta\frac{p^2+q^2+(p+q)^2}{p^2q^2(p+q)^2} + \mathcal{O}\left(\frac{1}{\lambda^4},\alpha_s\right)$$

which means that the formfactor $\mathcal{F}_{VVP}$ must satisfy the relation

$$\mathcal{F}_{VVP}(\lambda^2p^2,\lambda^2q^2,\lambda^2r^2) = \frac{B_0F_0^2}{2\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right) \tag{3.3.7}$$

The χPT Lagrangian is here enlarged by the Wess-Zumino term $\mathcal{L}_{WZ}$

$$\mathcal{L}^{(4)}_{\chi PT} = iL_{11}\langle\chi_-\left(D_\mu u^\mu + \frac{i}{2}\chi_-\right)\rangle - L_{12}\langle\left(D_\mu u^\mu + \frac{i}{2}\chi_-\right)^2\rangle, \tag{3.3.8}$$

$$L^{(4)}_{WZ} = -\frac{\sqrt{2}N_C}{8\pi^2F}d^{abc}\epsilon_{\mu\nu\alpha\beta}\langle\Phi\partial^\mu v^\nu\partial^\alpha v^\beta\rangle, \tag{3.3.9}$$

$$\mathcal{L}^{(6)}_{\chi PT} = ic^W_7\epsilon_{\mu\nu\rho\sigma}\langle\chi_- f^{\mu\nu}_+ f^{\rho\sigma}_+\rangle + c^W_{22}\epsilon_{\mu\nu\rho\sigma}\langle u^\mu\{D_\gamma f^{\gamma\nu}_+, f^{\rho\sigma}_+\}\rangle. \tag{3.3.10}$$

(3.3.11)

The formfactor for χPT yields

$$\mathcal{F}^\chi_{VVP}(p^2,q^2,r^2) = 32B_0c^W_7 - \frac{8B_0c^W_{22}(p^2+q^2)}{r^2} - \frac{N_CB_0}{8\pi^2r^2}\left(1-\frac{4(L_{11}-L_{12})r^2}{F^2}\right) \tag{3.3.12}$$

In the low energy limit we have

$$\mathcal{F}^\chi_{VVP}((\epsilon p)^2,(\epsilon q)^2,(\epsilon r)^2) = -\frac{1}{\epsilon^2}\frac{N_CB_0}{8\pi^2r^2} + 32B_0c^W_7 \tag{3.3.13}$$
$$-\frac{8B_0c^W_{22}(p^2+q^2)}{r^2} + \frac{N_CB_0(L_{11}-L_{12})}{2\pi^2F^2} + \epsilon^2\frac{32B_0c^W_{22}(p^2+q^2)}{F^2} + \mathcal{O}(\epsilon^2)$$

and this relation will be useful in the matching it with the result calculated in RχT.

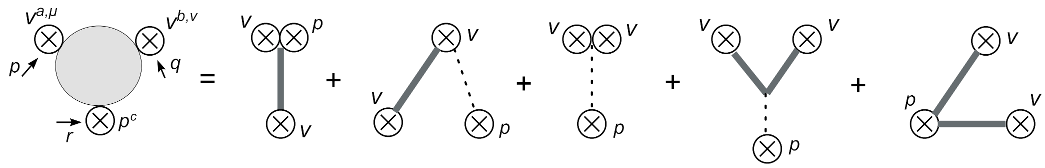


Figure 3.4: Diagram topologies contributing to VVP correlator.

Resonance contribution

Vector formalism

In the vector formalism we have the following $\mathcal{O}(p^6)$ resonance Lagrangians

$$\mathcal{L}_V^{(6)} = -\frac{f_V}{2\sqrt{2}}\langle \hat{V}_{\mu\nu} f_+^{\mu\nu}\rangle + h_V \epsilon_{\mu\nu\alpha\beta}\langle V^\mu\{u^\nu, f_+^{\alpha\beta}\rangle, \tag{3.3.14}$$

$$\mathcal{L}_V^{(8)} = \frac{1}{2}\sigma_V \epsilon_{\alpha\beta\mu\nu}\langle\{\hat{V}^{\mu\nu}, V^\alpha\}u^\beta\rangle \tag{3.3.15}$$

where we have explicitly introduced also one of the contributing $\mathcal{O}(p^8)$ terms that has the analogue in the $\mathcal{O}(p^6)$ term in the first order formalism. The formfactor then reads

$$\mathcal{F}_{VVP}^V(p^2,q^2,r^2) = \frac{B_0}{r^2}\left\{-\frac{2\sigma_V f_V^2 p^2 q^2}{(p^2-M^2)(q^2-M^2)} + \frac{4\sqrt{2}p^2 h_V f_V}{(p^2-M^2)}\right\} + 32B_0\tilde{c}_7^W - \frac{8\tilde{c}_{22}^W B_0(p^2+q^2)}{r^2} - \frac{N_C B_0}{16\pi^2 r^2}\left(1-\frac{4(\tilde{L}_{11}-\tilde{L}_{12})r^2}{F^2}\right) + (p\leftrightarrow q) \tag{3.3.16}$$

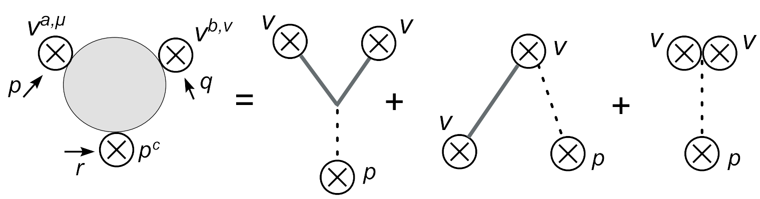


Figure 3.5: Diagrams in vector formalism.

In the high energy region we have

$$\begin{aligned}
&\lim_{\lambda\to\infty}\mathcal{F}_{VVP}^V((\lambda p)^2,(\lambda q)^2,(\lambda r)^2) = \\
&\quad \lambda^2\frac{32B_0(\tilde{L}_{11}-\tilde{L}_{12})\tilde{c}_{22}^W(p^2+q^2)}{F^2} + \frac{B_0 N_C(\tilde{L}_{11}-\tilde{L}_{12})}{2\pi^2F^2} \\
&\quad + 32\tilde{c}_7^W B_0 - \frac{8B_0\tilde{c}_{22}^W(p^2+q^2)}{r^2} + \frac{B_0}{\lambda^2}\left(-\frac{4\sigma_V f_V^2}{r^2} + \frac{8\sqrt{2}h_V f_V}{r^2} - \frac{N_C}{8\pi^2 r^2}\right) \\
&\quad + \frac{B_0}{\lambda^4}\left(\frac{4f_V(\sqrt{2}h_V - f_V\sigma_V)(p^2+q^2)M^2}{p^2q^2r^2}\right) + \mathcal{O}\left(\frac{1}{\lambda^6}\right)
\end{aligned} \tag{3.3.17}$$

By setting some relations between the coupling constants we can satisfy the high energy constraints up to the order $1/\lambda^2$ but the condition proportional to $1/\lambda^4$ can be satisfied in no way. Therefore, the result in the vector formalism is not consistent with high energy constraints and no saturation of χPT LECs can be found.

Antisymmetric tensor formalism

The Lagrangian in the antisymmetric tensor formalism that contribute to the $\langle VVP\rangle$ correlator is much richer than in the vector case so there is a chance that some relations between coupling constants allow us to satisfy the high energy constraints.

$$\mathcal{L}_R^{(4)} = \frac{F_V}{2\sqrt{2}}\langle R_{\mu\nu} f_+^{\mu\nu}\rangle, \tag{3.3.18}$$

$$\begin{aligned}\mathcal{L}_R^{(6)} = {} & \frac{c_1}{M}\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\nu}, f_+^{\rho\alpha}\}D_\alpha u^\sigma\rangle + \frac{c_2}{M}\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\alpha}, f_+^{\rho\sigma}\}D_\alpha u^\nu\rangle \\ & + \frac{ic_3}{M}\epsilon_{\mu\nu\rho\sigma}\langle\{R^{\mu\nu}, f_+^{\rho\sigma}\chi_-\rangle + \frac{c_4}{M}\epsilon_{\mu\nu\rho\sigma}\langle R^{\mu\nu}[f_-^{\rho\sigma}, \chi_+]\rangle \\ & + \frac{c_5}{M}\epsilon_{\mu\nu\rho\sigma}\langle\{D_\alpha R^{\mu\nu}, f_+^{\rho\alpha}\}u^\sigma\rangle + \frac{c_6}{M}\epsilon_{\mu\nu\rho\sigma}\langle\{D_\alpha R^{\mu\alpha}, f_+^{\rho\sigma}\}u^\nu\rangle \\ & + \frac{c_7}{M}\epsilon_{\mu\nu\rho\sigma}\langle\{D^\sigma R^{\mu\nu}, f_+^{\rho\alpha}\}u_\alpha\rangle \\ & + i\lambda_{21}^V\langle R_{\mu\nu}D_\alpha D^\alpha(u^\mu u^\nu)\rangle + \lambda_{22}^V\langle R_{\mu\nu}D_\alpha D^\alpha f_+^{\mu\nu}\rangle\end{aligned} \tag{3.3.19}$$

The result in this formalism has the form

$$\mathcal{F}_{VVP}^R(p^2, q^2, r^2) = \tag{3.3.20}$$

$$\begin{aligned} & B_0\Bigg\{2(F_V - 2\sqrt{2}\lambda_{22}^V p^2)(F_V - 2\sqrt{2}\lambda_{22}^V q^2)\frac{(d_1 - d_3)r^2 + d_3(p^2+q^2)}{(p^2-M^2)(q^2-M^2)r^2} \\ & \quad + \frac{16 d_2(F_V - 2\sqrt{2}\lambda_{22}^V p^2)^2}{(p^2-M^2)(q^2-M^2)} + \frac{16\sqrt{2}c_3(F_V - 2\sqrt{2}\lambda_{22}^V p^2)}{M(p^2-M^2)} \\ & \quad - \frac{N_C}{16\pi^2 r^2}\left(1 - \frac{4(\tilde{L}_{11} - \tilde{L}_{12})r^2}{F^2}\right) + \frac{2\sqrt{2}}{M}(F_V - 2\sqrt{2}\lambda_{22}^V p^2) \times \\ & \times \frac{r^2(c_1 + c_2 - c_5) + p^2(-c_1 + c_2 + c_5 - 2c_6) + q^2(c_1 - c_2 + c_5)}{r^2(p^2-M^2)} \\ & + 32\tilde{c}_7^W - \frac{8\tilde{c}_{22}^W(p^2+q^2)}{r^2} - \frac{N_C}{16\pi^2 r^2}\left(1 - \frac{4(\tilde{L}_{11} - \tilde{L}_{12})r^2}{F^2}\right)\Bigg\} + (p \leftrightarrow q)\end{aligned} \tag{3.3.21}$$

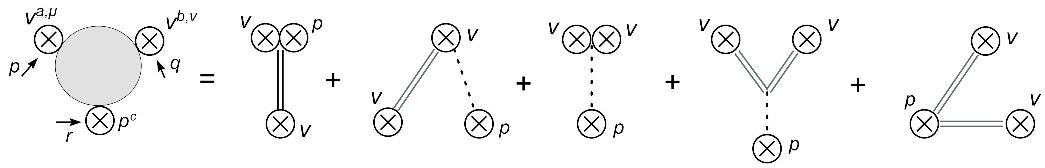


Figure 3.6: Diagrams in antisymmetric tensor formalism.

In the high energy region we have

$$
\begin{aligned}
&\lim_{\lambda\to\infty}\mathcal{F}^R_{VVP}((\lambda p)^2,(\lambda q)^2,(\lambda r)^2) = B_0 \times \Bigg\{ \\
&\lambda^2 \frac{32\tilde{c}^W_{22}(\tilde{L}_{11}-\tilde{L}_{12})}{F^2} \\
&+\frac{1}{\lambda^0}\Bigg[\frac{p^2+q^2}{r^2}\left(32d_3(\lambda^V_{22})^2+\frac{16(c_6-c_5)\lambda^V_{22}}{M}-8\tilde{c}^W_{22}\right)+32(\lambda^V_{22})^2(d_1+8d_2-d_3) \\
&\quad -\frac{16\lambda^V_{22}}{M}(c_1+c_2+8c_3-c_5)+32\tilde{c}^W_7+\frac{N_C(\tilde{L}_{11}-\tilde{L}_{12})}{2\pi^2F^2}\Bigg] \\
&+\frac{1}{\lambda^2}\Bigg[\frac{p^2+q^2}{p^2q^2}\Bigg\{2\sqrt{2}\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(c_1+c_2+8c_3-c_5) \\
&\quad -8\sqrt{2}M\lambda^V_{22}\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(d_1+8d_2-d_3)\Bigg\} \\
&\quad +\frac{p^4+q^4}{p^2q^2r^2}\Bigg\{2\sqrt{2}\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(c_1-c_2+c_5) \\
&\quad -8\sqrt{2}\lambda^V_{22}Md_3\left(\frac{F_V}{M}-2\sqrt{2}M\lambda^V_{22}\right)\Bigg\} \\
&\quad +\frac{1}{r^2}\Bigg\{-4\sqrt{2}\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(c_1-c_2-c_5+2c_6) \\
&\quad -16\sqrt{2}M\lambda^V_{22}d_3\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)-\frac{N_C}{8\pi^2}\Bigg\}\Bigg] \\
&+\frac{1}{\lambda^4}\Bigg[\frac{1}{p^2q^2}\Bigg\{4(F_V-2\sqrt{2}\lambda^V_{22}M^2)^2(d_1+8d_2-d_3)\Bigg\} \\
&\quad +\frac{p^2+q^2}{p^2q^2r^2}\Bigg\{4(F_V^2-6\sqrt{2}F_V\lambda^V_{22}M^2+16(\lambda^V_{22})^2M^4) \\
&\quad -2\sqrt{2}M^2\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(c_1-c_2-c_5+2c_6)\Bigg\} \\
&\quad +\frac{p^4+q^4}{p^4q^4}\Bigg\{2\sqrt{2}M^2\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(c_1+c_2+8c_3-c_5) \\
&\quad -8\sqrt{2}\lambda^V_{22}M^3\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(d_1+8d_2-d_3)\Bigg\} \\
&\quad +\frac{p^6+q^6}{p^4q^4r^2}\Bigg\{2\sqrt{2}M^2\left(\frac{F_V}{M}-2\sqrt{2}\lambda^V_{22}M\right)(c_1-c_2+c_5-4\lambda^V_{22}Md_3)\Bigg\}\Bigg]\Bigg\} \\
&+\mathcal{O}\left(\frac{1}{\lambda^6}\right) \qquad (3.3.22)
\end{aligned}
$$

To fulfill the high energy conditions we have to demand

$$c_1 = -4c_3 \tag{3.3.23}$$
$$c_2 = -4c_3 + c_5 \tag{3.3.24}$$
$$c_6 = c_5 - \frac{N_C M}{64\sqrt{2}\pi^2 F_V} \tag{3.3.25}$$
$$d_1 = -8d_2 - \frac{N_C M^2}{64\pi^2 F_V^2} + \frac{F^2}{4F_V^2} \tag{3.3.26}$$
$$d_3 = -\frac{N_C M^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2} \tag{3.3.27}$$
$$\lambda_{22}^V = 0 \tag{3.3.28}$$
$$\tilde{c}_7^W = -\frac{N_C(\tilde{L}_{11} - \tilde{L}_{12})}{64\pi^2 F^2} \tag{3.3.29}$$
$$\tilde{c}_{22}^W = 0. \tag{3.3.30}$$

If these conditions are satisfied, the result simplifies to

$$\mathcal{F}^R_{VVP}(p^2, q^2, r^2) = \frac{B_0 F^2}{2} \frac{p^2 + q^2 + r^2 - \frac{N_C}{4\pi^2}\frac{M^4}{F^2}}{(p^2 - M^2)(q^2 - M^2)r^2} \tag{3.3.31}$$

which coincides with the lowest meson dominance (LMD) approximation developed in [26, 9].

$$\begin{aligned}\mathcal{F}^R_{VVP}((\epsilon p)^2, (\epsilon q)^2, (\epsilon r)^2) &= -\frac{1}{\epsilon^2}\frac{B_0 N_C}{8\pi^2 r^2} + \frac{F^2}{2M^4} + \frac{p^2+q^2}{r^2}\left(\frac{F^2}{2M^4} - \frac{N_C}{8\pi^2 M^2}\right) \\ &+ \epsilon^2\left\{\frac{p^4+q^4}{r^2}\left(\frac{F^2}{2M^6} - \frac{N_C}{8\pi^2 M^4}\right) + \frac{p^2 q^2}{r^2}\left(\frac{F^2}{M^6} - \frac{N_C}{8\pi^2 M^4}\right) + (p^2+q^2)\frac{F^2}{2M^6}\right\} + \mathcal{O}(\epsilon^4)\end{aligned} \tag{3.3.32}$$

and comparing with the χPT prediction up to $\mathcal{O}(1)$ we finally get the relations

$$c_7^W = -\frac{N_C(L_{11} - L_{12})}{64\pi^2 F^2} + \frac{F^2}{64M^4}, \tag{3.3.33}$$
$$c_{22}^W = \frac{N_C}{64\pi^2 M^2} - \frac{F^2}{16M^4}. \tag{3.3.34}$$

The relation (3.3.33) confirms the well known fact that the $\mathcal{O}(p^4)$ constants L_{11} and L_{12} can be effectively included in the $\mathcal{O}(p^6)$ constant c_7^W. The reason is that the corresponding operators are proportional to the classical $\mathcal{O}(p^2)$ equations of motion and can be removed by means of the field redefinition.

First order formalism

The Lagrangian in the first order formalism up to $\mathcal{O}(p^6)$ is identical with the sum of Lagrangians from the vector and the antisymmetric tensor formalisms (up to $\mathcal{O}(p^6)$). Moreover, we have one mixing term that is analogous of the $\mathcal{O}(p^8)$ interaction term in the Proca field Lagrangian that was already mentioned. So we can write

$$\mathcal{L}_{RV} = \mathcal{L}_R^{(4)} + \mathcal{L}_R^{(6)} + \mathcal{L}_V^{(6)} + \frac{1}{2} M \sigma_V \epsilon_{\alpha\beta\mu\nu} \langle \{V^\alpha, R^{\mu\nu}\} u^\beta \rangle \tag{3.3.35}$$

The result for the formfactor then reads

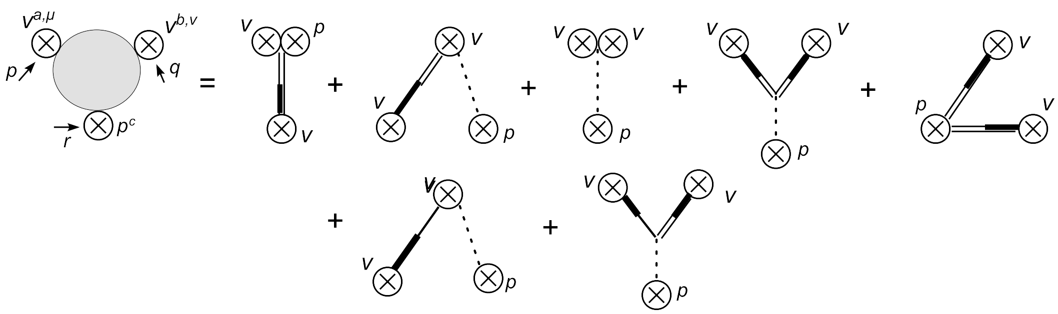


Figure 3.7: Diagrams in first order formalism. The thick line represents the sum of double line (antisymmetric tensor fields) and thin line (vector fields).

$$\begin{aligned}
\mathcal{F}^{RV}_{VVP}(p^2,q^2,r^2) = B_0 \Bigg\{ &+ 32 B_0 \tilde{c}_7^W - \frac{8B_0 \tilde{c}_{22}^W (p^2+q^2)}{r^2} \\
&- \frac{N_C}{16\pi^2 r^2}\left(1 - \frac{4(\tilde{L}_{11}-\tilde{L}_{12})r^2}{F^2}\right) + \frac{4\sqrt{2}h_V p^2}{(p^2-M^2)r^2}\left(f_V - \frac{F_V}{M} + 2\sqrt{2}\lambda_{22}^V \frac{p^2}{M}\right) \\
&+ 2F_V(p^2)F_V(q^2)\frac{(d_1+8d_2-d_3)r^2 + 2d_3 p^2}{(p^2-M^2)(q^2-M^2)r^2} \\
&+ \frac{2\sqrt{2}F_V(p^2)}{M}\frac{(c_1+c_2+8c_3-c_5)r^2 + (-c_1+c_2+c_5-2c_6)p^2 + (c_1-c_2+c_5)q^2}{(p^2-M^2)r^2} \\
&+ \frac{2\sigma_V M F_V(q^2)p^2}{(p^2-M^2)(q^2-M^2)r^2}\left(f_V - \frac{F_V}{M} + 2\sqrt{2}\lambda_{22}^V\frac{p^2}{M}\right)\Bigg\} + (p \leftrightarrow q).
\end{aligned} \tag{3.3.36}$$

where

$$F_V(q^2) = F_V - f_V \frac{q^2}{M} - 2\sqrt{2}\lambda_{22}^V q^2 \tag{3.3.37}$$

In the high energy region we have

$$
\begin{aligned}
&\lim_{\lambda\to\infty}\mathcal{F}^{RV}_{VVP}((\lambda p)^2,(\lambda q)^2,(\lambda r)^2) = B_0\times\\
&\quad\frac{1}{\lambda^0}\left[\frac{p^2+q^2}{r^2}\left(\frac{4d_3 f_V}{M^2}(f_V+4\sqrt{2}M\lambda^V_{22})+32(\lambda^V_{22})^2 d_3+\frac{16h_V\lambda^V_{22}}{M}\right.\right.\\
&\qquad\left.+\frac{4\sqrt{2}}{M^2}(c_6-c_5)(f_V+2\sqrt{2}\lambda^V_{22}M)-\frac{2\sqrt{2}\lambda^V_{22}\sigma_V}{M}(f_V+2\sqrt{2}\lambda^V_{22}M)\right)\\
&\qquad+\frac{4f_V}{M^2}(f_V+4\sqrt{2}\lambda^V_{22}M)(d_1+8d_2-d_3)+32(\lambda^V_{22})^2(d_1+8d_2-d_3)\\
&\qquad\left.-\frac{4\sqrt{2}}{M^2}(f_V+2\sqrt{2}\lambda^V_{22}M)(c_1+c_2+8c_3-c_5)+\frac{N_C(\tilde{L}_{11}-\tilde{L}_{12})}{2\pi^2F^2}\right]\\
&\quad+\frac{1}{\lambda^2}\left[\frac{p^2+q^2}{p^2q^2}\left\{4\left(f_V^2+4\sqrt{2}Mf_V\lambda^V_{22}-\frac{F_Vf_V}{M}+8(\lambda^V_{22})^2M^2-2\sqrt{2}F_V\lambda^V_{22}\right)\times\right.\right.\\
&\qquad\left.\times(d_1+8d_2-d_3)+\frac{2\sqrt{2}}{M}(F_V-f_VM-2\sqrt{2}\lambda^V_{22}M^2)(c_1+c_2+8c_3-c_5)\right\}\\
&\quad+\frac{p^4+q^4}{p^2q^2r^2}\left\{\frac{2\sqrt{2}}{M}(F_V-f_VM-2\sqrt{2}\lambda^V_{22}M^2)(c_1-c_2+c_5)\right.\\
&\qquad\qquad+4\sqrt{2}\lambda^V_{22}(F_V-f_VM-2\sqrt{2}\lambda^V_{22}M^2)\sigma_V\\
&\quad\left.+4d_3\left(8(\lambda^V_{22})^2M^3+4\sqrt{2}f_VL\lambda^V_{22}M^2+f_V^2M-2\sqrt{2}F_V\lambda^V_{22}M-f_VF_V\right)\right\}\\
&\quad+\frac{1}{r^2}\left\{-\frac{N_C}{8\pi^2}+32h_VM\lambda^V_{22}-32\sigma_VM^2(\lambda^V_{22})^2-4f_V^2\sigma_V+\frac{4f_VF_V\sigma_V}{M}\right.\\
&\qquad\qquad-\frac{8\sqrt{2}h_V}{M}(F_V-Mf_V)+8\sqrt{2}\lambda^V_{22}\sigma_V(F_V-2Mf_V)\\
&\qquad\qquad-\frac{4\sqrt{2}}{M}(F_V-f_VM-2\sqrt{2}M^2\lambda^V_{22})(c_1-c_2-c_5+2c_6)\\
&\quad\left.\left.+\frac{8d_3}{M}(8(\lambda^V_{22})^2M^3+4\sqrt{2}f_V\lambda^V_{22}M^2+f_V^2M-2\sqrt{2}F_V\lambda^V_{22}M-f_VF_V)\right\}\right]\\
&\quad+\frac{1}{\lambda^4}\left[\frac{1}{p^2q^2}\left\{4(d_1+8d_2-d_3)\left(F_V-M(f_V+2\sqrt{2}\lambda^V_{22}M)\right)^2\right\}\right.\\
&\quad+\frac{p^2+q^2}{p^2q^2r^2}\left\{-2\sqrt{2}M(c_1+c_2-c_5+2c_6+2h_V)(F_V-Mf_V-2\sqrt{2}\lambda^V_{22}M^2)\right.\\
&\qquad\qquad-2(\sigma_V-2d_3)\left(16(\lambda^V_{22})^2M^4+8\sqrt{2}f_V\lambda^V_{22}M^3+2f_V^2M^2\right.\\
&\qquad\qquad\qquad\left.\left.-6\sqrt{2}F_V\lambda^V_{22}M^2-3f_VF_VM+F_V^2\right)\right\}\\
&\quad+\frac{p^4+q^4}{p^4q^4}\left\{4M(d_1+8d_2-d_3)\left(8(\lambda^V_{22})^2M^3+4\sqrt{2}f_v\lambda^V_{22}M^2+f_V^2M\right.\right.\\
&\qquad\qquad\qquad\left.\left.-2\sqrt{2}F_V\lambda^V_{22}M-f_VF_V\right)\right\}\\
&\quad+\frac{p^6+q^6}{p^4q^4r^2}\left\{2\sqrt{2}M(c_1-c_2+c_5)(F_V-f_VM-2\sqrt{2}\lambda^V_{22}M^2)-4f_VMd_3(F_V-f_VM)\right.\\
&\qquad\left.\left.+4\sqrt{2}\lambda^V_{22}M^2(F_V-f_VM-2\sqrt{2}\lambda^V_{22}M^2)(\sigma_V-2d_3)\right\}\right]+\mathcal{O}\left(\frac{1}{\lambda^6}\right)
\end{aligned}
\tag{3.3.38}
$$

Now the situation is little bit different from the situation discussed in [A] where the coupling f_V had to be zero in order to satisfy the high energy constraints. In this calculation we have taken into account the complete $\mathcal{O}(p^6)$ Lagrangian and it is possible to preserve the nonzero value of this constant. Moreover, on the contrary to the antisymmetric tensor formalism we do not have to set the λ_{22}^V constant to zero, it is in some sense compensated by the constant f_V. Finally, we obtain

$$\lambda_{22}^V = -\frac{f_V}{2\sqrt{2}M} \tag{3.3.39}$$

$$c_1 = -4c_3 \tag{3.3.40}$$

$$c_2 = -4c_3 + c_5 \tag{3.3.41}$$

$$c_6 = c_5 - h_V - \frac{N_C M}{64\sqrt{2}\pi^2 F_V} \tag{3.3.42}$$

$$d_1 = -8d_2 + \frac{\sigma_V}{2} - \frac{N_C m^2}{64\pi^2 F_V^2} + \frac{F^2}{4F_V^2} \tag{3.3.43}$$

$$d_3 = \frac{\sigma_V}{2} - \frac{N_C m^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2} \tag{3.3.44}$$

$$\tilde{c}_7^W = -\frac{N_C(\tilde{L}_{11} - \tilde{L}_{12})}{64\pi^2 F^2} \tag{3.3.45}$$

$$\tilde{c}_{22}^W = 0. \tag{3.3.46}$$

Applying these constraints on the result we find the same relations of LEC as in the antisymmetric tensor formalism.

In the first order formalism, the bad high energy behavior connected with the vector formalism (f_V) is canceled by the similar unsuitable behavior from the antisymmetric tensor formalism (λ_{22}^V).

3.4 Summary of the chapter

First, we have briefly reminded the general properties of Green functions and the operator product expansion.

Then we have done the explicit calculation of $\langle PP \rangle$ and $\langle VV \rangle$ correlators, we have applied the high energy constraints and we have found the saturation of the $\mathcal{O}(p^6)$ LEC c_{93}. The study of the vector formfactor was already briefly mentioned in [B]. Here it provides us with the interesting relations between the constant F and $\mathcal{O}(p^4)$ LEC L_9 and the difference $c_{90} - c_{88}$ of $\mathcal{O}(p^6)$ constants.

More tricky example of the three point Green function is $\langle VVP \rangle$ correlator that was already discussed in [11, 9] and [A]. We have concluded that the high energy constraints cannot be satisfied in the vector formalism up to $\mathcal{O}(p^6)$ there is an inconsistency with the high energy behavior dictated by OPE. In the antisymmetric tensor formalism, we have repeated the calculation done in [11] and [A] with the more general Lagrangian containing all contributing $\mathcal{O}(p^6)$ terms but we have found that this extra term (with coupling λ_{22}^V) did not survive the restriction from the high energy constraints. However, we have found the relations for the $\mathcal{O}(p^6)$ LECs c_{22}^W and c_7^W. Some interesting facts have been observed in the first order formalism. The result presented in [A] required to set $f_V = 0$ in order to preserve the compatibility with OPE but in this more general case we can preserve f_V nonvanishing and we find the relation between this coupling from the vector formalism and $\mathcal{O}(p^6)$ coupling λ_{22}^V from the antisymmetric tensor formalism.

As a result, we can sum up this chapter: we have found the case where the compatibility of the vector field formalism with the high energy constraints is not preserved and we have also seen the nontrivial implication of the first order formalism.

CHAPTER 4

Compton-like scattering of Goldstone bosons

In this chapter we continue in studying processes within the framework of Resonance Chiral Theory. Now, we focus on the example of the four-point correlator $\langle VVPP \rangle$ that was already briefly mentioned in [27] but the detailed study is still missing. In the beginning we study the general properties of this correlator and its symmetries, then we restrict ourselves to the Compton-like process with the external legs of the pseudoscalar bosons which is the simplified version of the general case. Then we calculate the contribution of resonances to this process in the vector formalism up to $\mathcal{O}(p^6)$ and in the antisymmetric tensor formalism up to $\mathcal{O}(p^4)$. We also mention some aspects of the result in the antisymmetric tensor formalism up to $\mathcal{O}(p^6)$, but we skip the complete calculation because it is extremely long. The results will be then compared with the high energy constraints and some relations between coupling constants will be found.

4.1 Motivation

This chapter is motivated by the conjecture in [27] that for π^0 Compton scattering the antisymmetric tensor formalism violates the Froissart bound while the vector formalism preserves it. In this paper it was taken into account just one interaction term with the coupling h_V in the vector formalism and the analogous term in the tensor formalism (one of the terms with the coupling c_i).

4.2 Definition

The $\langle VVPP\rangle$ correlator is the four point Green function, its definition in momentum representation has the form

$$\begin{aligned} G^{abcd}_{\mu\nu}(p,q,r;s) &= \langle 0 \mid T\widetilde{V}^a_\mu(p)\widetilde{V}^b_\nu(-q)\widetilde{P}^c(r)P^d(0)\mid 0\rangle \\ &= \int \mathrm{d}^4x\mathrm{d}^4y\mathrm{d}^4z \mathrm{e}^{\mathrm{i}p\cdot x-\mathrm{i}q\cdot y+\mathrm{i}r\cdot z}\langle 0\mid TV^a_\mu(x)V^b_\nu(y)P^c(z)P^d(0)\mid 0\rangle \end{aligned} \tag{4.2.1}$$

where the conservation of momenta indicates $s = p + r - q$. The on-shell matrix element of the Compton-like process in the chiral limit can be written as

$$\begin{aligned} A^{abcd}_{\mu\nu}(p,q,r;s) &= \langle \phi^c(r)\mid T\widetilde{V}^a_\mu(p)V^b_\nu(0)\mid \phi^d(s)\rangle \\ &= -\lim_{r^2,s^2\to 0} r^2s^2F_0^{-2}B_0^{-2}\langle 0\mid T\widetilde{V}^a_\mu(p)\widetilde{V}^b_\nu(-q)\widetilde{P}^c(r)P^d(0)\mid 0\rangle \end{aligned} \tag{4.2.2}$$

where the pseudoscalar density satisfies

$$\langle 0|P^a(0)|\phi^b(s)\rangle = F_0B_0\delta^{ab} \tag{4.2.3}$$

and $|\phi^a(p)\rangle$ represents the Goldstone boson state. The amplitude of the Compton-like scattering of the Goldstone bosons is then

$$i\mathcal{M}^{abcd}_{\lambda\kappa}(p,q,r;s) = \lim_{p^2,q^2\to 0}\varepsilon^{*\mu}(p,\lambda)\varepsilon^{\nu}(q,\kappa)A^{abcd}_{\mu\nu}(p,q,r;s). \tag{4.2.4}$$

4.3 Symmetry properties

Now, we discuss the symmetry properties of $G^{abcd}_{\mu\nu}(p,q,r,s)$ and $A^{abcd}_{\mu\nu}(p,q,r,s)$ including Bose symmetry, gauge symmetry and the Ward identities.

Bose symmetry and crossing symmetry

Bose symmetry for the correlator reflects the symmetries under the interchange of $V^a_\mu(p) \leftrightarrow V^b_\nu(-q)$ or $\phi^c(r)\leftrightarrow\phi^d(-s)$. Then the corresponding relations are

$$G^{abcd}_{\mu\nu}(p,q,r;s) = G^{abdc}_{\mu\nu}(p,q,-s;-r) = G^{badc}_{\nu\mu}(-q,-p,r;s) \tag{4.3.1}$$

Analogously, Bose symmetry and crossing give

$$A^{abcd}_{\mu\nu}(p,q,r;s) = A^{abdc}_{\mu\nu}(p,q,-s;-r) = A^{bacd}_{\nu\mu}(-q,-p,r;s). \tag{4.3.2}$$

$SU(3)$ symmetry

Resonance Chiral Theory is $SU(3)$ invariant so all the correlators must show this symmetry. Let us therefore study the group structure of the correlator and find the basis of invariant group tensors. As it was shown in [D] the group structure is the linear combinations of the invariant tensors

$$\langle T^{\sigma(a)}T^{\sigma(b)}\rangle\langle T^{\sigma(c)}T^{\sigma(d)}\rangle$$
$$\langle T^{\sigma(a)}T^{\sigma(b)}T^{\sigma(c)}T^{\sigma(d)}\rangle$$

where σ is some permutation. The detailed discussion can be found in [D] and in the appendix C. The basis of tensors is

$$T_1^{abcd} = \delta^{ab}\delta^{cd} \tag{4.3.3}$$
$$T_{2,3}^{abcd} = \delta^{ac}\delta^{bd} \pm \delta^{ad}\delta^{bc} \tag{4.3.4}$$
$$T_4^{abcd} = f^{abl}f^{cdl} \tag{4.3.5}$$
$$T_5^{abcd} = d^{abl}d^{cdl} \tag{4.3.6}$$

The correlators can then be expanded as

$$G_{\mu\nu}^{abcd}(p,q,r;s) = \sum_{i=1}^{5} G_{\mu\nu}(p,q,r;s)^{(i)}T_i^{abcd}$$
$$A_{\mu\nu}^{abcd}(p,q,r;s) = \sum_{i=1}^{5} A_{\mu\nu}(p,q,r;s)^{(i)}T_i^{abcd}$$

Ward identities

Let us now apply the Ward identities in the momentum representation on the general case of the four point correlator

$$\begin{aligned} -ip^{\mu}G_{\mu\nu}^{abcd}(p,q,r;s) = {} & \mathrm{i}f^{abl}\langle 0|T\widetilde{V}_{\nu}^{l}(p-q)P^c(r)P^d(0)|0\rangle \\ & +\mathrm{i}f^{acl}\langle 0|T\widetilde{V}_{\nu}^{b}(-q)\widetilde{P}^{l}(r+p)P^d(0)|0\rangle \\ & +\mathrm{i}f^{adl}\langle 0|T\widetilde{V}_{\nu}^{b}(-q)\widetilde{P}^{c}(r)P^d(0)|0\rangle \end{aligned} \tag{4.3.7}$$

Analogously for the Compton-like scattering we obtain

$$-ip^{\mu}A_{\mu\nu}^{abcd}(p,q,r;s) = \mathrm{i}f^{abl}\langle\phi^c(r)\mid V_{\nu}^{l}(0)\mid\phi^d(s)\rangle \tag{4.3.8}$$
$$iq^{\nu}A_{\mu\nu}^{abcd}(p,q,r;s) = \mathrm{i}f^{bal}\langle\phi^c(r)\mid V_{\mu}^{l}(0)\mid\phi^d(s)\rangle \tag{4.3.9}$$

The right hand side of these identities is nothing else than the vector formfactor with one Goldstone boson in the initial state and one in the final state. The structure of such a matrix element is

$$\langle\phi^c(r)|V_{\nu}^{l}(0)|\phi^d(s)\rangle = \mathrm{i}f^{cdl}(r+s)_{\nu}F_V((r-s)^2) \tag{4.3.10}$$

where $F_V(p^2)$ is related to the vector formfactor from chapter 3 as $F_V(p^2) = -\mathcal{F}(p^2)$. We see that only the term $A_{\mu\nu}(p,q,r;s)^{(4)}$ survives the contraction with the momentum p^{μ}. This means,

$$-ip^{\mu}A_{\mu\nu}(p,q,r;s)^{(i)} = 0 \quad for\ i \neq 4 \tag{4.3.11}$$
$$-ip^{\mu}A_{\mu\nu}(p,q,r;s)^{(4)} = -(r+s)_{\nu}F_V((r-s)^2) \tag{4.3.12}$$

and similarly for second momentum q^ν

$$iq^\nu A_{\mu\nu}(p,q,r;s)^{(i)} = 0 \quad for\ i \neq 4 \tag{4.3.13}$$
$$iq^\nu A_{\mu\nu}(p,q,r;s)^{(4)} = -(r+s)_\mu F_V((r-s)^2) \tag{4.3.14}$$

4.4 Lorentz structure

Let us now focus on the Lorentz structure of the result. The conservation of momenta has the form $p + r = q + s$. Let us take the momenta p, q and $k = r + s$ as independent. Generally, there are six independent invariants: p^2, q^2, r^2, s^2 and

$$S = (p+r)^2 = (q+s)^2 = \frac{1}{4}(k+p+q)^2 \tag{4.4.1}$$
$$T = (p-q)^2 = (r-s)^2 \tag{4.4.2}$$
$$U = (p-s)^2 = (q-r)^2 = \frac{1}{4}(k-p-q)^2 \tag{4.4.3}$$

with $S + T + U = p^2 + q^2 + r^2 + s^2$. For our purpose $r^2 = s^2 = 0$ (in the chiral limit) and the only independent invariants are now p^2, q^2, S, U. T can be expressed as $T = p^2 + q^2 - S - U$. The independent transverse structures satisfy $p^\mu L^i_{\mu\nu} = q^\nu L^i_{\mu\nu} = 0$

$$L^1_{\mu\nu} = q_\mu p_\nu - (p\cdot q)g_{\mu\nu} \tag{4.4.4}$$
$$L^2_{\mu\nu} = (q\cdot k)p_\nu k_\mu + (p\cdot k)q_\mu k_\nu - (q\cdot k)(p\cdot k)g_{\mu\nu} - (p\cdot q)k_\mu k_\nu \tag{4.4.5}$$
$$L^3_{\mu\nu} = q^2 p_\mu p_\nu + p^2 q_\mu q_\nu - p^2 q^2 g_{\mu\nu} - (p\cdot q)p_\mu q_\nu \tag{4.4.6}$$
$$L^4_{\mu\nu} = p^2(q\cdot k)q_\nu k_\mu + q^2(p\cdot k)p_\mu k_\nu - (q\cdot k)(p\cdot k)p_\mu q_\nu - p^2 q^2 k_\mu k_\nu \tag{4.4.7}$$
$$\begin{aligned} L^5_{\mu\nu} = {} & p^2q^2\left[(p\cdot k)q_\mu k_\nu - (q\cdot k)p_\nu k_\mu\right] + (q\cdot k)(p\cdot k)\left[q^2 p_\mu p_\nu - p^2 q_\mu q_\nu\right] \\ & +(p\cdot q)\left[p^2(q\cdot k)q_\nu k_\mu - q^2(p\cdot k)p_\mu k_\nu\right] \end{aligned} \tag{4.4.8}$$

General structure

We have already seen that the Ward identities for $k = r + s$ and $T = (r-s)^2$ have the form

$$-ip^\mu A^{abcd}_{\mu\nu} = -f^{abl}f^{cdl}k_\nu F_V(T). \tag{4.4.9}$$

The solution of the Ward identities can be divided into two parts. The first one vanishes after contracting with p^μ and is constructed from the transverse structure $L^i_{\mu\nu}$. The second one is responsible for the right hand side of 4.4.9. Finally, using also the relation $(p\cdot k) = (q\cdot k) = (S-U)/2$ we can write

$$\begin{aligned} A^{abcd}_{\mu\nu}(p,q,r;s) &= \sum_{i=1}^{5} A_{\mu\nu}(p,q,r;s)^i T^{abcd}_i - \mathrm{i}\frac{2F_V(T)}{S-U} f^{abl}f^{cdl}k_\mu k_\nu \\ &= \sum_{i=1}^{5}\sum_{j=1}^{5} A(p^2,q^2,S,U;T)^i_j T^{abcd}_i L^j_{\mu\nu} - \mathrm{i}\frac{2F_V(T)}{S-U} f^{abl}f^{cdl}k_\mu k_\nu \end{aligned} \tag{4.4.10}$$

where we have introduced the formfactors $A(p^2, q^2, S, U; T)^i_j$ that posses no Lorentz and group structure. Under crossing symmetry $\phi^c(r) \leftrightarrow \phi^d(s)$ we obtain

$$\begin{aligned} A^{abcd}_{\mu\nu}(p,q,r;s) &= A^{abdc}_{\mu\nu}(p,q,-s;-r) \\ &= \sum_{i=1}^{5}\sum_{j=1}^{5} A(p^2,q^2,U,S;T)^i_j T^{abdc}_i L^j_{\mu\nu}(p,q,-k) - \mathrm{i}\frac{2F_V(T)}{S-U} f^{abl} f^{cdl} k_\mu k_\nu \\ &= \sum_{i=1}^{5}\sum_{j=1}^{5} \varepsilon^{(i)} A(p^2,q^2,U,S;T)^i_j T^{abcd}_i L^j_{\mu\nu}(p,q,k) - \mathrm{i}\frac{2F_V(T)}{S-U} f^{abl} f^{cdl} k_\mu k_\nu . \end{aligned} \tag{4.4.11}$$

The resulting symmetry relations for the formfactors are

$$A(p^2,q^2,S,U;T)^i_j = \varepsilon^{(i)} A(p^2,q^2,U,S;T)^i_j \tag{4.4.12}$$

where $\varepsilon^{(1)} = \varepsilon^{(2)} = -\varepsilon^{(3)} = -\varepsilon^{(4)} = \varepsilon^{(5)} = 1$. Analogously, Bose symmetry $V^a_\mu(p) \leftrightarrow V^b_\nu(q)$ implies

$$\begin{aligned} A^{abcd}_{\mu\nu}(p,q,r;s) &= A^{bacd}_{\nu\mu}(-q,-p,r;s) \\ &= \sum_{i=1}^{5}\sum_{j=1}^{5} A(q^2,p^2,U,S;T)^i_j T^{bacd}_i L^j_{\nu\mu}(-q,-p,k) - \mathrm{i}\frac{2F_V(T)}{S-U} f^{abl} f^{cdl} k_\mu k_\nu \\ &= \sum_{i=1}^{5}\sum_{j=1}^{5} \varepsilon^{(i)} \eta_{(j)} A(q^2,p^2,U,S;T)^i_j T^{abcd}_i L^j_{\mu\nu}(p,q,k) - \mathrm{i}\frac{2F_V(T)}{S-U} f^{abl} f^{cdl} k_\mu k_\nu \end{aligned} \tag{4.4.13}$$

which means

$$A(p^2,q^2,S,U;T)^i_j = \varepsilon^{(i)} \eta_{(j)} A(q^2,p^2,U,S;T)^i_j \tag{4.4.14}$$

where $\eta_{(1)} = \eta_{(2)} = \eta_{(3)} = \eta_{(4)} = -\eta_{(5)} = 1$ while the crossing symmetry gives further

$$A(p^2,q^2,S,U;T)^i_j = \eta_{(j)} A(q^2,p^2,S,U;T)^i_j . \tag{4.4.15}$$

Helicity amplitudes

For further calculation it is useful to introduce the helicity amplitudes. The result derived in [D] reads

$$\begin{aligned} &\mathcal{M}^{abcd}_{\lambda\kappa}(p,q,r;s) = \\ &\quad \frac{1}{2} F^*_{\mu\nu}(p,\lambda) F^{\mu\nu}(q,\kappa) \mathcal{M}^{abcd}_1(S,U;T) - F^*_{\mu\nu}(p,\lambda) F^{\mu\rho}(q,\kappa) k^\nu k_\rho \mathcal{M}^{abcd}_2(S,U;T) \\ &\quad - \frac{2F_V(T)}{S-U} f^{abl} f^{cdl} \left(\varepsilon^*(p,\lambda)\cdot k\right)\left(\varepsilon(q,\kappa)\cdot k\right) \end{aligned} \tag{4.4.16}$$

where

$$F^{\mu\nu}(l,\sigma) = -\mathrm{i}\left(l^\mu \varepsilon^\nu(l,\sigma) - l^\nu \varepsilon^\mu(l,\sigma)\right) \tag{4.4.17}$$

$$\mathrm{i}\mathcal{M}^{abcd}_j(p^2,q^2,S,U;T) = \sum_{i=1}^{6} A(0,0,S,U;T)^i_j T^{abcd}_i . \tag{4.4.18}$$

Further for external on-shell legs we have $p^2 = q^2 = 0$ and, therefore we can write in CMS

$$\frac{1}{2}F^*_{\mu\nu}(p,\lambda)F^{\mu\nu}(q,\kappa) = -\frac{1}{2}\delta_{\lambda,-\kappa}T \tag{4.4.19}$$

$$-F^*_{\mu\nu}(p,\lambda)F^{\mu\rho}(q,\kappa)k^\nu k_\rho = -\delta_{\lambda\kappa}SU + \frac{1}{4}\delta_{\lambda,-\kappa}T^2 \tag{4.4.20}$$

$$(\varepsilon^*(p,\lambda)\cdot k)\,(\varepsilon(q,\kappa)\cdot k) = -\frac{1}{2}\lambda\kappa\frac{TU}{S} \tag{4.4.21}$$

The final result for the helicity amplitudes in CMS

$$\mathcal{M}^{abcd}_{\pm\pm}(p,q,r;s) = -\,SU\mathcal{M}^{abcd}_2(S,U;T) + TU\frac{F_V(T)}{S(S-U)}f^{abl}f^{cdl} \tag{4.4.22}$$

$$\mathcal{M}^{abcd}_{\pm\mp}(p,q,r;s) = -\,\frac{1}{2}T\mathcal{M}^{abcd}_1(S,U;T) + \frac{1}{4}T^2\mathcal{M}^{abcd}_2(S,U;T) - TU\frac{F_V(T)}{S(S-U)}f^{abl}f^{cdl} \tag{4.4.23}$$

These expressions will be very useful in the future when we will compare the calculated results with the high energy constraints.

4.5 The high energy constraints

The Ward identities and the other symmetry properties indicate the conditions that are satisfied automatically without any other constraints on the coupling constants. On the contrary, the high energy constraints are not intrinsically contained in the results and do not have to be satisfied in all cases. The compatibility with these constraints shows the right high energy behavior of the theory.

Operator product expansion

The OPE reflects the high energy behavior of QCD. At the leading order in α_s we can write for $p \to \lambda p$, $q \to \lambda q$, $\lambda \to \infty$ and r, s fixed

$$A^{abcd}_{\mu\nu}(\lambda p, \lambda p + r - s, r; s) = -\frac{\mathrm{i}}{\lambda}f^{abl}f^{cdl}[p_\mu k_\nu + p_\nu k_\mu - (p\cdot k)g_{\mu\nu}]\frac{F_V(T)}{p^2} + O\left(\frac{1}{\lambda^2},\alpha_s\right) \tag{4.5.1}$$

The more symmetric form can be obtained when introducing the kinematic quantities $k = r+s$, $\Delta = r - s$, $\Sigma = p + q$. Then we have for $\Sigma \to \lambda\Sigma$ and k, Δ fixed

$$A^{abcd}_{\mu\nu}\left(\frac{1}{2}(\lambda\Sigma-\Delta),\frac{1}{2}(\lambda\Sigma+\Delta),\frac{1}{2}(k+\Delta);\frac{1}{2}(k-\Delta)\right) = -\frac{2\mathrm{i}}{\lambda}f^{abl}f^{cdl}[\Sigma_\mu k_\nu + \Sigma_\nu k_\mu - (\Sigma\cdot k)g_{\mu\nu}]\frac{F_V(T)}{\Sigma^2} + O\left(\frac{1}{\lambda^2},\alpha_s\right) \tag{4.5.2}$$

The leading behavior for $L^i_{\mu\nu}$ in the large λ limit reads

$$
\begin{aligned}
L^1_{\mu\nu} &= \frac{\lambda^2}{4}\left(\Sigma_\mu\Sigma_\nu - \Sigma^2 g_{\mu\nu}\right) + O(\lambda) && (4.5.3)\\
L^2_{\mu\nu} &= \frac{\lambda^2}{4}\left[(k\cdot\Sigma)(\Sigma_\mu k_\nu + \Sigma_\nu k_\mu - (\Sigma\cdot k)g_{\mu\nu} - \Sigma^2 k_\mu k_\nu\right] + O(\lambda) && (4.5.4)\\
L^3_{\mu\nu} &= \frac{\lambda^4}{16}\Sigma^2\left(\Sigma_\mu\Sigma_\nu - \Sigma^2 g_{\mu\nu}\right) + O(\lambda^3) && (4.5.5)\\
L^4_{\mu\nu} &= -\frac{\lambda^4}{16}\left(\Sigma_\mu(k\cdot\Sigma) - \Sigma^2 k_\mu\right)\left(\Sigma_\nu(k\cdot\Sigma) - \Sigma^2 k_\nu\right) + O(\lambda^3) && (4.5.6)\\
L^5_{\mu\nu} &= \frac{\lambda^5}{32}(k\cdot\Sigma)\left[\Sigma^2(\Sigma^2 + (k\cdot\Sigma))(k_\mu\Delta_\nu + k_\nu\Delta_\mu)\right. && (4.5.7)\\
&\qquad \left. -\Sigma^2(\Sigma\cdot\Delta)(\Sigma_\mu k_\nu + \Sigma_\nu k_\mu) + 2(\Sigma\cdot\Delta)(k\cdot\Sigma)\Sigma_\mu\Sigma_\nu\right] + O(\lambda^4)
\end{aligned}
$$

The constraints on the level of formfactors $A(p^2, q^2, S, U; T)^i_j$ can be found in appendix C.

Froissart bound

The Froissart bound is a high energy condition for four particle processes. The application on the Compton-like scattering helicity amplitudes $\mathcal{M}^{abcd}_{\lambda\kappa}(p, q, r; s)$ is detailed discussed [D]. The results are

$$
\begin{aligned}
|\mathcal{M}^{abcd}_{\lambda\kappa}(S, -S; 0)| &\lesssim \text{const. } S \ln^2 S && (4.5.8)\\
|\mathcal{M}^{abcd}_{\lambda\kappa}(S, -S-T; T)| &< \text{const. } S \ln^{3/2} S \quad \text{for} \quad T \le 0 \quad \text{fixed} && (4.5.9)
\end{aligned}
$$

These results make the implications for the constraints for the formfactors $A(p^2, q^2, S, U, T)^i_j$

$$
\begin{aligned}
A(0,0,S,-S;0)^i_2 &\lesssim \text{const. } \frac{1}{S}\ln^2 S && (4.5.10)\\
A(0,0,S,-S-T;T)^i_2 &< \text{const.} \frac{1}{S}\ln^{3/2} S \;\; \text{for} \quad T \le 0 \text{ fixed} && (4.5.11)\\
A(0,0,S,-S-T;T)^i_1 &< \text{const. } S \ln^{3/2} S \quad \text{for} \quad T \le 0 \text{ fixed} && (4.5.12)
\end{aligned}
$$

This is the last note on the general properties of the Compton-like scattering process. Let us now focus on the concrete calculations of the formfactors.

4.6 χPT contribution

First, we do the calculation in pure χPT without resonances. Let us assume only the leading $\mathcal{O}(p^2)$ Lagrangian

$$
\mathcal{L}^{(2)}_\chi = \frac{F^2}{4}\langle u^\mu u_\mu + \chi_+\rangle. \qquad (4.6.1)
$$

There are only two possible Feynman diagrams that contribute.

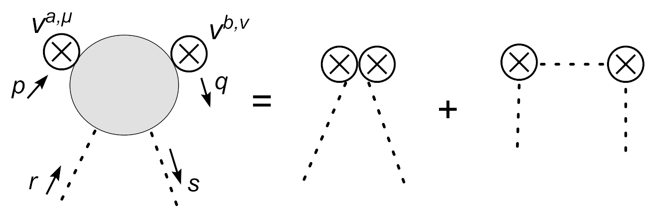


Figure 4.1: Contributing diagrams in pure χPT.

Helicity amplitudes

The results for the formfactors can be found in appendix. The amplitudes $\mathcal{M}_i^{abcd}(S,U;T)$ are then

$$\mathcal{M}_1^{abcd}(S,U;T) = \frac{(S+U)}{2SU}(T_1 - T_2 + 3T_5) + \frac{(U-S)}{2SU}T_4 \tag{4.6.2}$$

$$\mathcal{M}_2^{abcd}(S,U;T) = -\frac{1}{SU}(T_1 - T_2 + 3T_5) + \frac{(S+U)}{SU(S-U)}T_4 \tag{4.6.3}$$

In $\mathcal{O}(p^4)$ χPT we have $F_V(T) = -1$ and the helicity amplitudes are

$$\mathcal{M}_{\pm\pm}^{abcd}(S,U,T) = (T_1 - T_2 + 3T_5) + \frac{T}{S}T_4 \tag{4.6.4}$$

$$\mathcal{M}_{\pm\mp}^{abcd}(S,U,T) = -\frac{T}{S}T_4 \tag{4.6.5}$$

Constraints

Now we apply the Froissart bound on the previous results of the helicity amplitudes. Taking the limit $S \to \infty$, $T = konst$

$$\mathcal{M}_{\pm\pm}^{abcd}(S;T) = (T_1 - T_2 + 3T_5) + \mathcal{O}\left(\frac{1}{S}\right) \tag{4.6.6}$$

$$\mathcal{M}_{\pm\mp}^{abcd}(S;T) = -\frac{T}{S}T_4 + \mathcal{O}\left(\frac{1}{S^2}\right) \tag{4.6.7}$$

which means that the constraints coming from the Froissart bound are automatically satisfied. For $T = 0$ we have

$$\mathcal{M}_{\pm\pm}^{abcd}(S) = (T_1 - T_2 + 3T_5) + \mathcal{O}\left(\frac{1}{S}\right) \tag{4.6.8}$$

$$\mathcal{M}_{\pm\mp}^{abcd}(S) = 0 \tag{4.6.9}$$

and this result also satisfies given constraints.

We can also easily see that the high energy constraints coming from OPE and the results calculated in χPT up to $\mathcal{O}(p^2)$ (see appendix C) are not compatible.

4.7 Resonance contribution

In the following section we will study the contribution of vector resonances to the Compton-like scattering. For simplicity, in the following we neglect the contribution from $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ Goldstone boson Lagrangians because the corresponding coupling constants are usually assumed to be small at the resonance scale.

Topology of graphs with resonances

Before starting to do the calculations in the concrete formalism we first draw all possible diagrams with the vector resonance exchange that can appear. See figure 4.2.

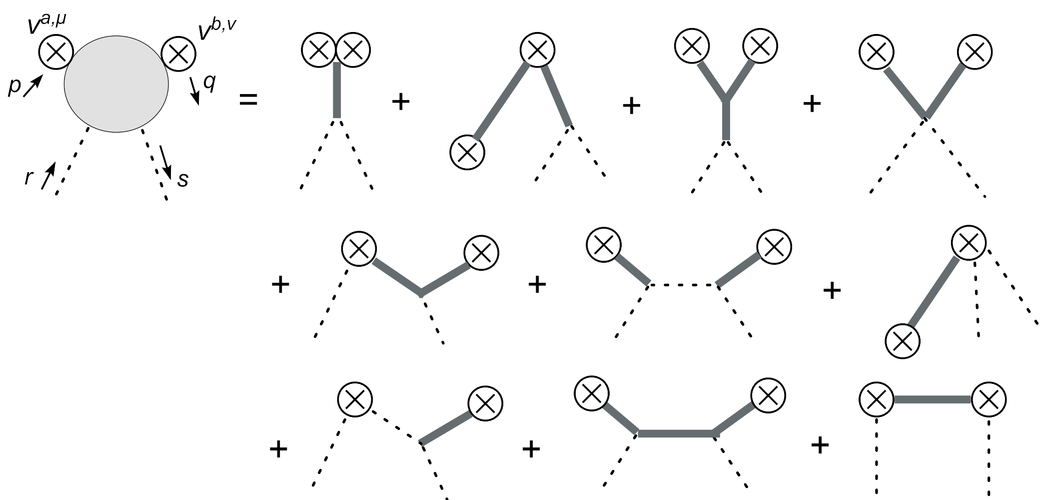


Figure 4.2: Diagram topologies contributing resonance exchanges of Compton pion scattering process.

Vector formalism

The interaction Lagrangian for vector resonances in the vector formalism up to $\mathcal{O}(p^6)$ that is important for this process has the form

$$\mathcal{L}_V^{(6)} = -\frac{f_V}{2\sqrt{2}}\langle \hat{V}^{\mu\nu} f_{+\mu\nu}\rangle - \frac{ig_V}{2\sqrt{2}}\langle \hat{V}^{\mu\nu}[u_\mu, u_\nu]\rangle + i\alpha_V \langle V^\mu [u^\nu, f_{-\mu\nu}]\rangle + \beta_V \langle V^\mu [u_\mu, \chi_-]\rangle + h_V \varepsilon_{\mu\nu\alpha\beta}\langle V^\mu \{u^\nu, f_+^{\alpha\beta}\}\rangle \tag{4.7.1}$$

There are only four possible Feynman diagrams that contribute (see figure 4.3.).

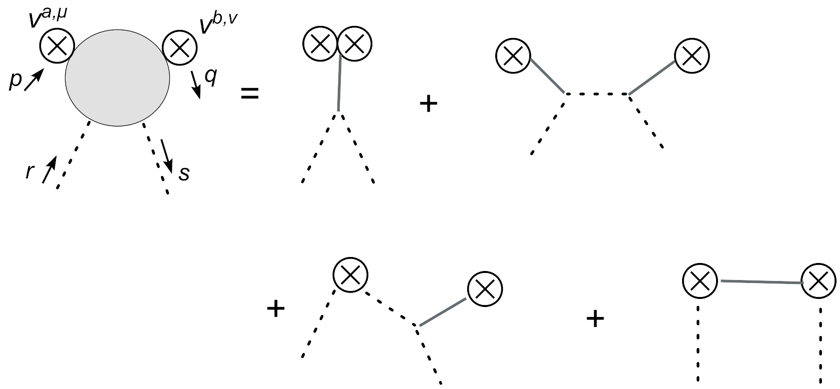


Figure 4.3: Contributing diagrams in the vector formalism (up to crossing).

Helicity amplitudes

For the vector formalism we have

$$F_V(T) = -1 + \frac{f_V g_V}{F^2}\frac{T^2}{T-M^2}.$$

The amplitudes calculated in this formalism read

$$\mathcal{M}_1^{abcd}(S,U;T) = \frac{(S+U)}{2SU}(T_1 - T_2 + 3T_5) + \frac{(U-S)}{2SU}T_4 - \frac{2h_V^2(S-U)(4M^2+S+U)}{3F^2(M^2-S)(M^2-U)}(2T_3 - 3T_4) \tag{4.7.2}$$

$$-\frac{2h_V^2(-6SM^2 - 6UM^2 + S^2 + U^2 + 10SU)}{3F^2(M^2-S)(M^2-U)}(T_1 + T_2 - 3T_5) \tag{4.7.3}$$

$$\mathcal{M}_2^{abcd}(S,U;T) = -\frac{1}{SU}(T_1 - T_2 + 3T_5) + \frac{(S+U)}{SU(S-U)}T_4 + \frac{4h_V^2(S+U-2M^2)}{3F^2(M^2-S)(M^2-U)}(T_1 + T_2 - 3T_5) \tag{4.7.4}$$

$$+\frac{4h_V^2(S-U)}{3F^2(M^2-S)(M^2-U)}(2T_3 - 3T_4) + \frac{4f_V g_V(S+U)}{F^2(S-U)(M^2-T)}T_4$$

The helicity amplitudes are

$$\mathcal{M}_{\pm\pm}^{abcd}(S,U,T) = -\frac{4h_V^2 SU(S+U-2M^2)}{3F^2(M^2-S)(M^2-U)}(T_1 + T_2 - 3T_5) \tag{4.7.5}$$

$$-\frac{4h_V^2 SU(S-U)}{3F^2(M^2-S)(M^2-U)}(2T_3 - 3T_4) + \frac{f_V g_V UT(2S-T)}{F^2 S(M^2-T)}T_4 + (T_1 - T_2 + 3T_5) + \frac{T}{S}T_4 \tag{4.7.6}$$

$$\mathcal{M}_{\pm\mp}^{abcd}(S,U,T) = -\frac{h_V^2 T\left[2M^2(2S+2U) - S^2 - S(T+9U)\right]}{3F^2(M^2-S)(M^2-U)}(T_1 + T_2 - 3T_5) \tag{4.7.7}$$

$$+\frac{4h_V^2 M^2 T(S-U)}{3F^2(M^2-S)(M^2-U)}(2T_3 - 3T_4) - \frac{f_V g_V T(S+U)}{F^2 S(T-M^2)}T_4 + \frac{T}{S}T_4 \tag{4.7.8}$$

High energy constraints

Taking the limit $S \to \infty$, $T = konst$

$$\mathcal{M}^{abcd}_{\pm\pm}(S;T) = -\left\{\frac{8h_V^2}{3F^2}(2T_3 - 3T_4) - \frac{2f_V g_V T}{F^2(T-M^2)}T_4\right\}S + \mathcal{O}(S^0) \tag{4.7.9}$$

$$\mathcal{M}^{abcd}_{\pm\mp}(S;T) = \frac{8h_V^2 T}{3F^2}(T_1 + T_2 - 3T_5) + \mathcal{O}\left(\frac{1}{S}\right) \tag{4.7.10}$$

For $T = 0$ we obtain

$$\mathcal{M}^{abcd}_{\pm\pm}(S) = -\left\{\frac{8h_V^2}{3F^2}(2T_3 - 3T_4)\right\}S + \mathcal{O}(S^0) \tag{4.7.11}$$

$$\mathcal{M}^{abcd}_{\pm\mp}(S) = \mathcal{O}\left(\frac{1}{S}\right) \tag{4.7.12}$$

The Froissart bound is satisfied automatically without any additional constraints on the coupling constants.

Application of the high energy constraints from OPE can be found in appendix C. Here we can see that the constraints are very strict and cannot be nontrivially satisfied (without including additional contact terms or other types of resonances).

Antisymmetric tensor formalism up to $\mathcal{O}(p^4)$

The interaction Lagrangian in the antisymmetric tensor formalism up to the order $\mathcal{O}(p^4)$ has the following form

$$\mathcal{L}^{(4)}_R = -\frac{F_V}{2\sqrt{2}}\langle R^{\mu\nu} f_{+\mu\nu}\rangle - \frac{iG_V}{2\sqrt{2}}\langle R^{\mu\nu}[u_\mu, u_\nu]\rangle \tag{4.7.13}$$

Helicity amplitudes

As was presented in chapter 3 the vector formfactor calculated in the antisymmetric tensor formalism is

$$F_V(T) = -1 + \frac{F_V G_V}{F^2}\frac{T}{T-M^2}. \tag{4.7.14}$$

The amplitudes in this formalism yield

$$\mathcal{M}^{abcd}_1(S,U;T) = -\frac{2F_V G_V}{F^2 M^2}(T_1 - T_2 + 3T_5) + \frac{(S+U)}{2SU}(T_1 - T_2 + 3T_5) + \frac{(U-S)}{2SU}T_4$$

$$\mathcal{M}^{abcd}_2(S,U;T) = -\frac{1}{SU}(T_1 - T_2 + 3T_5) + \frac{(S+U)}{SU(S-U)}T_4 - \frac{4F_V G_V}{F^2(M^2-T)(S-U)}T_4 \tag{4.7.15}$$

The helicity amplitudes are

$$\mathcal{M}^{abcd}_{\pm\pm}(S,U,T) = \frac{F_V G_V U(2S-T)}{F^2 S(M^2-T)}T_4 + (T_1 - T_2 + 3T_5) + \frac{T}{S}T_4 \tag{4.7.16}$$

$$\mathcal{M}^{abcd}_{\pm\mp}(S,U,T) = \frac{F_V G_V T}{F^2 M^2}(T_1 - T_2 + 3T_5) - \frac{F_V G_V T^2}{F^2 S(M^2-T)}T_4 - \frac{T}{S}T_4 \tag{4.7.17}$$

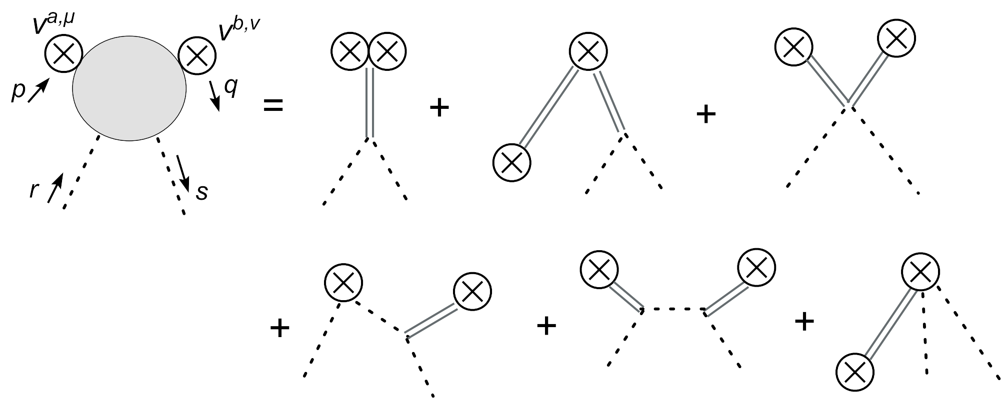


Figure 4.4: Contributing diagrams in antisymmetric tensor formalism (up to crossing). In comparison with the vector formalism there is no diagram with the resonance exchange in S and U channels.

High energy constraints

Taking the limit $S \to \infty$, $T = konst$ in the expressions for the helicity amplitudes we obtain

$$\mathcal{M}^{abcd}_{\pm\pm}(S;T) = -\left\{ \frac{2F_V G_V}{F^2(M^2 - T)} T_4 \right\} S + \mathcal{O}(S^0) \tag{4.7.18}$$

$$\mathcal{M}^{abcd}_{\pm\mp}(S;T) = \frac{F_V G_V T}{F^2 M^2}(T_1 - T_2 + 3T_5) + \mathcal{O}\left(\frac{1}{S}\right) \tag{4.7.19}$$

For $T = 0$ we obtain

$$\mathcal{M}^{abcd}_{\pm\pm}(S) = -\frac{2F_V G_V}{F^2 M^2} T_4 S + \mathcal{O}(S^0) \tag{4.7.20}$$

$$\mathcal{M}^{abcd}_{\pm\mp}(S) = 0 \tag{4.7.21}$$

As a result, Froissart bound is satisfied without any additional coupling constant constraints. If we add the analogue of the h_V term from the vector formalism (as it was proposed in [27]) we obtain the $\mathcal{O}(p^6)$ interaction term (with coupling c_6 which gives rise to the diagram with the resonance exchange in S and U channels). Including this term the Froissart bound is explicitly violated, as it will be seen in the next subsection.

The OPE high energy constraints cannot be again nontrivially satisfied (the proof can be found in appendix C).

Antisymmetric tensor formalism to $\mathcal{O}(p^6)$

The resonance Lagrangian in the antisymmetric tensor formalism up to $\mathcal{O}(p^6)$ is very rich, many terms contribute to this process and the complete expressions for the formfactors shall take plenty sheets of paper. Therefore, we will not investigate the high energy constraints coming from OPE. However, we know the result from $\mathcal{O}(p^4)$ case where these constraints are not nontrivially satisfied. The situation would not be better here and we hardly would obtain any nontrivial equation between coupling constants. Also the expressions for the helicity amplitudes are very long and we focus now only on the limits that can be used for applying the Froissart bound.

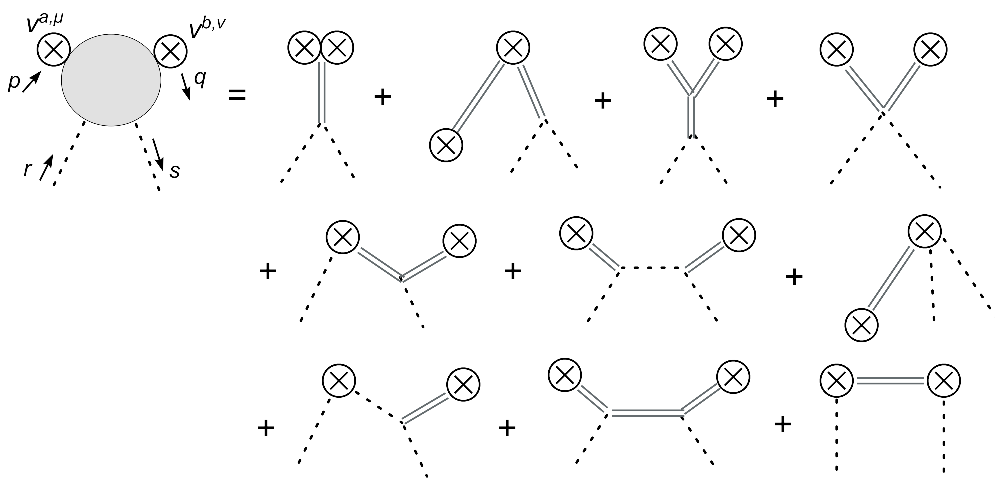


Figure 4.5: Contributing diagrams in antisymmetric tensor formalism.

Froissart bound

The helicity amplitudes in the limit $S \to \infty$ and $T = 0$ are

$$\begin{aligned}
\mathcal{M}^{abcd}_{\pm\pm}(S;T) = & \qquad\qquad (4.7.22) \\
& - \Bigg\{ \frac{\left[M^2(-c_1+c_2+c_5+2c_7)^2 + 2d_3(d_1+d_3)F_V^2\right]}{3F^2M^6}(T_1+T_2-3T_5) \\
& + \frac{\sqrt{2}F_V(-c_1+c_2+c_5+2c_7)d_4}{3F^2M^5}(T_1+T_2-6T_3+9T_4-3T_5)\Bigg\} S^3 \\
& - \Bigg\{ \frac{1}{6F^2M^4}\left[T(-c_1+c_2+c_5+2c_7)^2 - 2M^2(-c_1+c_2+c_5-2c_6)^2\right. \\
& \left. -d_3(d_1+d_3)F_V^2\right](T_1+T_2-3T_5) + \frac{T}{2F^2M^4}(-c_1+c_2+c_5+2c_7)^2(2T_3-3T_4) \\
& - \frac{3\sqrt{2}d_3F_V}{3F^2M^3}(c_1-c_2-c_5+2c_6)(T_1+T_2-3T_5) \\
& + \frac{\sqrt{2}F_V}{3F^2M^5}\left[(c_1-c_2-c_5+2c_6)M^2d_3 - 4(-c_1+c_2+c_5+2c_7)Td_4\right](2T_3-3T_4)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{F_V^2}{F^2M^4}(\lambda_3^{VV}+\lambda_4^{VV})(T_2-2T_5)+\frac{2F_V^2\lambda_5^{VV}}{3F^2M^4}(2T_1-3T_4+3T_5)\\
&-\frac{F_V}{3\sqrt{2}F^2M^2}\left[8\lambda_{13}-4\lambda_{14}-4\lambda_{15}-3(2\lambda_{16}+\lambda_{18}+4\lambda_{19})\right]T_1\\
&-\frac{F_V}{\sqrt{2}F^2M^2}(-4\lambda_{14}-4\lambda_{15}+2\lambda_{16}+\lambda_{18}+4\lambda_{19})T_2\\
&-\frac{F_V}{\sqrt{2}F^2M^2}\left[4\lambda_{13}+8\lambda_{14}+8\lambda_{15}-3(2\lambda_{16}+\lambda_{18}+4\lambda_{19})\right]T_5\\
&\left.+\frac{d_3(d_1+d_3)F_V^2T}{12F^2M^6}(2T_1+2T_2+6T_3-9T_4+6T_5)\right\}S^2+\mathcal{O}(S)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{\pm\mp}^{abcd}(S;T)= \qquad (4.7.23)\\
&-\left\{\frac{\left[2TM^2(-c_1+c_2+c_5+2c_7)^2-Td_3(d_1+d_3)F_V^2\right]}{6F^2M^6}(T_1+T_2-3T_5)\right.\\
&\left.-\frac{\sqrt{2}(-c_1+c_2+c_5+2c_7)Td_4F_V}{3F^2M^5}(3T_1+3T_2-2T_3+3T_4-9T_5)\right\}S^2\\
&-\left\{\frac{2T}{3F^2M^4}\left[M^2(-c_1+c_2+c_5-2c_6)^2+T(-c_1+c_2+c_5+2c_7)^2\right](2T_3-3T_4)\right.\\
&\quad-\frac{\sqrt{2}TF_V}{3F^2M^5}\left[(c_1-c_2-c_5+2c_6)d_3M^2+2(-c_1+c_2+c_5+2c_7)Td_4\right](T_1+T_2-3T_5)\\
&\quad-\frac{\sqrt{2}TF_V}{3F^2M^5}\left[3(c_1-c_2-c_5+2c_6)M^2d_3-2(-c_1+c_2+c_5+2c_7)Td_4\right](2T_3-3T_4)\\
&\quad+\frac{T}{2F^2M^4}(G_V-\sqrt{2}T\lambda_{21})\left[8M^2F_V\lambda_7^{VV}+4\sqrt{2}M^4\lambda_7+3\sqrt{2}F_V^2\kappa_V\right]T_4
\end{aligned}
\qquad (4.7.24)
$$

$$
\begin{aligned}
&+\frac{F_VT}{\sqrt{2}F^2M^2}(4\lambda_{17}-\lambda_{18}+4\lambda_{21})T_4+\frac{Td_3(d_1+d_3)F_V^2}{24F^2M^6}(4M^2-T)(2T_3-3T_4)\\
&\left.-\frac{T^2d_3(d_1+d_3)F_V^2}{6F^2M^6}(T_1+T_2-3T_5)+\frac{F_V^2T}{3F^2M^4}(\lambda_3^{VV}-\lambda_4^{VV})T_1\right\}S+\mathcal{O}\left(S^0\right)
\end{aligned}
$$

For $T=0$ we obtain

$$
\begin{aligned}
&\mathcal{M}_{\pm\pm}^{abcd}(S)= \qquad (4.7.25)\\
&-\left\{\frac{\left[M^2(-c_1+c_2+c_5+2c_7)^2+2d_3(d_1+d_3)F_V^2\right]}{3F^2M^6}(T_1+T_2-3T_5)\right.\\
&\left.+\frac{\sqrt{2}F_V(-c_1+c_2+c_5+2c_7)d_4}{3F^2M^5}(T_1+T_2-6T_3+9T_4-3T_5)\right\}S^3
\end{aligned}
$$

$$
\begin{aligned}
- \Bigg\{ & - \frac{1}{3F^2M^2}(-c_1+c_2+c_5-2c_6)^2(T_1+T_2-3T_5) \\
& - \frac{\sqrt{2}d_3F_V}{3F^2M^3}(c_1-c_2-c_5+2c_6)(3T_1+3T_2-2T_3+3T_4-9T_5) \\
& + \frac{F_V^2}{F^2M^4}(\lambda_3^{VV}+\lambda_4^{VV})(T_2-2T_5) + \frac{2F_V^2\lambda_5^{VV}}{3F^2M^4}(2T_1-3T_4+3T_5) \\
& - \frac{F_V}{3\sqrt{2}F^2M^2}\left[8\lambda_{13}-4\lambda_{14}-4\lambda_{15}-3(2\lambda_{16}+\lambda_{18}+4\lambda_{19})\right]T_1 \\
& - \frac{F_V}{\sqrt{2}F^2M^2}(-4\lambda_{14}-4\lambda_{15}+2\lambda_{16}+\lambda_{18}+4\lambda_{19})T_2 \\
& - \frac{F_V}{\sqrt{2}F^2M^2}\left[4\lambda_{13}+8\lambda_{14}+8\lambda_{15}-3(2\lambda_{16}+\lambda_{18}+4\lambda_{19})\right]T_5 \\
& - \frac{d_3(d_1+d_3)F_V^2}{6F^2M^4}(T_1+T_2-3T_5)\Bigg\}S^2+\mathcal{O}(S)
\end{aligned}
$$

$$
\mathcal{M}^{abcd}_{\pm\mp}(S) = \mathcal{O}\left(S^0\right) \tag{4.7.26}
$$

In order to kill $\mathcal{M}_{\pm\pm} \approx S^3$ terms we have to demand

$$
-c_1+c_2+c_5+2c_7 = 0, \tag{4.7.27}
$$

$$
d_1+d_3 = 0. \tag{4.7.28}
$$

In order to kill $\mathcal{M}_{\pm\pm} \approx S^2$ terms we obtain the additional relations

$$
c_1-c_2-c_5+2c_6 = 0, \tag{4.7.29}
$$

$$
\lambda_5^{VV} = 0, \tag{4.7.30}
$$

$$
8\lambda_{13}-4\lambda_{14}-4\lambda_{15}-3(2\lambda_{16}+\lambda_{18}+4\lambda_{19}) - \frac{\sqrt{2}F_V}{M^2}(\lambda_3^{VV}+\lambda_4^{VV}) = 0, \tag{4.7.31}
$$

$$
-4\lambda_{14}-4\lambda_{15}+2\lambda_{16}+\lambda_{18}+4\lambda_{19}+\frac{2\sqrt{2}F_V}{M^2}(\lambda_3^{VV}+\lambda_4^{VV}) = 0. \tag{4.7.32}
$$

The constraints (4.7.27) - (4.7.29) are compatible with those found in the case of $\langle VVP\rangle$ correlator. The other relations have no analogue in the previous calculations.

4.8 Summary of the chapter

In this chapter, we have investigated the process of Compton-like scattering (the four point correlator $\langle VVPP\rangle$ with on-shell Goldstone bosons). In the beginning we have discussed the properties of this process including all symmetries and the general form of the formfactor and their transformation relations.

Next, we have turned to the concrete computation of Feynman diagrams in RχT. The results for the formfactors in the vector formalism up to $\mathcal{O}(p^6)$ and the antisymmetric tensor formalism up to $\mathcal{O}(p^4)$ can be found in the appendix C. Then we have computed the helicity amplitudes corresponding to these results, some fragments from the antisymmetric tensor formalism up to $\mathcal{O}(p^6)$ are also included. Then we have applied the OPE high energy constraints and the Froissart bound. It is shown in appendix C that none of the results could non-trivially satisfy the OPE constraints. The Froissart bound is automatically satisfied in the vector formalism and in the antisymmetric tensor formalism up to $\mathcal{O}(p^4)$, in the antisymmetric tensor formalism up to $\mathcal{O}(p^6)$ some additional constraints on coupling constants must be set.

In the end, we mention some possibilities how to satisfy the high energy constraints for the case of this process.

- We can add some $\mathcal{O}(p^6)$ local contact terms or higher order terms with vector resonances that save the high energy behavior.
- Of course, the result is incomplete because we take into account only vector resonances. Maybe if we would include all types of resonances then the result would be already compatible with the high energy constraints.

CHAPTER 5

Renormalization of propagators

In this chapter we are focusing on the calculating loops in Resonance Chiral Theory. The renormalization procedure can lead to the presence of special type of counterterms that are responsible for the propagation of additional degrees of freedom. We will see that this really happens in the antisymmetric tensor and in the first order formalisms, the vector formalism is free of this feature (probably because we restrict only up to $\mathcal{O}(p^6)$ Lagrangians).

First, we mention some basics of renormalization in RχPT and then we focus on the very interesting example of the resonance propagator. We will study in more details (than in chapter 2) the properties of the propagators in all three formalisms and we will find the reasons why the new degrees of freedom, which were frozen at the tree level, could appear after the renormalization. Finally, we will do the complete renormalization procedure and we will find the concrete forms of the counterterm couplings, i.e. the coefficients of the beta functions. This will explicitly show that the new terms responsible for the propagation of the new degrees of freedom are generated in the RχT.

5.1 Tools of renormalization procedure

Feynman integrals and counterterms

The detailed discussion of Feynman integrals is done in the appendix A. In the following we will be interested only in the infinite parts of the Feynman integrals, so for our purpose we can write

$$A_0(M^2) = \frac{M^2}{16\pi^2}\lambda_\infty \tag{5.1.1}$$

$$B_0(p^2, M_1^2, M_2^2) = -\frac{1}{16\pi^2}\lambda_\infty \tag{5.1.2}$$

with

$$\lambda_\infty = \frac{2\mu^{d-4}}{d-4} + \gamma_E - \ln 4\pi - 1 \tag{5.1.3}$$

where $\gamma_E = 0.577\ldots$ is the Euler constant and μ is the renormalization scale.

These infinities will be canceled by the contribution of counterterm Lagrangian

$$\mathcal{L}_{ct} = \sum_i A_i \mathcal{O}^i \tag{5.1.4}$$

where $\mathcal{O}^i$ are operators and the bare coupling constants A^i have the form

$$A_i = \Gamma_i \lambda_\infty + A_i^r(\mu) \tag{5.1.5}$$

and the finite part $A_i^r(\mu)$ renormalized at scale μ satisfies the renormalization group equation

$$\mu \frac{\partial}{\partial\mu} A_i^r(\mu) = -\Gamma_i. \tag{5.1.6}$$

In the following we will calculate the infinite parts of A_i, i.e. the constants Γ_i.

Power counting

There are more types of power countings which can be used for our purpose:

- Chiral powercounting: This is a very similar way to that used in the Chiral Perturbation Theory. The diagrams are hierarchized by the Weinberg formula

$$D = 2 + 2L + \sum_V (D_V - 2) \tag{5.1.7}$$

  where D_V is the chiral order of the vertex. The resonance fields are of order $R, V = \mathcal{O}(1)$ and their masses $M = \mathcal{O}(p)$. But we are not in the low energy region as in χPT and therefore, this expansion is not physically meaningful.

- Expansion in $1/N_C$: This is the theoretically well-founded solution based on the expansion in the underlying theory - large N_C QCD. The index of the vertex d_V (of order $\mathcal{O}(N_C^{d_V})$) is

$$d_V = 1 - \frac{n^{(V)}}{2} - O^{(V)} \tag{5.1.8}$$

  where $n^{(V)}$ is the number of mesons in the vertex and $O^{(V)}$ is the additional suppression ($O^{(V)} = 0$ at the leading order, $O^{(V)} = 1$ at subleading etc.). We classify the graphs and the counterterms using the parameter

$$d = \sum_V d_V = 1 - \frac{1}{2}E - L - \sum_V O^{(V)} \tag{5.1.9}$$

  However, there is a problem with the terms with higher derivatives which are not suppressed in this power counting.

- Combined expansion: We will use it in the following.

We introduce the parameter δ so that

$$\begin{aligned} p &= \delta^{1/2} \quad \rightarrow \quad \mathcal{O}(p^D) = \mathcal{O}(\delta^{D/2}) && (5.1.10) \\ \frac{1}{N_C} &= \delta \quad \rightarrow \quad \mathcal{O}(N_C^D) = \mathcal{O}(\delta^{-D}) && (5.1.11) \\ & && (5.1.12) \end{aligned}$$

The index of a given diagram is then

$$\Delta = \frac{1}{2}D - d. \tag{5.1.13}$$

Substituting for D and d we obtain the analogue of the Weinberg formula for the combined power counting

$$\Delta = 1 + L + \sum_V (\Delta^{(V)} - 1) \tag{5.1.14}$$

with

$$\Delta^{(V)} = \frac{1}{2}D_V - 1 + \frac{n^{(V)}}{2} + O^{(V)}. \tag{5.1.15}$$

In the calculation we will use all three power countings described in this subsection. The corresponding indices will be written under the diagrams. If we restrict ourselves with the calculations up to the given order Δ_{max} we have to include all the diagrams and counterterms with $\Delta \leq \Delta_{max}$.

5.2 Propagator in vector formalism

General properties

Let us start with a Lagrangian of Proca field that can be written in the form

$$\mathcal{L}_V = -\frac{1}{4}\langle \widehat{V}_{\mu\nu}\widehat{V}^{\mu\nu}\rangle + \frac{1}{2}M^2\langle V_\mu V^\mu\rangle + \mathcal{L}_{int} \tag{5.2.1}$$

and introduce the usual longitudinal and transverse projectors

$$P^L_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} \tag{5.2.2}$$

$$P^T_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}. \tag{5.2.3}$$

Without any additional assumption on the form and symmetries of the interaction part of the Lagrangian $\mathcal{L}_{int}$, we can expect the following general form of the complete two-point 1PI Green function

$$\Gamma^{(2)}_{\mu\nu}(p) = (M^2 - p^2 + \Sigma^T(p^2))P^T_{\mu\nu} + (M^2 + \Sigma^L(p^2))P^L_{\mu\nu} \tag{5.2.4}$$

which corresponds to the propagator

$$\Delta_{\mu\nu}(p) = -\frac{1}{p^2 - M^2 - \Sigma^T(p^2)} P^T_{\mu\nu} + \frac{1}{M^2 + \Sigma^L(p^2)} P^L_{\mu\nu}. \tag{5.2.5}$$

The poles of such a propagator are situated at $p^2 = M^2_{V,S}$ where M_V^2 is given by the solutions of

$$M_V^2 - M^2 - \Sigma^T(M_V^2) = 0, \tag{5.2.6}$$
$$M_S^2 + \Sigma^L(M_S^2) = 0. \tag{5.2.7}$$

Assume (5.2.6) is satisfied for $p^2 = M_V^2 > 0$, the poles of this type then correspond to spin-one one particle poles,

$$\begin{aligned} \Delta_{\mu\nu}(p) &= \frac{Z_V}{p^2 - M_V^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}\right) + O(1) \\ &= \frac{Z_V}{p^2 - M_V^2}\sum_\lambda \varepsilon_\mu^{(\lambda)}(p)\varepsilon_\nu^{(\lambda)*}(p) + O(1) \end{aligned} \tag{5.2.8}$$

where

$$Z_V = \frac{1}{1 - \Sigma'^T(M_V^2)} \tag{5.2.9}$$

and where $\varepsilon_\mu^{(\lambda)}(p)$ are the usual spin-one polarization vectors. The corresponding spin-one particle state $|p, \lambda, V\rangle$ couples to the Proca field as

$$\langle 0|V_\mu(0)|p, \lambda, V\rangle = |Z_V|^{1/2}\varepsilon_\mu^{(\lambda)}(p). \tag{5.2.10}$$

At least one of these states is expected to be perturbative in the sense that its mass and coupling to V_μ can be written as

$$M_V^2 = M^2 + \delta M_V^2 \tag{5.2.11}$$
$$Z_V = 1 + \delta Z_V, \tag{5.2.12}$$

where δM_V^2 and δZ_V are small corrections vanishing in the free field limit. In the same limit $\Sigma^T(p^2) = 0$ and the other possible solutions of (5.2.6) corresponding to the additional spin-one one particle poles decouple. There are also another type of possible poles given by (intrinsically nonperturbative) solutions of (5.2.7). Suppose that the last condition is satisfied by $p^2 = M_S^2 > 0$. Such a pole

$$\Delta_{\mu\nu}(p) = \frac{Z_S}{p^2 - M_S^2 + \mathrm{i}0}\frac{p_\mu p_\nu}{M_S^2} + O(1) \tag{5.2.13}$$

where

$$Z_S = \frac{1}{\Sigma'^L(M_S^2)} \tag{5.2.14}$$

corresponds to the spin-zero one particle state $|p, S\rangle$ which couples to V_μ as

$$\langle 0|V_\mu(0)|p, S\rangle = \mathrm{i}p_\mu\frac{|Z_S|^{1/2}}{M_S}. \tag{5.2.15}$$

For the free field case this scalar mode is frozen and does not propagate according to the special form of the Proca field Lagrangian. Therefore, in the limit of the vanishing interaction the extra scalar state decouples. Without any additional assumptions on the symmetries of the interaction Lagrangian we can therefore expect appearance of the additional propagating degrees of freedom.

We see that the scalar mode will propagate only in the case $\Sigma_L(p^2) \neq 0$ and when this formfactor has explicit momentum dependence. The original free field Lagrangian has strictly $\Sigma_L(p^2) = 0$ but the counterterms necessary to be included in the renormalization procedure can lead to the non-trivial momentum dependence.

It was shown in [E] that the propagating scalar degrees of freedom are either ghosts or tachyons. The detailed study of the interpretation of this phenomenon is still missing.

One loop contribution

The interaction Lagrangian in the vector formalism up to $\mathcal{O}(p^6)$ is for our purpose

$$\mathcal{L}_V = -\frac{ig_V}{2\sqrt{2}}\langle \hat{V}^{\mu\nu}[u_\mu, u_\nu]\rangle + \frac{1}{2}\sigma_V \varepsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha, \hat{V}^{\mu\nu}\}u^\beta\rangle \tag{5.2.16}$$

The possible diagrams that contribute to the renormalization of vector resonance can be seen in the picture 5.1. The infinite part of the result is then

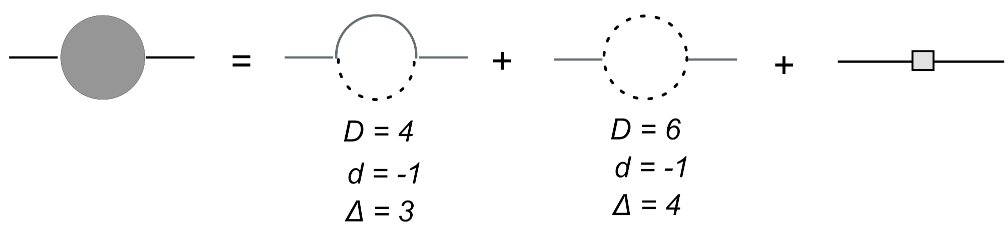


Figure 5.1: Vector propagator one loop diagrams in vector formalism.

$$\Sigma^T(p^2) = \frac{g_V^2 p^6 \lambda_\infty}{16\pi^2 F^4} + \frac{5\sigma_V^2 p^4 \lambda_\infty}{18\pi^2 F^2} - \frac{5M^2 p^2 \sigma_V^2 \lambda_\infty}{6\pi^2 F^2}, \tag{5.2.17}$$

$$\Sigma^L(p^2) = 0. \tag{5.2.18}$$

Restricting ourselves to $\Delta = 3$ $(D = 4)$ we obtain the complete result

$$\Sigma^T(p^2) = \frac{5\sigma_V^2 p^4 \lambda_\infty}{18\pi^2 F^2} - \frac{5M^2 p^2 \sigma_V^2 \lambda_\infty}{6\pi^2 F^2} \tag{5.2.19}$$

$$\Sigma^L(p^2) = 0 \tag{5.2.20}$$

Counterterms

The renormalization procedure requires to include the counterterms which kill the infinities in the results. The complete Lagrangian, which is necessary for the renormalization if we restrict ourselves to the order $\Delta = 3$, has the form

$$\mathcal{L}_V^{ct} = \frac{\delta M^2}{2}\langle V_\mu V^\mu\rangle \tag{5.2.21}$$

$$+ \frac{Z_V}{4}\langle \hat{V}_{\mu\nu}\hat{V}^{\mu\nu}\rangle + \frac{Y_V}{2}\langle (D_\mu V^\mu)^2\rangle \tag{5.2.22}$$

$$+\frac{X_{V1}}{4}\langle\{D_\alpha, D_\beta\}V_\mu\{D^\alpha, D^\beta\}V^\mu\rangle + \frac{X_{V2}}{4}\langle\{D_\alpha, D_\beta\}V_\mu\{D^\alpha, D^\mu\}V^\beta\rangle$$
$$+ \frac{X_{V4}}{2}\langle D^2 V_\mu\{D^\mu, D^\beta\}V_\beta\rangle + X_{V5}\langle D^2 V_\mu D^2 V^\mu\rangle$$
$$+ \frac{X_{V3}}{4}\langle\{D_\alpha, D_\beta\}V^\beta\{D^\alpha, D^\mu\}V_\mu\rangle \tag{5.2.23}$$

Only sums $X_V \equiv X_{V1} + X_{V5}$, $X'_V \equiv X_{V1} + X_{V2} + X_{V3} + X_{V4} + X_{V5}$ are relevant for our calculations. The counterterms contribute to the self-energies as

$$\Sigma_{ct}^T(p^2) = \delta M^2 + p^2 Z_V + p^4 X_V, \tag{5.2.24}$$
$$\Sigma_{ct}^L(p^2) = \delta M^2 - p^2 Y_V + p^4 X'_V. \tag{5.2.25}$$

Sum of the one loop contribution and the counterterms contribution must vanish

$$\Sigma^{T,L}(p^2) + \Sigma_{ct}^{T,L}(p^2) = 0 \tag{5.2.26}$$

that leads to the relations

$$\delta M^2 = (\delta M^2)^r(\mu), \tag{5.2.27}$$
$$Y_V = Y_V^r(\mu), \tag{5.2.28}$$
$$X'_V = X_V^{\prime r}(\mu), \tag{5.2.29}$$
$$Z_V = \frac{5M^2\sigma_V^2\lambda_\infty}{6\pi^2 F^2} + Z_V^r(\mu), \tag{5.2.30}$$
$$X_V = -\frac{5\sigma_V^2\lambda_\infty}{18\pi^2 F^2} + X_V^r(\mu). \tag{5.2.31}$$

We see that the infinite parts of Y_V and X'_V vanish. This means that we can fix the renormalized couplings $Y_V^r(\mu) = X_V^{\prime r}(\mu) = 0$ independently on the scale. Consequently, no additional degrees of freedom are generated in the vector formalism.

5.3 Propagator in antisymmetric tensor formalism

General properties

In this case the situation is quite analogous to the vector formalism. Let us write the Lagrangian in the form

$$\mathcal{L} = -\frac{1}{2}\langle(\partial_\mu R^{\mu\nu})(\partial^\rho R_{\rho\nu})\rangle + \frac{1}{4}M^2\langle R_{\mu\nu}R^{\mu\nu}\rangle + \mathcal{L}_{int}. \tag{5.3.1}$$

and introduce the projectors

$$\Pi^T_{\mu\nu\alpha\beta} = \frac{1}{2}\left(P^T_{\mu\alpha}P^T_{\nu\beta} - P^T_{\nu\alpha}P^T_{\mu\beta}\right) \tag{5.3.2}$$

$$\Pi^L_{\mu\nu\alpha\beta} = \frac{1}{2}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}\right) - \Pi^T_{\mu\nu\alpha\beta} \tag{5.3.3}$$

Again with completely general $\mathcal{L}_{int}$ we can assume the following general form of the complete two-point 1PI Green function

$$\Gamma^{(2)}_{\mu\nu\alpha\beta}(p) = \frac{1}{2}(M^2 + \Sigma^T(p^2))\Pi^T_{\mu\nu\alpha\beta} + \frac{1}{2}(M^2 - p^2 + \Sigma^L(p^2))\Pi^L_{\mu\nu\alpha\beta} \tag{5.3.4}$$

implying the propagator of the form

$$\Delta_{\mu\nu\alpha\beta}(p) = -\frac{2}{p^2 - M^2 - \Sigma^L(p^2)}\Pi^L_{\mu\nu\alpha\beta} + \frac{2}{M^2 + \Sigma^T(p^2)}\Pi^T_{\mu\nu\alpha\beta} \tag{5.3.5}$$

with the poles at $p^2 = M^2_{V,A}$ satisfying

$$M_V^2 - M^2 - \Sigma^L(M_V^2) = 0 \tag{5.3.6}$$

$$M^2 + \Sigma^T(M_A^2) = 0. \tag{5.3.7}$$

Assuming that the solution of (5.3.6) satisfies $M_V^2 > 0$, the propagator behaves at this pole according to

$$\begin{aligned}\Delta_{\mu\nu\alpha\beta}(p) &= \frac{Z_V}{p^2 - M_V^2}\frac{p_\mu g_{\nu\alpha}p_\beta - p_\nu g_{\mu\alpha}p_\beta - (\alpha \leftrightarrow \beta)}{M_V^2} + O(1) \\ &= \frac{Z_V}{p^2 - M_V^2}\sum_\lambda u^{(\lambda)}_{\mu\nu}(p)u^{(\lambda)}_{\alpha\beta}(p)^* + O(1)\end{aligned} \tag{5.3.8}$$

where

$$Z_V = \frac{1}{1 - \Sigma'^L(M_V^2)} \tag{5.3.9}$$

and the wave function $u^{(\lambda)}_{\mu\nu}(p)$ is expressed in terms of the spin-one polarization vectors as

$$u^{(\lambda)}_{\mu\nu}(p) = \frac{\mathrm{i}}{M_V}\left(p_\mu\varepsilon^{(\lambda)}_\nu(p) - p_\nu\varepsilon^{(\lambda)}_\mu(p)\right). \tag{5.3.10}$$

The pole corresponds therefore to the spin-one state $|p,\lambda,V\rangle$ which couples to $R_{\mu\nu}$ as

$$\langle 0|R_{\mu\nu}(0)|p,\lambda,V\rangle = |Z_V|^{1/2}u^{(\lambda)}_{\mu\nu}(p). \tag{5.3.11}$$

Analogously to the Proca case, at lest one of these poles is expected to be perturbative, *i.e.*

$$M_V^2 = M^2 + \delta M_V^2 \tag{5.3.12}$$

$$Z_V = 1 + \delta Z_V \tag{5.3.13}$$

with small corrections δM_V^2 and δZ_V vanishing in the free field limit; the other solutions decouple in this limit.

Provided there exists a solution of (5.3.7) for which $M_A^2 > 0$, we get at this pole

$$\begin{aligned}\Delta_{\mu\nu\alpha\beta}(p) &= \frac{Z_A}{p^2 - M_A^2}\left(g_{\mu\alpha}g_{\nu\beta} + \frac{p_\mu g_{\nu\alpha}p_\beta - p_\mu g_{\nu\beta}p_\alpha}{M_A^2} - (\mu \leftrightarrow \nu)\right) + O(1) \\ &= \frac{Z_A}{p^2 - M_A^2}\sum_\lambda w_{\mu\nu}^{(\lambda)}(p) w_{\alpha\beta}^{(\lambda)}(p)^* + O(1) \end{aligned} \tag{5.3.14}$$

where

$$Z_A = \frac{1}{\Sigma'^T(M_A^2)} \tag{5.3.15}$$

$$w_{\mu\nu}^{(\lambda)}(p) = \widetilde{u}_{\mu\nu}^{(\lambda)}(p) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}u^{(\lambda)\alpha\beta}(p). \tag{5.3.16}$$

These poles correspond to the spin-one particle states $|p, \lambda, A\rangle$ with opposite intrinsic parity which couple to the antisymmetric tensor field as

$$\langle 0|R_{\mu\nu}(0)|p, \lambda, A\rangle = |Z_A|^{1/2} w_{\mu\nu}^{(\lambda)}(p). \tag{5.3.17}$$

This degree of freedom is frozen in the free propagator due to the specific form of the free Lagrangian and it decouples in the limit of the vanishing interaction. As in the Proca field case, the additional degrees of freedom can be ghosts or tachyons.

One loop contribution

The Lagrangian contributing to the one loop correction of the propagator is

$$\begin{aligned}\mathcal{L}_R =& \frac{iG_V}{2\sqrt{2}}\langle R^{\mu\nu}[u_\mu, u_\nu]\rangle + i\lambda_{21}^V \langle R_{\mu\nu}D^2(u^\mu u^\nu)\rangle + d_1\epsilon_{\mu\nu\alpha\sigma}\langle D_\beta u^\sigma\{R^{\mu\nu}, R^{\alpha\beta}\}\rangle \\ &+ d_3\epsilon_{\rho\sigma\mu\lambda}\langle u^\lambda\{D_\nu R^{\mu\nu}, R^{\rho\sigma}\}\rangle + d_4\epsilon_{\rho\sigma\mu\alpha}\langle u_\nu\{D^\alpha R^{\mu\nu}, R^{\rho\sigma}\}\rangle \end{aligned} \tag{5.3.18}$$

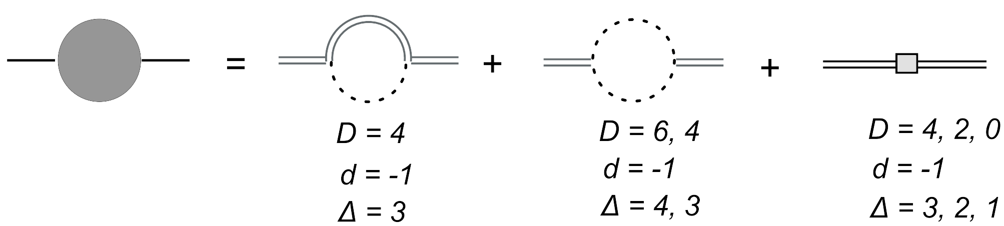


Figure 5.2: Tensor propagator one loop diagrams in antisymmetric tensor formalism.

The infinite parts of the result are

$$\begin{aligned}\Sigma^T(p^2) = \Bigg\{ &- \frac{5p^6}{18\pi^2F^2M^2}d_4(d_3 + d_4) - \frac{5p^4}{36\pi^2F^2}(d_3 + d_4)(3d_3 - 5d_4) \\ &- \frac{5p^2M^2}{12\pi^2F^2}\left[2d_1(d_1 + d_3 + d_4) - 3(d_3 + d_4)(3d_3 - d_4)\right] - \frac{5M^4}{6\pi^2F^2}d_1(d_1 - d_3 - d_4)\Bigg\}\lambda_\infty \end{aligned} \tag{5.3.19}$$

$$\Sigma^L(p^2) = \Bigg\{ \frac{p^8\lambda_{21}^2}{8\pi^2F^4} - \frac{\sqrt{2}p^6G_V\lambda_{21}}{8\pi^2F^4} + \frac{p^4G_V^2}{16\pi^2F^4} + \frac{5M^2p^2}{6\pi^2F^2}\left[(d_3+d_4)(d_1+3d_3-d_4)-d_1^2\right] - \frac{5M^4}{6\pi^2F^2}d_1(d_1-d_3-d_4)\Bigg\}\lambda_\infty \tag{5.3.20}$$

Restricting ourselves just to $\Delta = 3$ ($D = 4$) we obtain

$$\Sigma^T(p^2) = \Bigg\{ -\frac{5p^6}{18\pi^2F^2M^2}d_4(d_3+d_4) - \frac{5p^4}{36\pi^2F^2}(d_3+d_4)(3d_3-5d_4) - \frac{5p^2M^2}{12\pi^2F^2}\left[2d_1(d_1+d_3+d_4)-3(d_3+d_4)(3d_3-d_4)\right] - \frac{5M^4}{6\pi^2F^2}d_1(d_1-d_3-d_4)\Bigg\}\lambda_\infty \tag{5.3.21}$$

$$\Sigma^L(p^2) = \left\{ \frac{p^4G_V^2}{16\pi^2F^4} + \frac{5M^2p^2}{6\pi^2F^2}\left[(d_3+d_4)(d_1+3d_3-d_4)-d_1^2\right] - \frac{5M^4}{6\pi^2F^2}d_1(d_1-d_3-d_4)\right\}\lambda_\infty \tag{5.3.22}$$

Counterterms

We take into account all possible counterterms contributed to the diagrams up to the order $\Delta = 3$ and four derivatives[1]. The basis of these terms was already found in [13]

$$\mathcal{L}_R^{(0)} = \delta M^2\langle R_{\mu\nu}R^{\mu\nu}\rangle, \tag{5.3.23}$$

$$\mathcal{L}_R^{(2)} = 2Z_R\langle D^\alpha R_{\alpha\mu}D^\beta R_{\beta\mu}\rangle + Y_R\langle D^\alpha R_{\mu\nu}D_\alpha R^{\mu\nu}\rangle, \tag{5.3.24}$$

$$\begin{aligned}\mathcal{L}_R^{(4)} = {} & X_{R1}\langle D^2R^{\mu\nu}\{D_\nu,D^\sigma\}R_{\mu\sigma}\rangle + \frac{X_{2R}}{2}\langle\{D_\nu,D_\alpha\}R^{\mu\nu}\{D^\sigma,D^\alpha\}R_{\mu\sigma}\rangle \\ & + \frac{X_{3R}}{2}\langle\{D^\sigma,D^\alpha\}R^{\mu\nu}\{D_\nu,D_\alpha\}R_{\mu\sigma}\rangle + W_{R1}\langle D^2R_{\mu\nu}D^2R^{\mu\nu}\rangle \\ & + \frac{W_{R2}}{4}\langle\{D^\alpha,D^\beta\}R_{\mu\nu}\{D_\alpha,D_\beta\}R^{\mu\nu}\rangle\end{aligned} \tag{5.3.25}$$

Only the sums $X_R = X_{R1} + X_{R2} + X_{R3}$ and $W_R = W_{R1} + W_{R2}$ are relevant. Then the counterterms contribute to the self-energies as

$$\Sigma_{ct}^T(p^2) = 4(\delta M^2 + p^2Y_R + p^4W_R), \tag{5.3.26}$$

$$\Sigma_{ct}^L(p^2) = 4(\delta M^2 + p^2Z_R + p^2Y_R + p^4X_R + p^4W_R). \tag{5.3.27}$$

[1] The counterterms with six derivatives have not been classified yet.

Killing the infinities to the order p^4 in momenta we demand

$$X_R = -\frac{1}{576\pi^2 F^4}\left[20(3d_3-5d_4)(d_3+d_4)F^2+9G_V^2\right]+X_R^r(\mu), \tag{5.3.28}$$

$$Z_R = -\frac{5M^2}{48\pi^2 F^2}(d_3+d_4)(4d_1-3d_3+d_4)+Z_R^r(\mu), \tag{5.3.29}$$

$$W_R = \frac{5}{144\pi^2 F^2}(d_3+d_4)(3d_3-5d_4)+W_R^r(\mu), \tag{5.3.30}$$

$$Y_R = \frac{5M^2}{48\pi^2 F^2}\left[2d_1(d_1+d_3+d_4)-3(d_3+d_4)(3d_3-d_4)\right]+Y_R^r(\mu), \tag{5.3.31}$$

$$\delta M^2 = \frac{5M^4}{24\pi^2 F^2}d_1(d_1-d_3-d_4)+(\delta M^2)^r(\mu) \tag{5.3.32}$$

The non-vanishing infinite parts of Y_R and W_R indicate the non-trivial running of the corresponding renormalized couplings. This prevents us from fixing the finite parts of these couplings to zero. Consequently, this leads to the appearance of spin-1 particles with opposite parity as the propagating degrees of freedom in the antisymmetric tensor formalism.

5.4 Propagators in first order formalism

General properties

In this case, we write the relation for the Lagrangian

$$\mathcal{L} = M\langle V_\nu \partial_\mu R^{\mu\nu}\rangle + \frac{1}{2}M^2\langle V_\mu V^\mu\rangle + \frac{1}{4}M^2\langle R_{\mu\nu}R^{\mu\nu}\rangle + \mathcal{L}_{int}. \tag{5.4.1}$$

For this case the matrix of inverse propagators has the following general form

$$\Gamma^{(2)}_{RR}(p)_{\mu\nu\alpha\beta} = \frac{1}{2}(M^2+\Sigma^T_{RR}(p^2))\Pi^T_{\mu\nu\alpha\beta}+\frac{1}{2}(M^2+\Sigma^L_{RR}(p^2))\Pi^L_{\mu\nu\alpha\beta} \tag{5.4.2}$$

$$\Gamma^{(2)}_{VV}(p)_{\mu\nu} = (M^2+\Sigma^T_{VV}(p^2))P^T_{\mu\nu}+(M^2+\Sigma^L_{VV}(p^2))P^L_{\mu\nu} \tag{5.4.3}$$

$$\Gamma^{(2)}_{RV}(p)_{\mu\nu\alpha} = \frac{\mathrm{i}}{2}\left(M+\Sigma_{RV}(p^2)\right)\Lambda_{\mu\nu\alpha} \tag{5.4.4}$$

$$\Gamma^{(2)}_{VR}(p)_{\alpha\mu\nu} = \frac{\mathrm{i}}{2}\left(M+\Sigma_{VR}(p^2)\right)\Lambda^t_{\alpha\mu\nu} \tag{5.4.5}$$

where $\Sigma_{RV}(p^2)=\Sigma_{VR}(p^2)$ and

$$\Lambda_{\mu\nu\alpha} = -\Lambda^t_{\alpha\mu\nu} = p_\mu g_{\nu\alpha} - p_\nu g_{\mu\alpha} \tag{5.4.6}$$

This implies propagators

$$\Delta_{RR}(p)_{\mu\nu\alpha\beta} = \frac{2}{M^2+\Sigma^T_{RR}(p^2)}\Pi^T_{\mu\nu\alpha\beta}+2\frac{M^2+\Sigma^T_{VV}(p^2)}{D(p^2)}\Pi^L_{\mu\nu\alpha\beta} \tag{5.4.7}$$

$$\Delta_{VV}(p)_{\mu\nu} = \frac{1}{M^2+\Sigma^L_{VV}(p^2)}P^L_{\mu\nu}+\frac{M^2+\Sigma^L_{RR}(p^2)}{D(p^2)}P^T_{\mu\nu} \tag{5.4.8}$$

$$\Delta_{RV}(p)_{\mu\nu\alpha} = -\mathrm{i}\frac{M+\Sigma_{RV}(p^2)}{D(p^2)}\Lambda_{\mu\nu\alpha} \tag{5.4.9}$$

$$\Delta_{VR}(p)_{\alpha\mu\nu} = -\mathrm{i}\frac{M+\Sigma_{VR}(p^2)}{D(p^2)}\Lambda^t_{\alpha\mu\nu} \tag{5.4.10}$$

where

$$D(p^2) = (M^2 + \Sigma_{RR}^L(p^2))(M^2 + \Sigma_{VV}^T(p^2)) - p^2(M + \Sigma_{RV}(p^2))(M + \Sigma_{VR}(p^2)). \quad (5.4.11)$$

Let us now investigate the structure of the poles. These are situated at $p^2 = M_{V,A,S}^2$, being solutions of

$$\begin{aligned} D(M_V^2) &= 0 && (5.4.12) \\ M^2 + \Sigma_{RR}^T(M_A^2) &= 0 && (5.4.13) \\ M^2 + \Sigma_{VV}^L(M_S^2) &= 0. && (5.4.14) \end{aligned}$$

Assuming $M_V^2 > 0$, we get at this pole (as explained above)

$$\begin{aligned} \Delta_{RR}(p)_{\mu\nu\alpha\beta} &= \frac{Z_{RR}}{p^2 - M_V^2} \sum_\lambda u_{\mu\nu}^{(\lambda)}(p) u_{\alpha\beta}^{(\lambda)}(p)^* + O(1) && (5.4.15) \\ \Delta_{VV}(p)_{\mu\nu} &= \frac{Z_{VV}}{p^2 - M_V^2} \sum_\lambda \varepsilon_\mu^{(\lambda)}(p) \varepsilon_\nu^{(\lambda)*}(p) + O(1) && (5.4.16) \\ \Delta_{RV}(p)_{\mu\nu\alpha} &= \frac{Z_{RV}}{p^2 - M_V^2} \sum_\lambda u_{\mu\nu}^{(\lambda)}(p) \varepsilon_\alpha^{(\lambda)}(p)^* + O(1) && (5.4.17) \\ \Delta_{VR}(p)_{\alpha\mu\nu} &= \frac{Z_{VR}}{p^2 - M_V^2} \sum_\lambda \varepsilon_\alpha^{(\lambda)}(p) u_{\mu\nu}^{(\lambda)*}(p) + O(1) && (5.4.18) \end{aligned}$$

where

$$\begin{aligned} Z_{RR} &= \frac{M^2 + \Sigma_{VV}^T(M_V^2)}{D'(M_V^2)} && (5.4.19) \\ Z_{VV} &= \frac{M^2 + \Sigma_{RR}^L(M_V^2)}{D'(M_V^2)} && (5.4.20) \\ Z_{RV} &= \frac{M + \Sigma_{RV}(M_V^2)}{D'(M_V^2)} M_V = Z_{VR} = \frac{M + \Sigma_{VR}(M_V^2)}{D'(M_V^2)} M_V && (5.4.21) \end{aligned}$$

Note that, as a consequence of (5.4.14)

$$Z_{RR} Z_{VV} = Z_{RV}^2 = Z_{VR}^2, \quad (5.4.22)$$

(remember $\Sigma_{RV}(p^2) = \Sigma_{VR}(p^2)$), therefore the pole $p^2 = M_V^2 > 0$ corresponds to the spin-one one-particle state $|p, \lambda, V\rangle$ which couples to the fields as

$$\begin{aligned} \langle 0 | R_{\mu\nu}(0) | p, \lambda, V \rangle &= |Z_{RR}|^{1/2} u_{\mu\nu}^{(\lambda)}(p) && (5.4.23) \\ \langle 0 | V_\mu(0) | p, \lambda, V \rangle &= |Z_{VV}|^{1/2} \varepsilon_\mu^{(\lambda)}(p) && (5.4.24) \end{aligned}$$

and at least one of these states is expected to be perturbative as above; the others decouple when the interactions is switched off. The other possible poles, $p^2 = M_S^2$ and $p^2 = M_A^2$ are analogical to the spin-zero and spin-one (opposite parity) states mentioned in the previous two subsections, they correspond to the modes which are frozen at the leading order and decouple in the free field limit. Again, without further information, all the additional states can be also ghosts or tachyons.

One loop contribution

The contributing Lagrangian in the first order formalism is

$$\mathcal{L}_{RV} = \frac{iG_V}{2\sqrt{2}}\langle R^{\mu\nu}[u_\mu, u_\nu]\rangle + i\lambda_{21}^V\langle R_{\mu\nu}D^2(u^\mu u^\nu)\rangle - \frac{ig_V}{2\sqrt{2}}\langle \hat{V}^{\mu\nu}[u_\mu, u_\nu]\rangle$$
$$+ d_1\epsilon_{\mu\nu\alpha\sigma}\langle D_\beta u^\sigma\{R^{\mu\nu}, R^{\alpha\beta}\}\rangle + d_3\epsilon_{\rho\sigma\mu\lambda}\langle u^\lambda\{D_\nu R^{\mu\nu}, R^{\rho\sigma}\}\rangle$$
$$+ d_4\epsilon_{\rho\sigma\mu\alpha}\langle u_\nu\{D^\alpha R^{\mu\nu}, R^{\rho\sigma}\}\rangle + \frac{1}{2}M\sigma_V\varepsilon_{\alpha\beta\mu\nu}\langle\{V^\alpha, R^{\mu\nu}\}u^\beta\rangle \tag{5.4.25}$$

The self-energies have the form

$$\Sigma_{VV}^T(p^2) = \frac{g_V^2 p^6\lambda_\infty}{16\pi^2F^4} + \frac{5\sigma_V^2p^4\lambda_\infty}{72\pi^2F^2} - \frac{5M^2p^2\sigma_V^2\lambda_\infty}{24\pi^2F^2}, \tag{5.4.26}$$
$$\Sigma_{VV}^L(p^2) = 0. \tag{5.4.27}$$

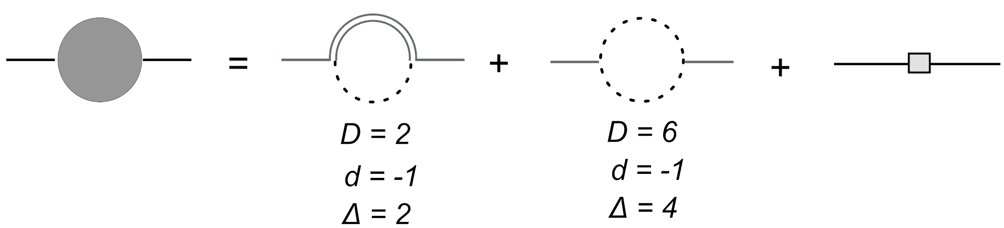


Figure 5.3: Vector propagator one loop diagrams in first order formalism.

Restricting ourselves to $\Delta = 3$ ($D = 4$) and taking into account also counterterms contributions we obtain the result

$$\Sigma_{VV}^T(p^2) = \frac{5\sigma_V^2p^4\lambda_\infty}{72\pi^2F^2} - \frac{5M^2p^2\sigma_V^2\lambda_\infty}{24\pi^2F^2} \tag{5.4.28}$$
$$\Sigma_{VV}^L(p^2) = 0 \tag{5.4.29}$$

The self-energies can be written as

$$\Sigma_{RR}^T(p^2) = \Bigg\{ -\frac{5p^6}{16\pi^2F^2M^2}d_4(d_3+d_4) - \frac{5p^4}{144\pi^2F^2}\left[\sigma_V^2 - 4\sigma_V(d_3+3d_4) + 4(d_3+d_4)(3d_3-5d_4)\right]$$
$$- \frac{5p^2M^2}{144\pi^2F^2}\left[24d_1(d_1+d_3+d_4) - 12(d_3+d_4)(3d_3-d_4) + 4\sigma_V(d_3+9d_4) - \sigma_V^2\right]$$
$$- \frac{5M^4}{24\pi^2F^2}\left[4(d_1+\sigma_V)(d_1-d_3-d_4)+\sigma_V^2\right]\Bigg\}\lambda_\infty \tag{5.4.30}$$

$$\Sigma_{RR}^L(p^2) = \Bigg\{\frac{\lambda_{21}^2p^8\lambda_\infty}{8\pi^2F^4} - \frac{p^6\sqrt{2}G_V\lambda_{21}^V\lambda_\infty}{18\pi^2F^4} + \frac{p^4G_V^2\lambda_\infty}{16\pi^2F^4} - \frac{5\sigma_Vd_3p^4\lambda_\infty}{18\pi^2F^4} \tag{5.4.31}$$
$$+ \frac{5M^2p^2}{6\pi^2F^2}\left[(d_3+d_4)(d_1+3d_3-d_4) - d_1^2\right] + \frac{5p^2M^2\lambda_\infty}{72\pi^2F^2}\sigma_V(\sigma_V+8d_3)$$
$$- \frac{5M^4}{24\pi^2F^2}\left[4(d_1+\sigma_V)(d_1-d_3-d_4)+\sigma_V^2\right]\Bigg\}\lambda_\infty$$

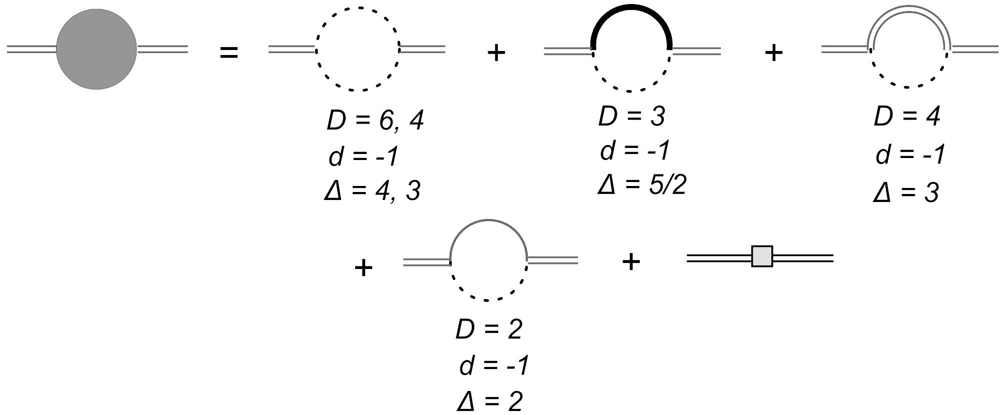


Figure 5.4: Tensor propagator one loop diagrams in first order formalism. They appear only in mixed formalism. The thick line stands for both mixed propagators = + .

Restricting to $\Delta = 3$ $(D = 4)$ we get

$$\begin{aligned}\Sigma^T_{RR}(p^2) = \Bigg\{ &-\frac{5p^6}{16\pi^2 F^2 M^2} d_4(d_3+d_4) \\ &-\frac{5p^4}{144\pi^2F^2}\left[\sigma_V^2 - 4\sigma_V(d_3+3d_4) + 4(d_3+d_4)(3d_3-5d_4)\right] \\ &-\frac{5p^2M^2}{144\pi^2F^2}\left[24d_1(d_1+d_3+d_4) - 12(d_3+d_4)(3d_3-d_4) + 4\sigma_V(d_3+9d_4) - \sigma_V^2\right] \\ &-\frac{5M^4}{24\pi^2F^2}\left[4(d_1+\sigma_V)(d_1-d_3-d_4)+\sigma_V^2\right]\Bigg\}\lambda_\infty\end{aligned} \tag{5.4.32}$$

$$\begin{aligned}\Sigma^L_{RR}(p^2) = \Bigg\{ &-\frac{p^6\sqrt{2}G_V\lambda^V_{21}\lambda_\infty}{18\pi^2F^4} + \frac{p^4G_V^2\lambda_\infty}{16\pi^2F^4} - \frac{5\sigma_V d_3 p^4\lambda_\infty}{18\pi^2F^4} \\ &+\frac{5M^2p^2}{6\pi^2F^2}\left[(d_3+d_4)(d_1+3d_3-d_4) - d_1^2\right] + \frac{5p^2M^2\lambda_\infty}{72\pi^2F^2}\sigma_V(\sigma_V+8d_3) \\ &-\frac{5M^4}{24\pi^2F^2}\left[4(d_1+\sigma_V)(d_1-d_3-d_4)+\sigma_V^2\right]\Bigg\}\lambda_\infty\end{aligned} \tag{5.4.33}$$

The mixed self-energy has the form

$$\begin{aligned}\Sigma_{RV}(p^2) = \Bigg\{ &-\frac{g_V\lambda_{21}p^6}{8\sqrt{2}\pi^2F^4} - \frac{p^4}{144\pi^2F^4M}(20d_3\sigma_VF^2 - 9Mg_VG_V) \\ &-\frac{5M\sigma_V(\sigma_V-4d_3)p^2}{72\pi^2F^2} - \frac{5M^3\sigma_V(2d_1-2d_3-2d_4-\sigma_V)}{24\pi^2F^2}\Bigg\}\lambda_\infty\end{aligned} \tag{5.4.34}$$

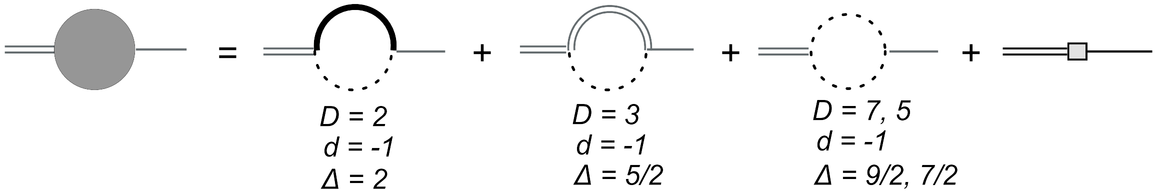


Figure 5.5: Mixed propagator one loop diagrams. The thick line stands for mixed propagator = .

Restricting to $\Delta = 5/2$ ($D = 3$) we obtain

$$\Sigma_{RV}(p^2) = \Bigg\{ -\frac{5d_3\sigma_V p^4}{36\pi^2F^2M} - \frac{5M\sigma_V(\sigma_V - 4d_3)p^2}{72\pi^2F^2} \tag{5.4.35}$$

$$- \frac{5M^3\sigma_V(2d_1 - 2d_3 - 2d_4 - \sigma_V)}{24\pi^2F^2}\Bigg\}\lambda_\infty \tag{5.4.36}$$

Counterterms

In the first order formalism we have to include both types of counterterms from the vector and the antisymmetric tensor formalisms. Moreover, we have also the mixing terms in this case,

$$\mathcal{L}^{ct}_{RV} = \mathcal{L}^{(1)}_{RV} + \mathcal{L}^{(3)}_{RV} \tag{5.4.37}$$

where

$$\mathcal{L}^{(1)}_{RV} = Z_{RV}\langle R_{\mu\nu}\hat{V}^{\mu\nu}\rangle, \tag{5.4.38}$$

$$\mathcal{L}^{(3)}_{RV} = X_{1RV}\langle D^\alpha R_{\mu\nu}D_\alpha\hat{V}^{\mu\nu}\rangle + \frac{X_{2RV}}{2}\langle D^\alpha R_{\mu\alpha}D_\beta\hat{V}^{\mu\beta}\rangle \tag{5.4.39}$$

Only the sum $X_{RV} = X_{1RV} + X_{2RV}$ is relevant in the calculations. The contribution to the self-energy are

$$\Sigma_{RV}(p^2) = -2Z_{RV} + 2p^2X_{RV}. \tag{5.4.40}$$

In the following we denote δM_V^2 and δM_R^2 the coupling constant standing by the renormalized mass term of V^μ and $R_{\mu\nu}$. If $\delta M_V^2 \neq \delta M_R^2$, this indicates the mass splitting of the resonance fields.

Matching of couplings in counterterms

Killing the infinities up to p^4 we get

$$Z_{RV} = -\frac{5M^3\sigma_V(2d_1 - 2d_3 - 2d_4 - \sigma_V)}{48\pi^2 F^2} + Z^r_{RV}(\mu), \tag{5.4.41}$$

$$X_{RV} = \frac{5M\sigma_V(\sigma_V - 4d_3)p^2}{144\pi^2 F^2} + X^r_{RV}(\mu), \tag{5.4.42}$$

$$\delta M_V^2 = (\delta M_V^2)^r(\mu), \tag{5.4.43}$$

$$Y_V = Y_V^r(\mu), \tag{5.4.44}$$

$$X'_V = (X'_V)^r(\mu), \tag{5.4.45}$$

$$Z_V = \frac{5M^2\sigma_V^2\lambda_\infty}{24\pi^2 F^2} + Z_V^r(\mu), \tag{5.4.46}$$

$$X_V = -\frac{5\sigma_V^2\lambda_\infty}{72\pi^2 F^2} + X_V^r(\mu), \tag{5.4.47}$$

$$Z_R = -\frac{5M^2\lambda_\infty}{576\pi^2 F^2}\left[12(d_3+d_4)(4d_1 - 3d_3 + d_4) + \sigma_V^2 + 4\sigma_V(5d_3 + 9d_4)\right] + Z_R^r(\mu), \tag{5.4.48}$$

$$X_R = -\frac{\lambda_\infty}{576\pi^2 F^4}\left[20(3d_3 - 5d_4)(d_3 + d_4)F^2\right. \tag{5.4.49}$$

$$\left. +5\sigma_V^2 F^2 - 60\sigma_V(d_3 + d_4) + 9G_V^2\right] + X_R^r(\mu), \tag{5.4.50}$$

$$\delta M_R^2 = \frac{5M^4}{96\pi^2 F^2}\left[4(d_1 + \sigma_V)(d_1 - d_3 - d_4) + \sigma_V^2\right] + (\delta M_R^2)^r(\mu), \tag{5.4.51}$$

$$Y_R = \frac{5M^2}{576\pi^2 F^2}\left[24d_1(d_1 + d_3 + d_4)\right. \tag{5.4.52}$$

$$\left. -12(d_3 + d_4)(3d_3 - d_4) + 4\sigma_V(d_3 + 9d_4) - \sigma_V^2\right], \tag{5.4.53}$$

$$W_R = \frac{5}{576\pi^2 F^2}\left[\sigma_V^2 - 4\sigma_V(d_3 + 3d_4) + 4(d_3 + d_4)(3d_3 - 5d_4)\right] \tag{5.4.54}$$

We have obtained not only the presence of spin-1 particles with opposite parity (that are either tachyons or ghosts) but also the dynamical generation of the kinetic and the mass terms for individual resonances V^μ and $R^{\mu\nu}$ where generally $\delta M_R^2 \neq \delta M_V^2$. The interpretation of this phenomenon is the task for future studies.

5.5 Summary of the chapter

In this chapter, we have discussed the question of the renormalization in Resonance Chiral Theory and its application on the concrete example of the resonance propagators. First, we have studied various possibilities of the power counting used in RχT and their disadvantages. Then we have concentrated on the self-energies and the propagators of vector resonances calculated in all three formalisms. Starting with the free field Lagrangians we have learned that at tree level only physical particles are propagated. However, if we add some special terms into Lagrangian the other degrees of freedom, which were frozen at tree level, are now propagated too. Unfortunately, these states have generally negative norm and refer to the appearance of ghosts in the spectrum.

We have done the one loop renormalization procedure restricting ourselves to the counterterms with maximally four derivatives and given Δ_{max} and we have found the concrete forms of the infinite parts of the counterterms couplings. In the vector formalism, no additional degrees of freedom are generated because related coupling constants can be fixed to zero independently on the renormalization scale. If we would enlarge the calculation up to the order $\mathcal{O}(p^8)$, these negative norm states would probably also appear. In the antisymmetric tensor and the first order formalisms, this pathology is present already at the order $\mathcal{O}(p^6)$.

CHAPTER 6

Conclusion

Resonance Chiral Theory is an effective theory for QCD for the intermediate energy region which interpolates between the Chiral Perturbation Theory (the limit for low energies) and perturbative regime of QCD in the limit of large N_C (at high energies). In the general case it would include the infinite tower of resonances in order to fully describe the spectrum of QCD with $N_C \to \infty$, but the relevant simplification for energies $1\,\mathrm{GeV} \leq E \leq 2\,\mathrm{GeV}$ takes into account only the lightest resonance in each channel.

In this thesis, we have restricted ourselves to the role of the vector resonances in the RχT. We have introduced two usual ways how to describe these particles - using the vector and the antisymmetric tensor fields. Then we have studied their equivalence and we have proposed also the third possibility that combines both previous - the first order formalism. It provides us with a method how to obtain the general effective chiral Lagrangian where no additional terms must be given by hand (which is not true for the vector and the antisymmetric tensor formalisms).

In chapter 3 and 4, we have done the calculation of concrete correlators together with their formal properties and the high energy constraints. As a result, we have first found the relations between resonance Lagrangian coupling constants and then after matching with χPT we have obtained the saturation of LECs. However, the more complicated example of Compton-like scattering indicates that in some situations the high energy constraints are very strict and cannot be non-trivially satisfied. Probably these lapses could be corrected if either other types of resonances or additional local contact terms would be added.

In chapter 5 we have found that one loop corrections to resonance propagators give rise to the problems relating with the possible appearance of ghosts (or tachyons) in the theory. The renormalization procedure in the antisymmetric tensor and the first order formalisms needs a presence of the new kinetic terms that lead to states with negative norm. Generally, it can probably happen also in the vector formalism if we include also $\mathcal{O}(p^8)$ Lagrangian terms.

APPENDIX A

Theoretical background

A.1 Notation

In order to simplify some long expressions we use the same short hand notation as in [A]. All used fields transform under adjoint representation of $SU(3)_V$. Using the normalisation of [11] we have $V_\mu = V_\mu^a T^a$ where $T^a = \lambda^a/\sqrt{2}$ and $T^0 = \mathbf{1}/\sqrt{3}$. The same is true about the antisymmetric tensor fields and pseudoscalar fields, $R_{\mu\nu} = R^a_{\mu\nu} T^a$ and $\phi = \phi^a T^a$ [1]. For sources v and p we have $p = p^a T^a/\sqrt{2}$ and $v_\mu = v_\mu^a T^a/\sqrt{2}$

The dot in brackets means the contraction of group and tensor indices, e.g.

$$(A \cdot B) \equiv A^a_\mu B^{a\mu}, \tag{A.1.1}$$

$$(V \cdot K \cdot V) \equiv V^a_\mu K^{ab\mu\nu} V^b_\nu. \tag{A.1.2}$$

For generic tensors we employ " : " for a pair of contracted antisymmetric indices, i.e.

$$R : J \equiv R_{\mu\nu} J^{\mu\nu} \tag{A.1.3}$$

We also use the symbol $\widehat{V}$ for an antisymmetric derivative of the vector field V, id. $\widehat{V}^{a\mu\nu} = D^{ab\mu}V^{b\nu} - D^{ab\nu}V^{b\mu}$ and W for a derivative of the antisymmetric tensor field $W^{a\beta} = D^{ab}_\alpha R^{b\alpha\beta}$.

A.2 Some remarks on SU(3)

The group SU(3) plays an important role in the concept of the Standard model, especially in the theories of strong interactions. In sixties it was used to construct the model of the Eightfold way

[1] The pseudoscalar mesons transform as an octet so there is no term $\phi^0 T^0$

where the particles were organized into multiplets corresponding to irreducible representations of this group. However, it was evident that this symmetry (flavour SU(3)) is broken due to mass differences among the particles in the multiplets. The fundamental theory of strong interactions, QCD, is based on the local $SU(3)_C$ and possesses chiral $SU(3)_L \times SU(3)_R$ in the massless quark limit. It is true only for massless quarks.

The element of the SU(3) group can be generally written in the form

$$U(\Theta) = \exp\left(-i\sum_{a=1}^{8}\Theta^a\frac{\lambda^a}{2}\right) \tag{A.2.1}$$

with eight real numbers Θ^a and eight linearly independent so-called Gell-Mann matrices satisfying

$$\lambda^a = \lambda_a^\dagger \tag{A.2.2}$$

$$\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab} \tag{A.2.3}$$

$$\mathrm{Tr}(\lambda^a) = 0. \tag{A.2.4}$$

An explicit form of the Gell-Mann matrices is

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \sqrt{\frac{1}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{A.2.5}$$

For our purpose we introduce the T^a matrices

$$T^a = \frac{\lambda^a}{\sqrt{2}}. \tag{A.2.6}$$

Next we define structure constants of SU(3). The commutator of two T's matrices has the form

$$[T^a, T^b] = \sqrt{2}if^{abc}T^c \tag{A.2.7}$$

where f^{abc} is totally antisymmetric object. The anticommutator is then

$$\{T^a, T^b\} = \frac{2}{3}\delta^{ab} + \sqrt{2}d^{abc}T^c \tag{A.2.8}$$

with d^{abc} being totally symmetric. The reverse relations are

$$f^{abc} = -\sqrt{2}i\mathrm{Tr}\left([T^a, T^b]T^c\right), \qquad d^{abc} = \sqrt{2}\mathrm{Tr}\left(\{T^a, T^b\}T^c\right) \tag{A.2.9}$$

A.3 Feynman integrals

We use same convention as in [13]. To calculate loop integrals we use Passarino-Veltman reductions method. The divergencies of the integrals can be collected in the factor

$$\lambda_\infty = \frac{2\mu^{d-4}}{d-4} + \gamma_E - \ln 4\pi - 1 \tag{A.3.1}$$

where $\gamma_E = 0.577\dots$ is the Euler constant and μ is the renormalization scale.

The notation of corresponding Feynman integrals is then

$$A_0(M^2) \equiv \int \frac{dk^d}{i(2\pi)^d} \frac{1}{k^2 - M^2 + i\epsilon} = \frac{M^2}{16\pi^2}\left\{\lambda_\infty + \ln\frac{M^2}{\mu^2}\right\} \tag{A.3.2}$$

$$\begin{aligned} B_0(p^2,M_1^2,M_2^2) &\equiv \int \frac{dk^d}{i(2\pi)^d} \frac{1}{(k^2 - M_1^2 + i\epsilon)[(p-k)^2 - M_2^2 + i\epsilon]} \\ &= -\frac{1}{16\pi^2}\left[\lambda_\infty + \frac{M_1^2}{M_1^2 - M_2^2}\ln\frac{M_1^2}{\mu^2} - \frac{M_2^2}{M_1^2 - M_2^2}\ln\frac{M_2^2}{\mu^2}\right] + \overline{J}(p^2, M_1^2, M_2^2), \end{aligned} \tag{A.3.3}$$

where the the finite function $\overline{J}(p^2, M_1^2, M_2^2)$ stands for

$$\begin{aligned} \overline{J}(p^2, M_1^2,M_2^2) &= \frac{1}{32\pi^2}\left\{2 + \left[\frac{M_1^2 - M_2^2}{p^2} - \frac{M_1^2 + M_2^2}{M_1^2 - M_2^2}\right]\ln\frac{M_2^2}{M_1^2}\right. \\ &\left. - \frac{\lambda^{1/2}(p^2, M_1^2, M_2^2)}{p^2}\ln\left[\frac{(q^2 + \lambda^{1/2}(p^2, M_1^2, M_2^2))^2 - (M_1^2 - M_2^2)^2}{(q^2 - \lambda^{1/2}(p^2, M_1^2, M_2^2))^2 - (M_1^2 - M_2^2)^2}\right]\right. \end{aligned} \tag{A.3.4}$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. Some useful particular cases of B_0 integrals are:

$$B_0(p^2, 0, 0) = -\frac{\lambda_\infty}{16\pi^2} + \hat{B}_0(p^2/\mu^2), \tag{A.3.5}$$

$$B_0(p^2, 0, M^2) = -\frac{1}{16\pi^2}\left\{\lambda_\infty + \ln\frac{M^2}{\mu^2}\right\} + \overline{J}(p^2, 0, M^2), \tag{A.3.6}$$

$$B_0(p^2, M^2, M^2) = -\frac{1}{16\pi^2}\left\{\lambda_\infty + \ln\frac{M^2}{\mu^2} + 1\right\} + \overline{B}_0(p^2, M^2), \tag{A.3.7}$$

with the finite parts

$$\hat{B}_0(p^2/\mu^2) = \frac{1}{16\pi^2}\left\{1 - \ln\left(-\frac{p^2}{\mu^2}\right)\right\}, \tag{A.3.8}$$

$$\overline{B}_0(p^2, M^2) = \overline{J}(p^2, M^2, M^2) = \frac{1}{16\pi^2}\left\{2 - \sigma_M \ln\left(\frac{\sigma_M + 1}{\sigma_M - 1}\right)\right\}, \tag{A.3.9}$$

$$\overline{J}(p^2, 0, M^2) = \frac{1}{16\pi^2}\left\{1 - \left(1 - \frac{M^2}{p^2}\right)\ln\left(1 - \frac{p^2}{M^2}\right)\right\} \tag{A.3.10}$$

where $\sigma_M = \sqrt{1 - 4M^2/p^2}$.

The three-propagator Feynman integral is defined as

$$\begin{aligned} &C_0(q^2,M_1^2, M_2^2, M_3^2) \equiv \\ &\int \frac{dk^d}{i(2\pi)^d} \frac{1}{[(p_1 - k)^2 - M_1^2 + i\epsilon][(p_2 + k)^2 - M_2^2 + i\epsilon](k^2 - M_3^2 + i\epsilon)}, \end{aligned} \tag{A.3.11}$$

where $q = p_1 + p_2$.

A.4 OPE for Green functions

As was mentioned earlier, the interpolating fields for external sources in QCD are defined as

$$V^{a,\mu}(x) = \overline{q}(x)\gamma^{\mu}\frac{T^a}{\sqrt{2}}q(x), \tag{A.4.1}$$

$$P^b(y) = i\overline{q}(y)\gamma^5\frac{T^b}{\sqrt{2}}q(y). \tag{A.4.2}$$

In OPE calculation we will use the propagator of quark fields

$$S(x) = \frac{x^{\mu}\gamma_{\mu}}{2\pi^2 x^4} \tag{A.4.3}$$

$\langle PP \rangle$

The operator product expansion for the $\langle PP \rangle$ correlator is

$$\begin{aligned}\langle 0|T[P^a(x)P^b(0)]|0\rangle = -\frac{1}{2}\mathrm{Tr}\Big\{&\langle 0|iS(-x)\gamma_5 T^a\, iS(x)\gamma_5 T^b|0\rangle \\ &+ \langle 0|iS(-x)\gamma_5 T^a : \overline{q}(0)q(x)\gamma_5 T^b|0\rangle \\ &+ \langle 0| : \overline{q}(x)q(0) : \gamma_5 T^a\, iS(x)\gamma_5 T^b|0\rangle \\ &+ \langle 0| : \overline{q}(x)q(0)\gamma_5 T^a\, \overline{q}(0)q(x) : \gamma_5 T^b|0\rangle\Big\} + \mathcal{O}(\alpha_s)\end{aligned} \tag{A.4.4}$$

Calculating Dirac traces the leading contribution of this expression can be written in the form

$$\langle 0|T[P^a(x)P^b(0)]|0\rangle = -\frac{3\delta^{ab}}{2\pi^4 x^6} + \mathcal{O}\left(\frac{1}{x^4}, \alpha_s\right) \tag{A.4.5}$$

$\langle VV \rangle$

The operator product expansion for the $\langle VV \rangle$ correlator is

$$\begin{aligned}\langle 0|T[V^a_{\mu}(x)V^b_{\nu}(0)]|0\rangle = -\frac{1}{2}\mathrm{Tr}\Big\{&\langle 0|iS(-x)\gamma^{\mu} T^a\, iS(x)\gamma^{\nu} T^b|0\rangle \\ &+ \langle 0|iS(-x)\gamma^{\mu} T^a : \overline{q}(0)q(x)\gamma^{\nu} T^b|0\rangle \\ &+ \langle 0| : \overline{q}(x)q(0) : \gamma^{\mu} T^a\, iS(x)\gamma^{\nu} T^b|0\rangle \\ &+ \langle 0| : \overline{q}(x)q(0)\gamma^{\mu} T^a\, \overline{q}(0)q(x) : \gamma^{\nu} T^b|0\rangle\Big\} + \mathcal{O}(\alpha_s)\end{aligned} \tag{A.4.6}$$

Calculating Dirac traces the leading contribution of this expression can be written in the form

$$\langle 0|T[V^a_{\mu}(x)V^b_{\nu}(0)]|0\rangle = -\frac{3\delta^{ab}}{2}\frac{2x_{\mu}x_{\nu} - g_{\mu\nu}x^2}{\pi^4 x^8} + \mathcal{O}\left(\frac{1}{x^6}, \alpha_s\right) \tag{A.4.7}$$

$\langle VVP \rangle$

The operator product expansion then reads

$$
\begin{aligned}
\langle 0|T[V^{a,\mu}(x)V^{b,\nu}(y)P^c(0)|0\rangle =& \langle 0|T\left(\overline{q}\gamma^\mu \frac{T^a}{\sqrt{2}}q\right)(x)\left(\overline{q}\gamma^\nu \frac{T^b}{\sqrt{2}}q\right)(y)\left(i\overline{q}\gamma^5 \frac{T^c}{\sqrt{2}}q\right)(0)|0\rangle \\
=& \frac{-i}{(\sqrt{2})^3}\mathrm{Tr}\Big\{ :q(0)\overline{q}(y):\gamma^\nu T^b\, iS(y-x)\gamma^\mu T^a\, iS(x)\gamma^5 T^c \\
& + iS(-y)\gamma^\nu T^b\, iS(y-x)\gamma^\mu T^a :\overline{q}(0)\overline{q}(x):\gamma^5 T^c \\
& + iS(-x)\gamma^\mu T^a\, iS(x-y)\gamma^\nu T^b :\overline{q}(0)\overline{q}(y):\gamma^5 T^c \\
& + iS(-x)\gamma^\mu T^a\ :\overline{q}(y)q(x):\gamma^\nu T^b\, iS(y)\gamma^5 T^c \\
& + iS(-y)\gamma^\nu T^b\ :\overline{q}(x)q(y):\gamma^\mu T^a\, iS(x)\gamma^5 T^c \\
& + :\overline{q}(x)q(0):\gamma^\mu T^a\, iS(x-y)\gamma^\nu T^b\, iS(y)\gamma^5 T^c \Big\} + \mathcal{O}(\alpha_s) \qquad \text{(A.4.8)}
\end{aligned}
$$

where we use the fact that only terms with two contractions can contribute due to properties of traces of Dirac matrices. Calculating it to the leading order we get

$$
\begin{aligned}
&\langle 0|T[V^{a,\mu}(x)V^{b,\nu}(y)P^c(0)|0\rangle \\
&= \frac{B_0F_0^2}{6\pi^4}d^{abc}\epsilon^{\mu\nu\alpha\beta}\left[\frac{x_\alpha y_\beta}{x^4(x-y)^4}+\frac{x_\alpha y_\beta}{x^4y^4}+\frac{x_\alpha y_\beta}{y^4(x-y)^4}\right]+\mathcal{O}\left(\frac{1}{x^8},\alpha_s\right) \qquad \text{(A.4.9)}
\end{aligned}
$$

with $-B_0F_0^2 = \langle 0|\overline{q}q|0\rangle$.

The OPE of Compton-like scattering is analogous to the case of $\langle VV \rangle$ correlator calculated to the next to leading order where the vacuum states are replaced by the external Goldsone boson states.

The Fourier transformations of these results are presented in the main text are used for determining of high energy constraints.

APPENDIX B

Feynman rules

In this appendix we provide the complete list of used Feynman rules.

Factor in Feynman rules

The generating functional can be written in the form

$$Z[v,a,p,s] = e^{iW[v,a,p,s]} = \left\langle 0|T\exp\left\{i\int j_V + j_A + j_P - j_S\right\}|0\right\rangle \tag{B.0.1}$$

where $W[v,a,s,p]$ is the generating functional of connected Green functions. By definition we give for Green function

$$\begin{aligned}&\langle 0|T(j_V(x_{V_1})...j_V(x_{A_1})...j_S(x_{P_1})...j_P(x_{S_1})...)|0\rangle\\&= (-i)^{\#v+\#p+\#a-\#s}\frac{\delta}{\delta v}...\frac{\delta}{\delta p}...\frac{\delta}{\delta a}...\frac{\delta}{\delta s}...\left(iW[v,a,s,p]\right)\end{aligned}$$

So for each vertex we have the sign rule

$$\text{sign} = \frac{i}{i^{\#v+\#p}}. \tag{B.0.2}$$

Factor i in numerator comes from iW. We can leave the factor in denominator just multiplying the expression for the correlators by

$$\text{sign of correlator} = \frac{1}{i^{\#v+\#p}}. \tag{B.0.3}$$

Chiral building blocks

In the concrete calculations it is neccesary to expand the usual chiral building blocks in terms of fields, currents and densities and their derivatives. Generally we have the infinite number of terms but for our purpose it is sufficient to take only few terms. By the definition

$$u = \exp\left(i\frac{\Phi}{\sqrt{2}F}\right) \approx 1 + i\frac{\Phi}{\sqrt{2}F} - \frac{\Phi^2}{4F^2} - i\frac{\Phi^3}{12\sqrt{2}F^3}, \tag{B.0.4}$$

$$u^\dagger = \exp\left(-i\frac{\Phi}{\sqrt{2}F}\right) \approx 1 - i\frac{\Phi}{\sqrt{2}F} - \frac{\Phi^2}{4F^2} + i\frac{\Phi^3}{12\sqrt{2}F^3} \tag{B.0.5}$$

Then we can write for the chiral building blocks

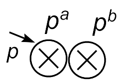
$$\begin{aligned}
u_\mu &= i(u^\dagger(\partial_\mu - iv_\mu)u - u(\partial_\mu - iv_\mu)u^\dagger) \\
&\approx -\frac{\sqrt{2}}{F}\partial_\mu\phi + \frac{\sqrt{2}i}{F}[v_\mu, \phi] + \frac{1}{6\sqrt{2}F^3}\{\phi^2, \partial_\mu\phi\} - \frac{1}{3\sqrt{2}F^3}\phi(\partial_\mu\phi)\phi, && \text{(B.0.6)} \\
\Gamma_\mu &= \frac{1}{2}\left\{u^\dagger(\partial_\mu - iv_\mu)u + u(\partial_\mu - iv_\mu)\right\} \\
&\approx \frac{1}{4F^2}[\phi, \partial_\mu\phi] - iv_\mu + \frac{i}{4F^2}\{v_\mu, \phi^2\}, && \text{(B.0.7)} \\
f_{+\mu\nu} &= uf_{\mu\nu}u^\dagger + u^\dagger f_{\mu\nu}u \\
&\approx 2(\partial_\mu v_\nu - \partial_\nu v_\mu) - 2i[v_\mu, v_\nu] - \frac{1}{2F^2}\{\partial_\mu v_\nu - \partial_\nu v_\mu, \phi^2\} + \frac{i}{2F^2}\{[v_\mu, v_\nu], \phi^2\} \\
&\quad + \frac{1}{F^2}\phi(\partial_\mu v_\nu - \partial_\nu v_\mu)\phi - \frac{i}{F^2}\phi[v_\mu, v_\nu]\phi, && \text{(B.0.8)} \\
f_{-\mu\nu} &= uf_{\mu\nu}u^\dagger - u^\dagger f_{\mu\nu}u \\
&\approx \frac{\sqrt{2}i}{F}[\phi, \partial_\mu v_\nu - \partial_\nu v_\mu] + \frac{\sqrt{2}}{F}[\phi, [v_\mu, v_\nu]], && \text{(B.0.9)} \\
\chi_+ &= u^\dagger\chi u^\dagger + u\chi^\dagger u \\
&\approx \frac{2\sqrt{2}B_0}{F}\{p, \phi\} - \frac{B_0}{3\sqrt{2}F^3}\{p, \phi^3\} - \frac{B_0}{\sqrt{2}F^3}\phi\{p, \phi\}\phi, && \text{(B.0.10)} \\
\chi_- &= u^\dagger\chi u^\dagger - u\chi^\dagger u \\
&\approx 4iB_0p - \frac{iB_0}{F^2}\{\phi^2, p\} - \frac{2iB_0}{F^2}\phi p\phi && \text{(B.0.11)} \\
\chi_{-\mu} &= u^\dagger D_\mu\chi u^\dagger + uD_\mu\chi^\dagger u = \nabla_\mu\chi_+ - \frac{i}{2}\{\chi_-, u_\mu\} && \text{(B.0.12)} \\
&\approx 4iB_0\pi_\mu p + 4B_0[v_\mu, p], && \text{(B.0.13)} \\
\chi_{+\mu} &= \frac{2\sqrt{2}B_0}{F}\{\pi_\mu, p\} - \frac{2\sqrt{2}iB_0}{F}\{\phi, [v_\mu, p]\}. && \text{(B.0.14)}
\end{aligned}$$

B.1 χPT vertices

In some results we use following simply notation

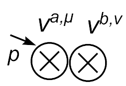
$$
\begin{aligned}
G_1 &= 2d^{abe}d^{cde} + \frac{4}{3}\delta^{ab}\delta^{cd} = 2T_5 + \frac{4}{3}T_1 \\
G_2(a,b,c,d) &= -\frac{1}{3}\delta^{ab}\delta^{cd} + (\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd}) - 2d^{abe}d^{cde} = -\frac{1}{3}T_1 + T_2 - 2T_5
\end{aligned}
$$

Vertex 1: pp



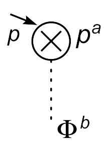
$$= -4iB_0^2\left[2L_8 - 2L_{11} + L_{12} - H_2 - c_{91}p^2\right]\delta^{ab} \tag{B.1.1}$$

Vertex 2: vv



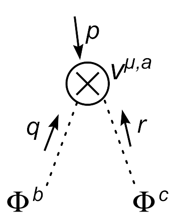
$$= 2i(L_{10} + 2H_1 + 2c_{93}p^2)(p^2 g_{\alpha\beta} - p_\alpha p_\beta)\delta^{ab} \tag{B.1.2}$$

Vertex 3: $p\phi$



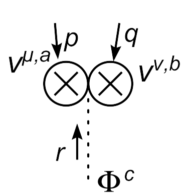
$$= iB_0F\left[1 - \frac{4(L_{11} - L_{12})p^2}{F^2}\right]\delta^{ab} \tag{B.1.3}$$

Vertex 5: $v\phi\phi$



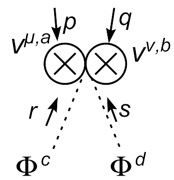
$$
\begin{aligned}
=&f^{abc}(r-q)_\alpha\Bigg\{1 + \frac{2L_9p^2 - 4L_{12}(r^2+q^2)}{F^2} \\
&\qquad - \frac{4c_{88}p^4 - 8c_{90}(q\cdot r)p^2}{F^2}\Bigg\} \\
&- f^{abc}p_\alpha\frac{2(q^2-r^2)}{F^2}\left[L_9 - 2c_{88}p^2 - 4c_{90}(q\cdot r)\right]
\end{aligned} \tag{B.1.4}
$$

Vertex 4: $vv\phi$



$$= -\frac{iN_C}{8\pi^2F}\epsilon_{\alpha\beta\mu\nu}p^\mu q^\nu d^{abc} \tag{B.1.5}$$

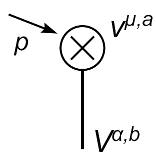
Vertex 5: $vv\phi\phi$ **(up to $\mathcal{O}(p^2)$)**



$$= ig_{\alpha\beta}(f^{ace}f^{bde} + f^{ade}f^{bce}) \tag{B.1.6}$$

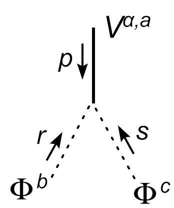
B.2 Vector formalism

Vertex 1 : Vv



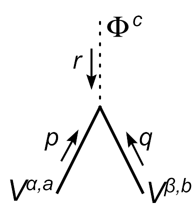
$$= -if_V\delta^{ab}(p^2 g_{\mu\alpha} - p_\alpha p_\mu) \tag{B.2.1}$$

Vertex 2 : $V\phi\phi$



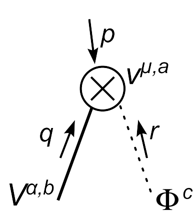
$$= -\frac{2g_V}{F^2} f^{abc}\left[(p\cdot r)s_\alpha - (p\cdot s)r_\alpha\right] \tag{B.2.2}$$

Vertex 3 : $VV\phi$



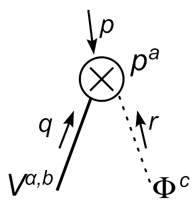
$$= \frac{2i\sigma_V}{F} d^{abc}\epsilon_{\alpha\beta\mu\nu} r^\mu (q-p)^\nu \tag{B.2.3}$$

Vertex 4 : $Vv\phi$



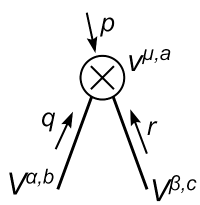
$$= \frac{4\sqrt{2}ih_V}{F} d^{abc}\epsilon_{\mu\alpha\rho\sigma} p^\rho r^\sigma \tag{B.2.4}$$

Vertex 5 : $Vp\phi$



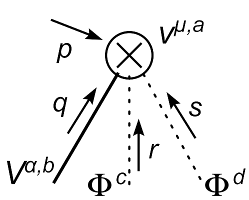
$$= \frac{4\sqrt{2}\beta_V B_0}{F} f^{abc} r_\alpha \tag{B.2.5}$$

Vertex 6 : VVv



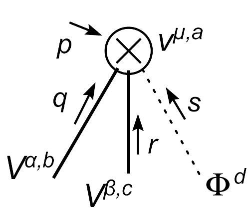
$$= f^{abc}\big[g_{\alpha\beta}(q-r)_\mu - g_{\mu\alpha} q_\beta + g_{\mu\beta} r_\alpha\big] \tag{B.2.6}$$

Vertex 7 : $Vv\phi\phi$



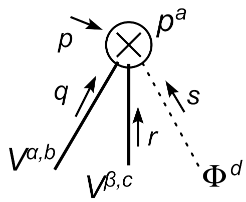
$$\begin{aligned} = \quad & \frac{i}{2F^2} f^{abe} f^{cde}\Big\{ f_V[g_{\alpha\mu}(p\cdot s) - \gamma_{\alpha\mu}(p\cdot r) + p_\alpha r_\mu - p_\alpha s_\mu] \\ & +4g_V r_\alpha s_\mu - 4g_V s_\alpha r_\mu \Big\} \\ & -\frac{i}{2F^2} f^{ace} f^{bde}\Big\{ 4\sqrt{2}\alpha_V[g_{\alpha\mu}(p\cdot s) - p_\alpha s_\mu] \\ & +f_V[g_{\alpha\mu}(p\cdot q) - p_\alpha q_\mu] + 4g_V(g_{\alpha\mu}(s\cdot q) - s_\alpha q_\mu]\Big\} \\ & -\frac{i}{2F^2} f^{ade} f^{bce}\Big\{ 4\sqrt{2}\alpha_V[g_{\alpha\mu}(p\cdot r) - p_\alpha r_\mu] \\ & +f_V[g_{\alpha\mu}(p\cdot q) - p_\alpha q_\mu] + 4g_V(g_{\alpha\mu}(r\cdot q) - r_\alpha q_\mu]\Big\} \end{aligned} \tag{B.2.7}$$

Vertex 8 : $VVv\phi$



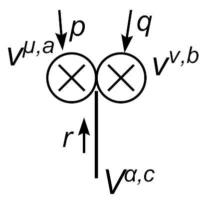
$$= \quad \frac{2\sigma_V}{F}\epsilon_{\mu\alpha\beta\rho}\left[f^{ade} d^{bce}(q-r)^\rho + s^\rho(f^{ace} d^{bde} - f^{abe} d^{cde})\right] \tag{B.2.8}$$

Vertex 9 : $VVp\phi$



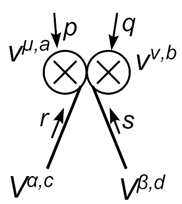
$$= d^{ade} d^{bce} \frac{2i\kappa_V B_0}{F} g_{\alpha\beta} + \delta^{ad}\delta^{bc} \frac{4i\kappa_V B_0}{3F} g_{\alpha\beta} \tag{B.2.9}$$

Vertex 10 : Vvv



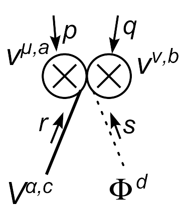
$$= f_V f^{abc} \left[(p-q)_\alpha g_{\mu\nu} + (r-p)_\nu g_{\mu\alpha} - (r-q)_\mu g_{\nu\alpha}\right] \tag{B.2.10}$$

Vertex 11 : $VVvv$



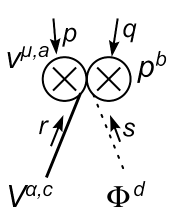
$$\begin{aligned} =& i f^{ace} f^{bde} (g_{\mu\nu} g_{\alpha\beta} - g_{\mu\beta} g_{\nu\alpha}) + i f^{ade} f^{bce} (g_{\mu\nu} g_{\alpha\beta} - g_{\mu\alpha} g_{\nu\beta}) \\ & - 2i f^{abe} f^{cde} (g_{\mu\alpha} g_{\nu\beta} - g_{\nu\alpha} g_{\mu\beta}) \end{aligned} \tag{B.2.11}$$

Vertex 12 : $Vvv\phi$



$$\begin{aligned} = \quad & f^{ade} d^{bce} \frac{4\sqrt{2} h_V}{F} \epsilon_{\mu\nu\alpha\sigma} q^\sigma - f^{bde} d^{ace} \frac{4\sqrt{2} h_V}{F} \epsilon_{\mu\nu\alpha\sigma} p^\sigma \\ & - f^{abe} d^{cde} \frac{4\sqrt{2}}{F} \epsilon_{\mu\nu\alpha\sigma} s^\sigma \end{aligned}$$

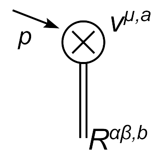
Vertex 13 : $Vvp\phi$



$$= \frac{4\sqrt{2} i \beta_V B_0}{F} f^{ade} f^{bce} g_{\mu\alpha} \tag{B.2.12}$$

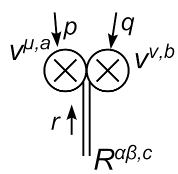
B.3 Antisymmetric tensor formalism

Vertex 1 : Rv



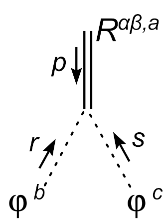
$$= \frac{1}{2}\delta^{ab}(p_\alpha g_{\mu\beta} - p_\beta g_{\mu\alpha})(F_V - 2\sqrt{2}\lambda_{22}^V p^2) \tag{B.3.1}$$

Vertex 2 : Rvv



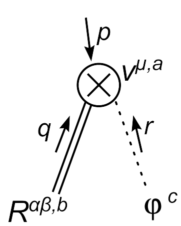
$$\begin{aligned} = \quad & i\left(\frac{F_V}{2} - \sqrt{2}\lambda_{22}^V r^2\right) f^{abc}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) \\ & +\sqrt{2}i\lambda_7^V f^{abc}\left[g_{\mu\nu}(p_\alpha q_\beta - p_\beta q_\alpha) + g_{\mu\beta}(q_\alpha p_\nu - (p\cdot q)g_{\alpha\nu})\right. \\ & \left. -g_{\mu\alpha}(p_\nu q_\beta - (p\cdot q)g_{\nu\beta}) + q_\mu(p_\beta g_{\alpha\nu} - p_\alpha g_{\beta\nu})\right] \end{aligned} \tag{B.3.2}$$

Vertex 3 : Rvv



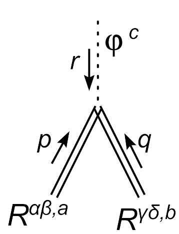
$$= \frac{i}{F^2} f^{abc}(r_\alpha s_\beta - r_\beta s_\alpha)(G_V - \sqrt{2}\lambda_{21}^V p^2) \tag{B.3.3}$$

Vertex 4 : $Rv\pi$



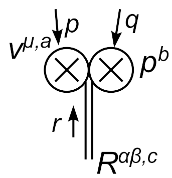
$$\begin{aligned} = \quad & \frac{2\sqrt{2}}{MF} d^{abc}\left[c_1\epsilon_{\alpha\beta\rho\sigma}p^\rho r^\sigma r_\mu - c_1\epsilon_{\alpha\beta\mu\sigma}r^\sigma(p\cdot r)\right. \\ & +c_2\epsilon_{\alpha\sigma\rho\mu}p^\rho r_\beta r^\sigma - c_2\epsilon_{\beta\sigma\rho\mu}p^\rho r_\alpha r^\sigma - c_5\epsilon_{\alpha\beta\mu\delta}r^\delta(p\cdot q) \\ & +c_5\epsilon_{\alpha\beta\rho\sigma}q_\mu p^\rho r^\sigma + c_6\epsilon_{\alpha\rho\sigma\mu}q_\beta p^\sigma r^\rho - c_6\epsilon_{\beta\rho\sigma\mu}q_\alpha p^\sigma r^\rho \\ & \left. -c_7\epsilon_{\alpha\beta\mu\sigma}q^\sigma(p\cdot r) + c_7\epsilon_{\alpha\beta\sigma\rho}q^\rho p^\sigma r_\mu\right] \end{aligned} \tag{B.3.4}$$

Vertex 5 : $RR\pi$



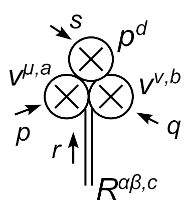
$$\begin{aligned} = \quad & \frac{i}{F} d^{abc}\left[d_1\epsilon_{\alpha\beta\gamma\sigma}r^\sigma r_\delta - d_1\epsilon_{\alpha\beta\delta\sigma}r^\sigma r_\gamma + d_1\epsilon_{\gamma\delta\alpha\sigma}r^\sigma r_\beta\right. \\ & -d_1\epsilon_{\gamma\delta\beta\sigma}r^\sigma r^\alpha + d_3\epsilon_{\alpha\beta\gamma\sigma}r^\sigma p_\delta - d_3\epsilon_{\alpha\beta\delta\sigma}r^\sigma p_\gamma + d_3\epsilon_{\gamma\delta\alpha\sigma}r^\sigma q_\beta \\ & -d_3\epsilon_{\gamma\delta\beta\sigma}r^\sigma q_\alpha + d_4\epsilon_{\gamma\delta\alpha\sigma}q^\sigma r_\beta - d_4\epsilon_{\gamma\delta\beta\sigma}q^\sigma r_\alpha + d_4\epsilon_{\alpha\beta\gamma\sigma}p^\sigma r_\delta \\ & \left. -d_4\epsilon_{\alpha\beta\delta\sigma}p^\sigma r_\gamma\right] \end{aligned} \tag{B.3.5}$$

Vertex 6 : *Rvp*



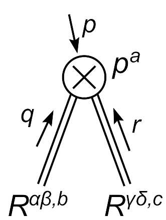
$$= -\frac{8\sqrt{2}c_3}{M} d^{abc} \epsilon_{\alpha\beta\sigma\mu} p^\sigma \tag{B.3.6}$$

Vertex 7 : *Rvvp*



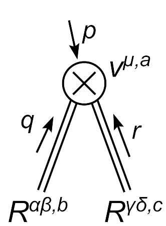
$$= -\frac{8ic_3}{M} f^{abe} d^{cde} \epsilon_{\mu\nu\alpha\beta} \tag{B.3.7}$$

Vertex 8 : *RRp*



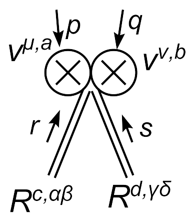
$$= -8d_2 B_0 d^{abc} \epsilon_{\alpha\beta\gamma\delta} \tag{B.3.8}$$

Vertex 9 : *RRv*



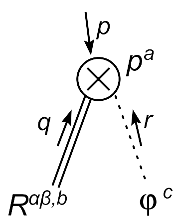
$$\begin{aligned} &= \frac{1}{4} f^{abc} \Big\{ q_\alpha (g_{\beta\delta} g_{\mu\gamma} - g_{\beta\gamma} g_{\kappa\delta}) - q_\beta (g_{\alpha\delta} g_{\mu\gamma} - g_{\alpha\gamma} g_{\mu\delta}) \\ &\qquad - r_\gamma (g_{\mu\alpha} g_{\beta\delta} - g_{\mu\beta} g_{\alpha\delta}) + r_\delta (g_{\beta\gamma} g_{\mu\alpha} - g_{\alpha\gamma} g_{\mu\beta}) \Big\} \\ &- \frac{\lambda_7^{VV}}{2} f^{abc} \Big\{ p_\alpha (g_{\beta\gamma} g_{\mu\delta} - g_{\beta\delta} g_{\mu\gamma}) - p_\beta (g_{\alpha\gamma} g_{\mu\delta} - g_{\alpha\delta} g_{\mu\gamma}) \\ &\qquad + p_\gamma (g_{\beta\delta} g_{\mu\alpha} - g_{\alpha\delta} g_{\mu\beta}) - p_\delta (g_{\mu\alpha} g_{\beta\gamma} - g_{\alpha\gamma} g_{\mu\beta}) \Big\} \end{aligned} \tag{B.3.9}$$

Vertex 10 : $RRvv$



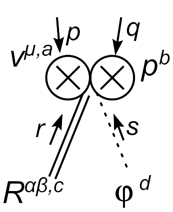
$$= -\frac{i}{4} f^{ace} f^{bde} \Big\{ g_{\mu\alpha}(g_{\beta\delta}g_{\gamma\nu} - g_{\beta\gamma}g_{\nu\delta}) - g_{\mu\beta}(g_{\alpha\delta}g_{\gamma\nu} - g_{\alpha\gamma}g_{\nu\delta}) \Big\}$$
$$- \frac{i}{4} f^{bce} f^{ade} \Big\{ g_{\nu\alpha}(g_{\beta\delta}g_{\gamma\mu} - g_{\beta\gamma}g_{\mu\delta}) - g_{\nu\beta}(g_{\alpha\delta}g_{\gamma\mu} - g_{\alpha\gamma}g_{\mu\delta}) \Big\}$$
$$+ \frac{i\lambda_7^{VV}}{2} f^{abe} f^{cde} \Big\{ g_{\mu\alpha}(g_{\beta\delta}g_{\gamma\nu} - g_{\beta\gamma}g_{\nu\delta}) - g_{\mu\beta}(g_{\alpha\delta}g_{\gamma\nu} - g_{\alpha\gamma}g_{\nu\delta}) \Big\}$$
$$- \frac{i\lambda_7^{VV}}{2} f^{abe} f^{cde} \Big\{ g_{\nu\alpha}(g_{\beta\delta}g_{\gamma\mu} - g_{\beta\gamma}g_{\mu\delta}) - g_{\nu\beta}(g_{\alpha\delta}g_{\gamma\mu} - g_{\alpha\gamma}g_{\mu\delta}) \Big\} \tag{B.3.10}$$

Vertex 11 : $Rp\phi$



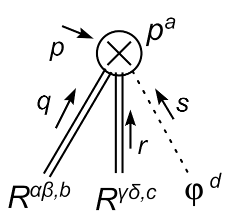
$$= \frac{2\sqrt{2} i B_0 \lambda_{10}^V}{F} f^{abc} (r_\alpha q_\beta - r_\beta q_\alpha) \tag{B.3.11}$$

Vertex 12 : $Rvp\phi$



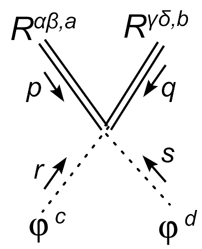
$$= \frac{2\sqrt{2} B_0 \lambda_{10}^V}{F} \Big[f^{ade} f^{bce} (g_{\mu\alpha} q_\beta - g_{\mu\beta} q_\alpha) + f^{abe} f^{cde} (g_{\mu\beta} s_\alpha - g_{\mu\alpha} s_\beta) \Big] \tag{B.3.12}$$

Vertex 13 : $RRp\phi$



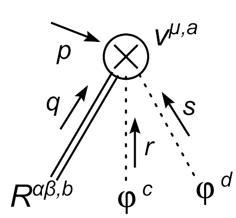
$$= \frac{i B_0 \lambda_6^{VV}}{F} \left(2 d^{ade} d^{bce} + \frac{4}{3} \delta^{ad} \delta^{bc} \right) (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}) \tag{B.3.13}$$

Vertex 14 : $RR\phi\phi$



$$
\begin{aligned}
&= -\frac{i}{F^2}(r\cdot s)\Big\{g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}\Big\}\left[\lambda_1^{VV}G_1 + 2\lambda_2^{VV}G_2(a,c,b,d)\right]\\
&\quad - \frac{i}{2F^2}\Big\{r_\alpha(s_\gamma g_{\beta\delta} - s_\delta g_{\gamma\beta}) - r_\beta(s_\gamma g_{\alpha\delta} - s_\delta g_{\alpha\gamma})\Big\}\\
&\qquad \times\left[\lambda_3^{VV}G_2(a,b,c,d) + \lambda_4^{VV}G_2(a,b,d,c) + 2\lambda_5^{VV}G_2(a,c,b,d)\right]\\
&\quad - \frac{i}{2F^2}\Big\{s_\alpha(r_\gamma g_{\beta\delta} - r_\delta g_{\gamma\beta}) - s_\beta(r_\gamma g_{\alpha\delta} - r_\delta g_{\alpha\gamma})\Big\}\\
&\qquad \times\left[\lambda_3^{VV}G_2(a,b,d,c) + \lambda_4^{VV}G_2(a,b,c,d) + 2\lambda_5^{VV}G_2(a,c,b,d)\right]\\
&\quad - \frac{i}{8F^2}f^{abe}f^{cde}\Big\{(s-r)_\alpha(q_\gamma g_{\beta\delta} - q_\delta g_{\beta\gamma}) - (s-r)_\beta(q_\gamma g_{\alpha\delta} - q_\delta g_{\alpha\gamma})\\
&\qquad - (s-r)_\gamma(p_\alpha g_{\beta\delta} - p_\beta g_{\alpha\delta}) + (s-r)_\delta(p_\alpha g_{\beta\gamma} - p_\beta g_{\alpha\gamma})\Big\}
\end{aligned}
\tag{B.3.14}
$$

Vertex 15 : $Rv\phi\phi$



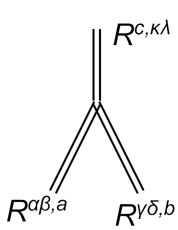
$$
\begin{aligned}
&= -\frac{F_V}{8F^2}(p_\alpha g_{\mu\beta} - p_\beta g_{\mu\alpha})\left[G_1 - 2G_2(a,d,b,c)\right]\\
&\quad + \frac{G_V}{F^2}f^{ace}f^{bde}(s_\beta g_{\mu\alpha} - s_\alpha g_{\mu\beta}) + \frac{G_V}{F^2}f^{ade}f^{bce}(r_\beta g_{\mu\alpha} - r_\alpha g_{\mu\beta})\\
&\quad - \frac{2\sqrt{2}}{F^2}(r\cdot s)(p_\alpha g_{\mu\beta} - p_\beta g_{\mu\alpha})\left[\lambda_{11}^V G_1 + \lambda_{12}^V G_2(a,d,b,c)\right]\\
&\quad - \frac{\sqrt{2}}{F^2}\Big\{p_\beta(r_\alpha s_\mu + r_\mu s_\alpha) - g_{\mu\beta}[(p\cdot s)r_\alpha + (p\cdot r)s_\alpha]\Big\}\\
&\qquad \times\left[\lambda_{13}^V G_2(a,d,b,c) - \lambda_{14}^V G_2(a,b,c,d) - \lambda_{15}^V G_2(a,b,d,c)\right]\\
&\quad + \frac{\sqrt{2}}{F^2}\Big\{p_\alpha(r_\beta s_\mu + r_\mu s_\beta) - g_{\mu\alpha}[(p\cdot s)r_\beta + (p\cdot r)s_\beta]\Big\}\\
&\qquad \times\left[\lambda_{13}^V G_2(a,d,b,c) - \lambda_{14}^V G_2(a,b,d,c) - \lambda_{15}^V G_2(a,b,c,d)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\sqrt{2}\lambda_{16}^{V}}{F^2} f^{ace} f^{bde}\Big\{ s_\mu (r_\alpha p_\beta - r_\beta p_\alpha) + (p\cdot s)[g_{\mu\alpha}(p+r)_\beta - g_{\mu\beta}(p+r)_\alpha] \Big\} \\
&+\frac{\sqrt{2}\lambda_{16}^{V}}{F^2} f^{ade} f^{bce}\Big\{ r_\mu (s_\alpha p_\beta - s_\beta p_\alpha) + (p\cdot r)[g_{\mu\alpha}(p+s)_\beta - g_{\mu\beta}(p+s)_\alpha] \Big\} \\
&+\frac{2\sqrt{2}\lambda_{17}^{V}}{F^2} f^{ace} f^{bde}\Big\{ (p+r)\cdot s\,(p_\alpha g_{\mu\beta} - p_\beta g_{\mu\alpha}) \Big\} \\
&+\frac{2\sqrt{2}\lambda_{17}^{V}}{F^2} f^{ade} f^{bce}\Big\{ (p+s)\cdot r\,(p_\alpha g_{\mu\beta} - p_\beta g_{\mu\alpha}) \Big\} \\
&+\frac{\sqrt{2}\lambda_{18}^{V}}{F^2} f^{ace} f^{bde}\Big\{ (p+r)\cdot p\,(s_\beta g_{\mu\alpha} - s_\alpha g_{\mu\beta}) + (p+r)_\mu (p_\beta s_\alpha - p_\alpha s_\beta) \Big\} \\
&+\frac{\sqrt{2}\lambda_{18}^{V}}{F^2} f^{ade} f^{bce}\Big\{ (p+s)\cdot p\,(r_\beta g_{\mu\alpha} - r_\alpha g_{\mu\beta}) + (p+s)_\mu (p_\beta r_\alpha - p_\alpha r_\beta) \Big\} \\
&+\frac{2\sqrt{2}\lambda_{19}^{V}}{F^2} f^{ace} f^{bde}\Big\{ s_\beta (s_\mu p_\alpha - (p\cdot s) g_{\mu\alpha}) - s_\alpha (s_\mu p_\beta - (p\cdot s) g_{\mu\beta}) \Big\} \\
&+\frac{2\sqrt{2}\lambda_{19}^{V}}{F^2} f^{ade} f^{bce}\Big\{ r_\beta (r_\mu p_\alpha - (p\cdot r) g_{\mu\alpha}) - r_\alpha (r_\mu p_\beta - (p\cdot r) g_{\mu\beta}) \Big\} \\
&+\frac{\sqrt{2}\lambda_{21}^{V}}{F^2} f^{abe} f^{cde} (r_\alpha s_\beta - r_\beta s_\alpha)(q-r-s)_\mu + \frac{\sqrt{2}\lambda_{21}^{V} q^2}{F^2} f^{ace} f^{bde} (s_\beta g_{\mu\alpha} - s_\alpha g_{\mu\beta}) \\
&+\frac{\sqrt{2}\lambda_{21}^{V} q^2}{F^2} f^{ade} f^{bce} (r_\beta g_{\mu\alpha} - r_\alpha g_{\mu\beta}) \qquad \text{(B.3.15)}
\end{aligned}
$$

where

$$
\begin{aligned}
G_1 &= 2d^{abe}d^{cde} + \frac{4}{3}\delta^{ab}\delta^{cd} = 2T_5 + \frac{4}{3}T_1 \\
G_2(a,b,c,d) &= -\frac{1}{3}\delta^{ab}\delta^{cd} + (\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd}) - 2d^{abe}d^{cde} = -\frac{1}{3}T_1 + T_2 - 2T_5
\end{aligned}
$$

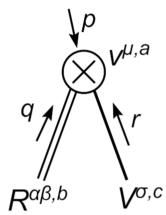
Vertex 16 : RRR



$$
\begin{aligned}
= \frac{3\sqrt{2}i\lambda^{VVV}}{8} f^{abc}\,(&g_{\alpha\lambda}g_{\beta\delta}g_{\gamma\kappa} - g_{\alpha\delta}g_{\beta\lambda}g_{\gamma\kappa} - g_{\alpha\kappa}g_{\beta\delta}g_{\gamma\lambda} + g_{\alpha\delta}g_{\beta\kappa}g_{\lambda\gamma} \\
&-g_{\alpha\lambda}g_{\beta\gamma}g_{\delta\kappa} + \gamma_{\alpha\gamma}g_{\beta\lambda}g_{\kappa\delta} + g_{\alpha\kappa}g_{\beta\gamma}g_{\lambda\delta} - g_{\alpha\gamma}g_{\beta\kappa}g_{\delta\lambda})
\end{aligned}
\qquad \text{(B.3.16)}
$$

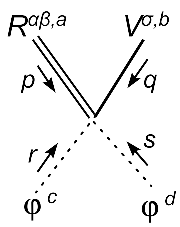
B.4 First order formalism

Vertex 1 : RVv



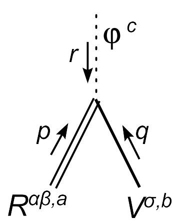
$$= \frac{iM}{2} f^{abc} (g_{\mu\alpha} g_{\beta\sigma} - g_{\mu\beta} g_{\alpha\sigma}) \tag{B.4.1}$$

Vertex 2 : $RV\phi\phi$



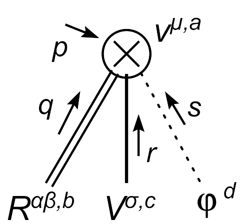
$$= -\frac{M}{4F^2} f^{abe} f^{cde} \left[(s-r)_\alpha g_{\beta\sigma} - (s-r)_\beta g_{\alpha\sigma} \right] \tag{B.4.2}$$

Vertex 3 : $RV\phi$



$$= -\frac{M\sigma_V}{F} d^{abc} \epsilon_{\alpha\beta\sigma\rho} r^\rho \tag{B.4.3}$$

Vertex 4 : $RVv\phi$



$$= -\frac{iM\sigma_V}{F} f^{ade} d^{bce} \epsilon_{\mu\alpha\beta\sigma} \tag{B.4.4}$$

APPENDIX C

Compton scattering

In this appendix we propose some technical calculations that were not included in the main text of thesis.

C.1 Basis of tensors

This gives 9 independent invariant tensors for general $SU(N)$, we can choose the basis as

$$
\begin{aligned}
T_1^{abcd} &= \langle T^a T^b \rangle \langle T^c T^d \rangle \\
T_{2,3}^{abcd} &= \langle T^a T^c \rangle \langle T^b T^d \rangle \pm \langle T^a T^d \rangle \langle T^b T^c \rangle \\
T_4^{abcd} &= -\frac{1}{2} \langle [T^a, T^b][T^c, T^d] \rangle \\
T_5^{abcd} &= \frac{1}{2} \langle \{T^a, T^b\}\{T^c, T^d\} \rangle - \frac{2}{3} \langle T^a T^b \rangle \langle T^c T^d \rangle \\
T_6^{abcd} &= \langle T^a T^c T^b T^d \rangle + \langle T^a T^d T^b T^c \rangle + \frac{1}{2} \langle \{T^a, T^b\}\{T^c, T^d\} \rangle \\
&\quad - \frac{2}{3} \langle T^a T^c \rangle \langle T^b T^d \rangle - \frac{2}{3} \langle T^a T^d \rangle \langle T^b T^c \rangle \\
T_7^{abcd} &= -\frac{\mathrm{i}}{2} \langle \{T^a, T^b\}[T^c, T^d] \rangle \\
T_8^{abcd} &= -\frac{\mathrm{i}}{2} \langle [T^a, T^b]\{T^c, T^d\} \rangle \\
T_9^{abcd} &= -\mathrm{i} \langle T^a T^c T^b T^d \rangle + \mathrm{i} \langle T^a T^d T^b T^c \rangle
\end{aligned}
$$

or, in terms of invariant tensors δ^{ab},f^{abc} and d^{abc}

$$
\begin{aligned}
T_1^{abcd} &= \delta^{ab}\delta^{cd} \\
T_{2,3}^{abcd} &= \delta^{ac}\delta^{bd} \pm \delta^{ad}\delta^{bc} \\
T_4^{abcd} &= f^{abl}f^{cdl} \\
T_5^{abcd} &= d^{abl}d^{cdl} \\
T_6^{abcd} &= d^{acl}d^{bdl} + d^{bcl}d^{adl} \\
T_7^{abcd} &= d^{abl}f^{cdl} \\
T_8^{abcd} &= f^{abl}d^{cdl} \\
T_9^{abcd} &= \frac{1}{2}\left(f^{acl}d^{bdl} + d^{acl}f^{bdl} - d^{adl}f^{bcl} - f^{adl}d^{bcl}\right)
\end{aligned}
$$

For general $SU(N)$ we the general result for $G_{\mu\nu}^{abcd}(p,q,r;s)$ and $A_{\mu\nu}^{abcd}(p,q,r;s)$ have the given forms

$$
\begin{aligned}
G_{\mu\nu}^{abcd}(p,q,r;s) &= \sum_{i=1}^{9} G_{\mu\nu}(p,q,r;s)^{(i)}T_i^{abcd} \\
A_{\mu\nu}^{abcd}(p,q,r;s) &= \sum_{i=1}^{9} A_{\mu\nu}(p,q,r;s)^{(i)}T_i^{abcd}
\end{aligned}
$$

For $SU(3)$ we have the additional Cayley-Hamilton identity

$$
\begin{aligned}
0 &= T^aT^bT^c + T^aT^cT^b + T^cT^aT^b + T^bT^aT^c + T^bT^cT^a + T^cT^bT^a \\
&\quad -T^a\langle T^bT^c\rangle - T^b\langle T^aT^c\rangle - T^c\langle T^aT^b\rangle - \langle T^aT^bT^c\rangle - \langle T^aT^cT^b\rangle
\end{aligned}
$$

that helps us to express T_6^{abcd} in terms of other group structures

$$
\begin{aligned}
d^{abl}d^{cdl} &= \frac{1}{3}\delta^{ab}\delta^{cd} + \frac{1}{3}(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd}) - (d^{adl}d^{bcl} + d^{acl}d^{bdl}). \\
T_6^{abcd} &= \frac{1}{3}T_1^{abcd} + \frac{1}{3}T_2^{abcd} - T_5^{abcd}
\end{aligned}
$$

which reduces the number of independent group structures to eight. Moreover, the invariances under C, P and T allow us to eliminate the structures T_7^{abcd}, T_8^{abcd} and T_9^{abcd}. As a result we have five independent group structures T_i^{abcd}, $i = 1, 2, \ldots 5$. In some cases we will use the short notation $T_i^{\overline{q}} = T_i^{abcd}$.

C.2 Formfactors

ChPT contribution

The list of the formfactors up to $\mathcal{O}(p^2)$ is following

$$\mathcal{A}(p^2,q^2,S,U)_1^1 = -\frac{i(S+U)}{2SU}, \tag{C.2.1}$$
$$\mathcal{A}(p^2,q^2,S,U)_1^2 = \frac{i(S+U)}{2SU}, \tag{C.2.2}$$
$$\mathcal{A}(p^2,q^2,S,U)_1^3 = 0, \tag{C.2.3}$$
$$\mathcal{A}(p^2,q^2,S,U)_1^4 = -\frac{i(U-S)}{2SU}, \tag{C.2.4}$$
$$\mathcal{A}(p^2,q^2,S,U)_1^5 = -\frac{3i(S+U)}{2SU}. \tag{C.2.5}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^1 = \frac{i}{SU}, \tag{C.2.6}$$
$$\mathcal{A}(p^2,q^2,S,U)_2^2 = -\frac{i}{SU}, \tag{C.2.7}$$
$$\mathcal{A}(p^2,q^2,S,U)_2^3 = 0, \tag{C.2.8}$$
$$\mathcal{A}(p^2,q^2,S,U)_2^4 = -\frac{i(S+U)}{SU(S-U)}, \tag{C.2.9}$$
$$\mathcal{A}(p^2,q^2,S,U)_2^5 = \frac{3i}{SU}. \tag{C.2.10}$$

Other formfactors vanish identically.

Vector formalism

$\underline{\mathcal{A}(p^2,q^2,S,U)_1^i}$

$$\begin{aligned}
\mathcal{A}(p^2,q^2,S,U)_1^1 = {} & \frac{2ih_V^2\left[6M^2(S+U)-S^2-U^2-10SU-2(2M^2-S-U)(p^2+q^2)\right]}{3F^2(M^2-S)(M^2-U)} \\
& + \frac{if_V(M^2-p^2)q^2\left[SU(2g_V-f_V+2\sqrt{2}\alpha_V)-(S+U)q^2g_V\right]}{2F^2SU(M^2-p^2)(M^2-q^2)} \\
& + \frac{if_V(M^2-q^2)p^2\left[SU(2g_V-f_V+2\sqrt{2}\alpha_V)-(S+U)p^2g_V\right]}{2F^2SU(M^2-p^2)(M^2-q^2)} \\
& + \frac{if_V^2g_V^2p^4q^4(S+U)}{2F^4SU(M^2-p^2)(M^2-q^2)} - \frac{i(S+U)}{2SU}
\end{aligned} \tag{C.2.11}$$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_1^2 =& \frac{2ih_V^2\left[6M^2(S+U)-S^2-U^2-10SU-2(2M^2-S-U)(p^2+q^2)\right]}{3F^2(M^2-S)(M^2-U)}\\ &+\frac{if_V(M^2-p^2)q^2\left[(S+U)q^2g_V+SU(f_V-2g_V-2\sqrt{2}\alpha_V)\right]}{2F^2SU(M^2-p^2)(M^2-q^2)}\\ &+\frac{if_V(M^2-q^2)p^2\left[(S+U)p^2g_V+SU(f_V-2g_V-2\sqrt{2}\alpha_V)\right]}{2F^2SU(M^2-p^2)(M^2-q^2)}\\ &+\frac{if_V^2g_V^2p^4q^4(S+U)}{2F^4SU(M^2-p^2)(M^2-q^2)}+\frac{i(S+U)}{2SU}\end{aligned} \tag{C.2.12}$$

$$\mathcal{A}(p^2,q^2,S,U)_1^3 = \frac{4ih_V^2(S-U)(4M^2+S+U-2p^2-2q^2)}{3F^2(M^2-S)(M^2-U)} \tag{C.2.13}$$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_1^4 =& \frac{2ih_V^2(S-U)(4M^2+S+U-2p^2-2q^2)}{F^2(M^2-S)(M^2-U)}\\ &+\frac{if_V^2g_V^2(S-U)p^4q^4}{2F^4SU(M^2-p^2)(M^2-q^2)}+\frac{if_Vg_V(S-U)\left[(M^2-p^2)q^4+(M^2-q^2)p^4\right]}{2F^2SU(M^2-p^2)(M^2-q^2)}-\frac{i(U-S)}{2SU}\end{aligned} \tag{C.2.14}$$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_1^5 =& \frac{2ih_V^2\left[6M^2(S+U)-S^2-U^2-10SU-2(2M^2-S-U)(p^2+q^2)\right]}{F^2(M^2-S)(M^2-U)}\\ &+\frac{3if_V(M^2-p^2)q^2\left[-(S+U)q^2g_V+SU(-f_V+2g_V+2\sqrt{2}\alpha_V)\right]}{2F^2SU(M^2-p^2)(M^2-q^2)}\\ &+\frac{3if_V(M^2-q^2)p^2\left[-(S+U)p^2g_V+SU(-f_V+2g_V+2\sqrt{2}\alpha_V)\right]}{2F^2SU(M^2-p^2)(M^2-q^2)}\\ &-\frac{3if_V^2g_V^2p^4q^4(S+U)}{2F^4SU(M^2-p^2)(M^2-q^2)}-\frac{3i(S+U)}{2SU}\end{aligned} \tag{C.2.15}$$

$\mathcal{A}(p^2,q^2,S,U)_2^i$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_2^1 =& \frac{4ih_V^2(2M^2-S-U+p^2+q^2)}{3F^2(M^2-S)(M^2-U)}\\ &+\frac{if_Vg_V\left[(M^2-q^2)p^4+(M^2-p^2)q^4\right]}{F^2SU(M^2-p^2)(M^2-q^2)}+\frac{if_V^2g_V^2p^4q^4}{F^4SU(M^2-p^2)(M^2-q^2)}+\frac{i}{SU}\end{aligned} \tag{C.2.16}$$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_2^2 =& \frac{4ih_V^2(2M^2-S-U+p^2+q^2)}{3F^2(M^2-S)(M^2-U)}-\frac{if_Vg_V\left[(M^2-q^2)p^4+(M^2-p^2)q^4\right]}{F^2SU(M^2-p^2)(M^2-q^2)}\\ &-\frac{if_V^2g_V^2p^4q^4}{F^4SU(M^2-p^2)(M^2-q^2)}-\frac{i}{SU}\end{aligned} \tag{C.2.17}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^3 = -\frac{8ih_V^2\left[(S-U)^2+(2M^2-S-U)(p^2+q^2)\right]}{3F^2(S-U)(M^2-S)(M^2-U)} \tag{C.2.18}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^4 = \frac{4ih_V^2\left[(S-U)^2+(2M^2-S-U)(p^2+q^2)\right]}{F^2(M^2-S)(M^2-U)(S-U)} - \frac{i(S+U)p^4q^4f_V^2g_V^2}{F^4(M^2-p^2)(M^2-q^2)SU(S-U)} - \frac{if_Vg_V\left[q^4(M^2-p^2)+p^4(M^2-q^2)\right](S+U)}{F^2(M^2-p^2)(M^2-q^2)SU(S-U)} - \frac{4if_Vg_V\left[p^4(M^2-q^2)+q^4(M^2-p^2)+(M^2-p^2)(M^2-q^2)(S+U)\right]}{F^2(M^2-p^2)(M^2-q^2)SU(S-U)(M^2+S+U-p^2-q^2)} + \frac{7if_Vg_VM^2\left[p^2(M^2-q^2)+q^2(M^2-p^2)\right]}{F^2(M^2-p^2)(M^2-q^2)(S-U)(M^2+S+U-p^2-q^2)} + \frac{if_V(f_V-2\sqrt{2}\alpha_V)\left[(M^2-q^2)p^2+(M^2-p^2)q^2\right]}{2F^2(M^2-p^2)(M^2-q^2)(S-U)} - \frac{i(S+U)}{SU(S-U)} \tag{C.2.19}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^5 = \frac{4ih_V^2(2M^2-S-U+p^2+q^2)}{F^2(M^2-S)(M^2-U)} + \frac{3if_V^2g_V^2p^4q^4}{F^4(M^2-p^2)(M^2-q^2)SU} + \frac{3if_Vg_V\left[(M^2-p^2)q^4+(M^2-q^2)p^4\right]}{F^2(M^2-p^2)(M^2-q^2)SU} + \frac{3i}{SU} \tag{C.2.20}$$

<u>$\mathcal{A}(p^2,q^2,S,U)_3^i$</u>

$$\mathcal{A}(p^2,q^2,S,U)_3^1 = \frac{4ih_V^2(2M^2-S-U)}{3F^2(M^2-S)(M^2-U)} + \frac{ih_V^2(S-U)^2(p^2+q^2)}{3F^2(M^2-S)(M^2-U)p^2q^2} - \frac{\sqrt{2}i\alpha_V(2M^2-p^2-q^2)}{F^2(M^2-p^2)(M^2-q^2)} + \frac{if_V^2g_V^2p^2q^2}{F^4(M^2-p^2)(M^2-q^2)} \tag{C.2.21}$$

$$\mathcal{A}(p^2,q^2,S,U)_3^2 = \frac{4ih_V^2(2M^2-S-U)}{3F^2(M^2-S)(M^2-U)} + \frac{ih_V^2(S-U)^2(p^2+q^2)}{3F^2(M^2-S)(M^2-U)p^2q^2} + \frac{\sqrt{2}i\alpha_V(2M^2-p^2-q^2)}{F^2(M^2-p^2)(M^2-q^2)} - \frac{if_V^2g_V^2p^2q^2}{F^4(M^2-p^2)(M^2-q^2)} \tag{C.2.22}$$

$$\mathcal{A}(p^2,q^2,S,U)_3^3 = \frac{2ih_V^2(S-U)\left[(S+U-2M^2)(p^2+q^2)-4p^2q^2\right]}{3F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.23}$$

$$\mathcal{A}(p^2,q^2,S,U)_3^4 = -\frac{ih_V^2(S-U)\left[(S+U-2M^2)(p^2+q^2)-4p^2q^2\right]}{F^2p^2q^2(M^2-S)(M^2-U)} + \frac{if_V(f_V-2\sqrt{2}\alpha_V)\left[(M^2-p^2)q^2+(M^2-q^2)p^2\right](S-U)}{8F^2p^2q^2(M^2-p^2)(M^2-q^2)} - \frac{if_Vg_VM^2(S-U)\left[(M^2-p^2)q^2+(M^2-q^2)p^2\right]}{4F^2(M^2-p^2)(M^2-q^2)p^2q^2(M^2+S+U-p^2-q^2)} \tag{C.2.24}$$

$$\mathcal{A}(p^2,q^2,S,U)_3^5 = \frac{4ih_V^2(S+U-2M^2)}{F^2(M^2-S)(M^2-U)} - \frac{ih_V^2(S-U)^2(p^2+q^2)}{F^2p^2q^2(M^2-S)(M^2-U)} - \frac{3\sqrt{2}if_V\alpha_V(2M^2-p^2-q^2)}{F^2(M^2-p^2)(M^2-q^2)} - \frac{3if_V^2g_V^2p^2q^2}{F^4(M^2-p^2)(M^2-q^2)} \tag{C.2.25}$$

$\underline{\mathcal{A}(p^2,q^2,S,U)_4^i}$

$$\mathcal{A}(p^2,q^2,S,U)_4^1 = -\frac{2ih_V^2(S+U)(p^2+q^2)}{3F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.26}$$

$$\mathcal{A}(p^2,q^2,S,U)_4^2 = -\frac{2ih_V^2(S+U)(p^2+q^2)}{3F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.27}$$

$$\mathcal{A}(p^2,q^2,S,U)_4^3 = \frac{4ih_V^2(2M^2-S-U)(S+U)(p^2+q^2)}{3F^2p^2q^2(M^2-S)(M^2-U)(S-U)} \tag{C.2.28}$$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_4^4 &= -\frac{2ih_V^2(2M^2-S-U)(S+U)(p^2+q^2)}{F^2(M^2-S)(M^2-U)p^2q^2}\\ &+\frac{2if_V^2g_V^2p^2q^2}{F^4(M^2-p^2)(M^2-q^2)(S-U)}\\ &-\frac{if_V(f_V-2\sqrt{2}\alpha_V)(S+U)\left[(M^2-q^2)p^2+(M^2-p^2)q^2\right]}{4F^2(M^2-p^2)(M^2-q^2)(S-U)p^2q^2}\\ &+\frac{if_Vg_V\left(2p^2q^2\left[2M^2(p^2+q^2-2M^2)-p^2q^2\right]+(S+U)M^2\left[M^2(p^2+q^2)-2p^2q^2\right]\right)}{2F^2(M^2-p^2)(M^2-q^2)p^2q^2(M^2+S+U-p^2-q^2)(S-U)}\end{aligned} \tag{C.2.29}$$

$$\mathcal{A}(p^2,q^2,S,U)_4^5 = \frac{2ih_V^2(S+U)(p^2+q^2)}{F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.30}$$

$\underline{\mathcal{A}(p^2,q^2,S,U)_5^i}$

$$\mathcal{A}(p^2,q^2,S,U)_5^1 = \frac{4ih_V^2(p^2-q^2)}{3F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.31}$$

$$\mathcal{A}(p^2,q^2,S,U)_5^2 = \frac{4ih_V^2(p^2-q^2)}{3F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.32}$$

$$\mathcal{A}(p^2,q^2,S,U)_5^3 = -\frac{8ih_V^2(2M^2-S-U)(p^2-q^2)}{3F^2p^2q^2(M^2-S)(M^2-U)(S-U)} \tag{C.2.33}$$

$$\begin{aligned}\mathcal{A}(p^2,q^2,S,U)_5^4 &= -\frac{4ih_V^2(S+U-2M^2)(p^2-q^2)}{F^2(M^2-S)(M^2-U)(S-U)p^2q^2}\\ &+\frac{if_V(f_V-2\sqrt{2}\alpha_V)M^2(p^2-q^2)}{2F^2(S-U)p^2q^2(M^2-p^2)(M^2-q^2)}\\ &-\frac{if_Vg_VM^4(p^2-q^2)}{F^2(S-U)p^2q^2(M^2-p^2)(M^2-q^2)(M^2+S+U-p^2-q^2)}\end{aligned} \tag{C.2.34}$$

$$\mathcal{A}(p^2,q^2,S,U)_5^5 = -\frac{4ih_V^2(p^2-q^2)}{F^2p^2q^2(M^2-S)(M^2-U)} \tag{C.2.35}$$

Antisymmetric tensor formalism up to $\mathcal{O}(p^4)$

$\underline{\mathcal{A}(p^2,q^2,S,U)_1^i}$

$$\mathcal{A}(p^2,q^2,S,U)_1^1 = -\frac{iF_V^2G_V^2p^2q^2(S+U)}{2F^4SU(M^2-p^2)(M^2-q^2)} - \frac{iF_VG_V\left[2SU(p^2+q^2-2M^2)-(S+U)(2p^2q^2-M^2p^2-M^2q^2)\right]}{2F^2SU(M^2-p^2)(M^2-q^2)} - \frac{i(S+U)}{2SU} \quad \text{(C.2.36)}$$

$$\mathcal{A}(p^2,q^2,S,U)_1^2 = \frac{iF_V^2G_V^2p^2q^2(S+U)}{2F^4SU(M^2-p^2)(M^2-q^2)} - \frac{iF_VG_V\left[2SU(p^2+q^2-2M^2)-(S+U)(2p^2q^2-M^2p^2-M^2q^2)\right]}{2F^2SU(M^2-p^2)(M^2-q^2)} + \frac{i(S+U)}{2SU} \quad \text{(C.2.37)}$$

$$\mathcal{A}(p^2,q^2,S,U)_1^3 = 0 \quad \text{(C.2.38)}$$

$$\mathcal{A}(p^2,q^2,S,U)_1^4 = -\frac{iF_V^2G_V^2p^2q^2(U-S)}{2F^4SU(M^2-p^2)(M^2-q^2)} - \frac{iF_V^2(p^2-q^2)}{2F^2(M^2-p^2)(M^2-q^2)} - \frac{iF_VG_V(U-S)(M^2p^2+M^2q^2-2p^2q^2)}{2F^2SU(M^2-p^2)(M^2-q^2)} - \frac{i(U-S)}{2SU} \quad \text{(C.2.39)}$$

$$\mathcal{A}(p^2,q^2,S,U)_1^5 = -\frac{3iF_V^2G_V^2p^2q^2(S+U)}{2F^4SU(M^2-p^2)(M^2-q^2)} + \frac{3iF_VG_V(2M^2-p^2-q^2)}{F^2(M^2-p^2)(M^2-q^2)} - \frac{3iF_VG_V(S+U)(M^2p^2+M^2q^2-2p^2q^2)}{2F^2SU(M^2-p^2)(M^2-q^2)} - \frac{3i(S+U)}{2SU} \quad \text{(C.2.40)}$$

$\underline{\mathcal{A}(p^2,q^2,S,U)_2^i}$

$$\mathcal{A}(p^2,q^2,S,U)_2^1 = -\frac{iF_VG_V(2p^2q^2-M^2p^2-M^2q^2)}{F^2SU(M^2-p^2)(M^2-q^2)} + \frac{iF_V^2G_V^2p^2q^2}{F^4SU(M^2-q^2)(M^2-p^2)} + \frac{i}{SU} \quad \text{(C.2.41)}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^2 = \frac{iF_VG_V(2p^2q^2-M^2p^2-M^2q^2)}{F^2SU(M^2-p^2)(M^2-q^2)} - \frac{iF_V^2G_V^2p^2q^2}{F^4SU(M^2-q^2)(M^2-p^2)} - \frac{i}{SU} \quad \text{(C.2.42)}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^3 = 0 \quad \text{(C.2.43)}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^4 = -\frac{iF_V^2G_V^2(S+U)p^2q^2}{F^4SU(M^2-p^2)(M^2-q^2)(S-U)}$$
$$-\frac{iF_VG_V}{F^2SU(M^2-p^2)(M^2-q^2)(M^2+S+U-p^2-q^2)(S-U)}\times$$
$$\Big\{M^4(S+U)(p^2+q^2)-4M^4SU-M^2(S+U)(p^4+q^4)$$
$$+M^2(p^2+q^2)(S^2+U^2)+3M^2SU(p^2+q^2)-4M^2p^2q^2(S+U)$$
$$+2p^2q^2(S+U)(p^2+q^2)-2p^2q^2(S^2+U^2+SU)\Big\}-\frac{i(S+U)}{SU(S-U)} \quad \text{(C.2.44)}$$

$$\mathcal{A}(p^2,q^2,S,U)_2^5 = -\frac{3iF_VG_V(2p^2q^2-M^2p^2-M^2q^2)}{F^2SU(M^2-p^2)(M^2-q^2)}+\frac{3iF_V^2G_V^2p^2q^2}{F^4SU(M^2-q^2)(M^2-p^2)}+\frac{3i}{SU} \quad \text{(C.2.45)}$$

<u>$\mathcal{A}(p^2,q^2,S,U)_3^i$</u>

$$\mathcal{A}(p^2,q^2,S,U)_3^1 = \frac{iF_V^2G_V^2}{F^4(M^2-p^2)(M^2-q^2)} \quad \text{(C.2.46)}$$
$$\mathcal{A}(p^2,q^2,S,U)_3^2 = -\frac{iF_V^2G_V^2}{F^4(M^2-p^2)(M^2-q^2)} \quad \text{(C.2.47)}$$
$$\mathcal{A}(p^2,q^2,S,U)_3^3 = 0 \quad \text{(C.2.48)}$$

$$\mathcal{A}(p^2,q^2,S,U)_3^4 = -\frac{iF_V^2(p^2+q^2)(U-S)}{8F^2p^2q^2(M^2-p^2)(M^2-q^2)}$$
$$-\frac{iF_VG_V(S-U)(M^2p^2+M^2q^2-2p^2q^2)}{4F^2(M^2-p^2)(M^2-q^2)p^2q^2(M^2+S+U-p^2-q^2)} \quad \text{(C.2.49)}$$
$$\mathcal{A}(p^2,q^2,S,U)_3^5 = \frac{3iF_V^2G_V^2}{F^4(M^2-p^2)(M^2-q^2)} \quad \text{(C.2.50)}$$

<u>$\mathcal{A}(p^2,q^2,S,U)_4^i$</u>

$$\mathcal{A}(p^2,q^2,S,U)_4^1 = \mathcal{A}(p^2,q^2,S,U)_4^2 = \mathcal{A}(p^2,q^2,S,U)_4^3 = \mathcal{A}(p^2,q^2,S,U)_4^5 = 0 \quad \text{(C.2.51)}$$
$$\mathcal{A}(p^2,q^2,S,U)_4^4 = \quad \text{(C.2.52)}$$
$$-\frac{iF_VG_V\left[2p^2q^2(S+U+4M^2)-M^2(S+U)(p^2+q^2)-4p^2q^2(p^2+q^2)\right]}{2F^2(S-U)(M^2-p^2)(M^2-q^2)p^2q^2(M^2+S+U-p^2-q^2)}$$

$\mathcal{A}(p^2, q^2, S, U)_5^i$

$$\mathcal{A}(p^2,q^2,S,U)_5^1 = \mathcal{A}(p^2,q^2,S,U)_5^2 = \mathcal{A}(p^2,q^2,S,U)_5^3 = \mathcal{A}(p^2,q^2,S,U)_5^5 = 0 \quad \text{(C.2.53)}$$

$$\mathcal{A}(p^2,q^2,S,U)_5^4 = \frac{i(p^2-q^2)F_V^2}{2F^2(S-U)p^2q^2(M^2-q^2)(M^2-p^2)} - \frac{iM^2F_VG_V(p^2-q^2)}{F^2(S-U)p^2q^2(M^2-p^2)(M^2-q^2)(M^2+S+U-p^2-q^2)} \quad \text{(C.2.54)}$$

C.3 Application of high energy constraints

The following asymptotical behaviour of formfactors for $i \neq 4$

$$A(p^2,q^2,S,U;T)_1^i = O\left(\frac{1}{\lambda^4}\right) \quad \text{(C.3.1)}$$

$$A(p^2,q^2,S,U;T)_2^i = O\left(\frac{1}{\lambda^4}\right) \quad \text{(C.3.2)}$$

$$A(p^2,q^2,S,U;T)_3^i = O\left(\frac{1}{\lambda^6}\right) \quad \text{(C.3.3)}$$

$$A(p^2,q^2,S,U;T)_4^i = O\left(\frac{1}{\lambda^6}\right) \quad \text{(C.3.4)}$$

$$A(p^2,q^2,S,U;T)_5^i = O\left(\frac{1}{\lambda^7}\right) \quad \text{(C.3.5)}$$

while for others

$$A(p^2,q^2,S,U;T)_1^4 = O\left(\frac{1}{\lambda^3}\right) \quad \text{(C.3.6)}$$

$$A(p^2,q^2,S,U;T)_2^4 = O\left(\frac{1}{\lambda^3}\right) \quad \text{(C.3.7)}$$

$$A(p^2,q^2,S,U;T)_3^4 = O\left(\frac{1}{\lambda^5}\right) \quad \text{(C.3.8)}$$

$$A(p^2,q^2,S,U;T)_4^4 = O\left(\frac{1}{\lambda^5}\right) \quad \text{(C.3.9)}$$

$$A(p^2,q^2,S,U;T)_5^4 = O\left(\frac{1}{\lambda^7}\right) \quad \text{(C.3.10)}$$

The results are compared with the OPE constraints (4.5.2). Therefore we write the formfactors $A(p^2,q^2,S,U;T)_j^i$ in terms of the kinematic values $\Sigma = p+q$, $\Delta = r-s$.

ChPT contribution $\mathcal{O}(p^2)$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^1 = \frac{4i}{\Sigma^2\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.11}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^2 = -\frac{4i}{\Sigma^2\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.12}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^3 = 0, \tag{C.3.13}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^4 = -\frac{8i(k\cdot\Sigma)}{\Sigma^4\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.14}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^5 = \frac{12i}{\Sigma^2\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.15}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^1 = -\frac{16i}{\Sigma^4\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.16}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^2 = \frac{16i}{\Sigma^4\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.17}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^3 = 0, \tag{C.3.18}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^4 = \frac{8i}{(\Sigma\cdot k)\Sigma^2\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.19}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^5 = -\frac{48i}{\Sigma^4\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right). \tag{C.3.20}$$

Vector formalism

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^1 = -\frac{if_V^2 g_V^2 \Sigma^2}{4F^4}\lambda^2 \tag{C.3.21}$$
$$+\left\{\frac{8ih_V^2}{3F^4} + \frac{if_V(f_V - 2\sqrt{2}\alpha_V)}{F^2} - \frac{if_V^2 g_V^2(3k^2 + 8M^2 - 2\Delta^2)}{4F^4}\right\} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^2 = \frac{if_V^2 g_V^2 \Sigma^2}{4F^4}\lambda^2 \tag{C.3.22}$$
$$+\left\{\frac{8ih_V^2}{3F^4} - \frac{if_V(f_V - 2\sqrt{2}\alpha_V)}{F^2} + \frac{if_V^2 g_V^2(3k^2 + 8M^2 - 2\Delta^2)}{4F^4}\right\} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^3 = -\frac{32ih_V^2(k\cdot\Sigma)}{3F^2\Sigma^2\lambda} + \mathcal{O}\left(\frac{1}{\lambda^3}\right), \tag{C.3.23}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^4 = \frac{if_V^2 g_V^2 (k\cdot\Sigma)}{2F^4}\lambda \tag{C.3.24}$$
$$+\left\{\frac{16ih_V^2(k\cdot\Sigma)}{F^2\Sigma^2} + \frac{if_V^2 g_V^2(k\cdot\Sigma)(k^2+4M^2-\Delta^2)}{F^4\Sigma^2} - \frac{4if_V g_V(k\cdot\Sigma)}{F^4\Sigma^2}\right\}\frac{1}{\lambda} + \mathcal{O}\left(\frac{1}{\lambda^3}\right),$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_1^5 = -\frac{3if_V^2 g_V^2\Sigma^2}{4F^4}\lambda^2 \tag{C.3.25}$$
$$-\left\{\frac{8ih_V^2}{F^2} + \frac{3if_V^2 g_V^2(3k^2+8M^2-2\Delta^2)}{4F^4} - \frac{3if_V(f_V-2\sqrt{2}\alpha_V)}{F^2}\right\} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^1 = \frac{if_V^2 g_V^2}{F^4} + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \tag{C.3.26}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^2 = -\frac{if_V^2 g_V^2}{F^4} + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \tag{C.3.27}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^3 = \frac{32ih_V^2}{3F^2(k\cdot\Sigma)\lambda} + \mathcal{O}\left(\frac{1}{\lambda^3}\right), \tag{C.3.28}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^4 = -\frac{if_V^2 g_V^2(k\cdot\Sigma)}{2F^4 k^2}\lambda \tag{C.3.29}$$
$$+\left\{-\frac{16ih_V^2}{F^2(k\cdot\Sigma)} + \frac{if_V g_V(k^2-2M^2+\Delta^2)}{F^2(k\cdot\Sigma)(M^2-\Delta^2)} - \frac{if_V(f_V-2\alpha_V)}{F^2(k\cdot\Sigma)}\right.$$
$$\left. - \frac{if_V^2 g_V^2(3k^2+8M^2-2\Delta^2)}{2F^4(k\cdot\Sigma)}\right\}\frac{1}{\lambda} + \mathcal{O}\left(\frac{1}{\lambda^3}\right),$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_2^5 = \frac{3if_V^2 g_V^2}{F^4} + \mathcal{O}\left(\frac{1}{\lambda^2}\right) \tag{C.3.30}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^1 = \frac{i f_V^2 g_V^2}{F^4} + \left\{ \frac{8 i f_V (\sqrt{2}\alpha_V F^2 + M^2 f_V g_V^2)}{F^4 \Sigma^2} - \frac{32 i h_V^2}{3 F^2 \Sigma^2} \right\} \frac{1}{\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.31}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^2 = -\frac{i f_V^2 g_V^2}{F^4} - \left\{ \frac{8 i f_V (\sqrt{2}\alpha_V F^2 + M^2 f_V g_V^2)}{F^4 \Sigma^2} + \frac{32 i h_V^2}{3 F^2 \Sigma^2} \right\} \frac{1}{\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.32}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^3 = \frac{128 i h_V^2 (k\cdot\Sigma)(k^2 - 4M^2 + 3\Delta^2)}{3 F^2 \Sigma^6 \lambda^5} + \mathcal{O}\left(\frac{1}{\lambda^7}\right), \tag{C.3.33}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^4 = - \left\{ \frac{4 i f_V g_V (k\cdot\Sigma)(k^2 - 2M^2 + \Delta^2)}{F^2 \Sigma^4 (M^2 - \Delta^2)} + \frac{4 i f_V (f_V - 2\sqrt{2}\alpha_V)(k\cdot\Sigma)}{F^2 \Sigma^4} \right\} \frac{1}{\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.34}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^5 = \frac{3 i f_V^2 g_V^2}{F^4} + \left\{ \frac{32 i h_V^2}{F^2 \Sigma^2 \lambda^2} + \frac{24 i f_V (\sqrt{2}\alpha_V F^2 + M^2 f_V g_V^2)}{F^4 \Sigma^2} \right\} \frac{1}{\lambda^2} + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.35}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^1 = -\frac{128 i h_V^2}{3 F^2 \Sigma^4 \lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right) \tag{C.3.36}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^2 = -\frac{128 i h_V^2}{3 F^2 \Sigma^4 \lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right) \tag{C.3.37}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^3 = -\frac{128 i h_V^2}{3 F^2 (k\cdot\Sigma)\Sigma^2 \lambda^3} - \frac{128 i h_V^2 (3\Delta^2 + 4k^2 + 4M^2)}{3 F^2 (k\cdot\Sigma)\Sigma^4 \lambda^5} + \mathcal{O}\left(\frac{1}{\lambda^7}\right), \tag{C.3.38}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^4 = \frac{2 i f_V^2 g_V^2}{F^4 (k\cdot\Sigma)\lambda} + \left\{ \frac{64 i h_V^2}{F^4 (k\cdot\Sigma)\Sigma^2} + \frac{4 i f_V (f_V - 2\sqrt{2}\alpha_V)}{F^2 (k\cdot\Sigma)\Sigma^2} - \frac{4 i f_V g_V (k^2 - 2M^2 + \Delta^2)}{F^2 (k\cdot\Sigma)\Sigma^2 (M^2 - \Delta^2)} + \frac{16 i M^2 f_V^2 g_V^2}{F^4 (k\cdot\Sigma)\Sigma^2} \right\} \frac{1}{\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.39}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_4^5 = \frac{128ih_V^2}{F^2\Sigma^4\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.40}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_5^1 = -\frac{1024ih_V^2(\Delta\cdot\Sigma)}{3F^2\Sigma^8\lambda^7} + \mathcal{O}\left(\frac{1}{\lambda^9}\right) \tag{C.3.41}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_5^2 = -\frac{1024ih_V^2(\Delta\cdot\Sigma)}{3F^2\Sigma^8\lambda^7} + \mathcal{O}\left(\frac{1}{\lambda^9}\right) \tag{C.3.42}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_5^3 = -\frac{1024ih_V^2(\Delta\cdot k)}{3F^2k^2\Sigma^6\lambda^6} + \mathcal{O}\left(\frac{1}{\lambda^8}\right), \tag{C.3.43}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_5^4 = \frac{512ih_V^2(\Delta\cdot k)}{F^2k^2\Sigma^6\lambda^6} + \mathcal{O}\left(\frac{1}{\lambda^8}\right),$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_5^5 = \frac{1024ih_V^2(\Delta\cdot\Sigma)}{F^2\Sigma^8\lambda^7} + \mathcal{O}\left(\frac{1}{\lambda^9}\right) \tag{C.3.44}$$

Antisymmetric tensor formalism

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_1^1 = -\frac{4i}{F^4\Sigma^2\lambda^2}\left(F_V^2G_V^2-F^4\right) + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.45}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_1^2 = \frac{4i}{F^4\Sigma^2\lambda^2}\left(F_V^2G_V^2-F^4\right) + \mathcal{O}\left(\frac{1}{\lambda^4}\right) \tag{C.3.46}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_1^3 = 0, \tag{C.3.47}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_1^4 = \left\{-\frac{8i(k\cdot\Sigma)}{\Sigma^4} + \frac{8iF_V^2(k\cdot\Delta)}{F^2\Sigma^4} + \frac{8iF_V^2G_V^2(k\cdot\Sigma)}{F^4\Sigma^4} - \frac{16iF_VG_V(k\cdot\Sigma)}{F^2\Sigma^4}\right\}\frac{1}{\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.48}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_1^5 = -\frac{12i}{F^4\Sigma^2\lambda^2}\left(F_V^2G_V^2-F^4\right) + \mathcal{O}\left(\frac{1}{\lambda^4}\right), \tag{C.3.49}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_2^1 = \left\{\frac{16i}{F^4\Sigma^4}\left(F_V^2G_V^2-F^4\right) - \frac{32iF_VG_V}{F^2\Sigma^4}\right\}\frac{1}{\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.50}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_2^2 = -\left\{\frac{16i}{F^4\Sigma^4}\left(F_V^2G_V^2-F^4\right) - \frac{32iF_VG_V}{F^2\Sigma^4}\right\}\frac{1}{\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.51}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_2^3 = 0, \tag{C.3.52}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_2^4 = \frac{2iF_VG_V}{F^2(k\cdot\Sigma)(\Delta^2-M^2)\lambda} + \left\{\frac{4iF_VG_V(k^2-2M^2-3\Delta^2)}{F^2(k\cdot\Sigma)\Sigma^2(M^2-\Delta^2)} + \frac{4iF_V^2}{F^2(k\cdot\Sigma)\Sigma^2} - \frac{8i}{F^4(k\cdot\Sigma)\Sigma^2}\left(F_V^2G_V^2-F^4\right)\right\}\frac{1}{\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.53}$$

$$\mathcal{A}(\lambda^2p^2,\lambda^2q^2,S,U)_2^5 = \left\{\frac{48i}{F^4\Sigma^4}\left(F_V^2G_V^2-F^4\right) - \frac{96iF_VG_V}{F^2\Sigma^4}\right\}\frac{1}{\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.54}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^1 = \frac{16iF_V^2 G_V^2}{F^4\Sigma^4\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.55}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^2 = -\frac{16iF_V^2 G_V^2}{F^4\Sigma^4\lambda^4} + \mathcal{O}\left(\frac{1}{\lambda^6}\right), \tag{C.3.56}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^3 = 0, \tag{C.3.57}$$

$$\begin{aligned}\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^4 &= \frac{8iF_V G_V (k\cdot\Sigma)}{F^2\Sigma^4(M^2-\Delta^2)\lambda^3} \\ &+ \left\{\frac{16iF_V^2(k\cdot\Sigma)}{F^2\Sigma^6} + \frac{16iF_V G_V(k\cdot\Sigma)(2M^2+\Delta^2)}{F^2\Sigma^6(M^2-\Delta^2)}\right\}\frac{1}{\lambda^5} + \mathcal{O}\left(\frac{1}{\lambda^7}\right),\end{aligned} \tag{C.3.58}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_3^5 = \frac{48iF_V^2 G_V^2}{F^4\Sigma^4\lambda^4} + \frac{96iF_V^2 G_V^2(\Delta^2+4M^2)}{F^4\Sigma^6\lambda^6} + \mathcal{O}\left(\frac{1}{\lambda^8}\right). \tag{C.3.59}$$

$$\begin{aligned}A(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^1 &= \mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^2 \\ &= \mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^3 = \mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^5 = 0,\end{aligned} \tag{C.3.60}$$

$$\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_4^4 = \frac{8iF_V G_V}{F^2(k\cdot\Sigma)\Sigma^2(M^2-\Delta^2)\lambda^3} + \mathcal{O}\left(\frac{1}{\lambda^5}\right), \tag{C.3.61}$$

$$\begin{aligned}A(\lambda^2 p^2, \lambda^2 q^2, S, U)_5^1 &= \mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_5^2 \\ &= \mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_5^3 = \mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_5^5 = 0,\end{aligned} \tag{C.3.62}$$

$$\begin{aligned}&\mathcal{A}(\lambda^2 p^2, \lambda^2 q^2, S, U)_5^4 = \\ &\left\{-\frac{128iF_V^2(\Delta\cdot k)}{F^2k^2\Sigma^8} + \frac{128iF_V G_V(\Delta\cdot k)(k^2-2M^2+\Delta^2)}{F^2k^2\Sigma^8(\Delta^2-M^2)}\right\}\frac{1}{\lambda^8} + \mathcal{O}\left(\frac{1}{\lambda^{10}}\right).\end{aligned} \tag{C.3.63}$$

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