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Hedging with interest rate derivatives: Estimation of hedge ratio & hedging effectiveness

Master thesis

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Abstract

The thesis investigates the effectiveness of several hedging strategies and inspects whether advanced econometric models contribute to lower portfolio risk and offer advantages over simple constant hedges. Focused on the German bond market, Euro-Bund and Euro-Bobl futures traded at Eurex are employed to determine which hedging strategy performs best in the fixed-income framework. The hedge ratio is estimated with the OLS, VAR, VECM, and GARCH models, as well as with the duration-based approach. The hedging effectiveness is subsequently measured in terms of percentage variance reduction of a portfolio's returns relative to an unhedged bond, while also considering risk-return trade-off. The analysis showed that the hedging strategies are, in almost all cases, effective in risk minimization though the degree of variance reduction does differ. The duration method decreases the variance by as much as 99% while mostly resulting in low or negative returns. Relative to other constant strategies, the time-varying hedge ratio, estimated by the GARCH, limits the variance least, nonetheless, mostly it provided a variance reduction of at least 65% while also delivering one of the highest returns. Whereas the dynamic strategy did not outperform constant hedges in terms of risk protection, the choice of hedge ratio eventually depends on an investor's risk appetite and potential costs of portfolio rebalancing when employing a dynamic approach.

JEL Classification	C13, C22, G11, G23
Keywords	hedging strategy, hedge ratio, hedging effectiveness, interest rate futures, bond portfolio, German bond market

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Abstrakt

Práce se zabývá efektivitou několika konkrétních hedgingových strategií a zkoumá, zda pokročilé ekonometrické modely přispívají k nižšímu riziku portfolia a nabízejí výhody oproti jednoduchým konstantním metodám. Práce se zaměřuje na německý dluhopisový trh, přičemž termínové kontrakty Euro-Bund a Euro-Bobl, které jsou obchodovány na Eurexu, jsou použity k určení nejvhodnější hedgingové strategie k zajištění dluhopisového portfolia. Hedgingový poměr je odhadnutý pomocí modelů OLS, VAR, VECM a GARCH a také podle přístupu založeného na duraci dluhopisu. Efektivita strategie je následně měřena pomocí procentuálního snížení variance výnosů portfolia oproti nezajištěnému dluhopisu, přičemž se také bere v úvahu výše výnosu. Analýza ukázala, že hedgingové strategie jsou téměř ve všech případech účinné v minimalizaci rizika, i když se míra redukce variance liší. Metoda využívající duraci dluhopisu snižuje varianci až o 99%, přičemž většinou vede k nízkým nebo negativním výnosům. V porovnání s konstantními metodami omezuje varianci nejméně časově variabilní hedgingový poměr odhadnutý pomocí modelu GARCH. Na druhou stranu ve většině případů přináší jeden z nejvyšších výnosů při redukci variance nejméně 65%. Zatímco dynamická strategie nepřekonala konstantní hedgingové strategie z hlediska rizika, volba hedgingového poměru nakonec závisí na averzi investora vůči riziku a potenciálních nákladech na změny vyvážení portfolia při použití dynamického přístupu.

JEL klasifikace	C13, C22, G11, G23
Klíčová slova	hedgingová strategie, hedgingový poměr, hedgingová efektivita, úrokové termínové kontrakty, dluhopisové portfolio, německý dluhopisový trh

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Declaration of Authorship

I hereby proclaim that I wrote my master thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, 25 December 2019

Signature

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Master's Thesis Proposal

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Proposed Topic

Hedging with interest rate derivatives: Estimation of hedge ratio & hedging effectiveness

Motivation

Interest rate risk is one of the most significant risks market participants face in the contemporary financial marketplace. One of the options to protect the value of a fixed-income portfolio is with interest rate futures contracts. When combining cash assets with futures contracts to offset the change in the asset's price it is crucial to determine a suitable number of futures contracts that should be held for each unit of a cash asset, specifically it is important to determine an appropriate hedge ratio.

Ederington (1979) applies portfolio theory to derive the minimum-variance hedge ratio that minimizes the variance of the cash portfolio of an investor. Such a hedge ratio is considered as optimal and can be estimated by standard OLS regression. When hedging bond portfolios specifically, another risk minimization technique that has its advocates uses the concept of duration where the hedge ratio is determined based on the relative durations of the bond and futures contract as per Pepić (2014).

Over the years the development of time-series models gave rise to more advanced methods for hedge ratio estimation with the aim to provide the gain in risk reduction over simple constant hedges. Designed to overcome the drawbacks of the OLS estimation, the more advanced methods aim to capture the time-varying nature of the relationships among financial time-series, the crucial notion that OLS-based methods fail to capture as per Hatemi-J & El-Khatib (2012). With an aim to determine the most effective hedging strategy, several authors employed methodologies involving GARCH-type models (Park & Jei (2010), Caporin, Jimenez-Martin & Gonzalez-Serrano (2014)), ECM (Kenourgios, Samitas & Drosos (2008)), VECM (Li (2010)) and wild bootstrap (Nguyen, Kim & Henry (2017)) among others. While some approaches provided portfolio's variance reduction, others failed to supply any evidence of the superiority of advanced methods over the simple OLS.

Despite academics and practitioners have been searching for an optimal hedge ratio over the last years, the superiority of one method over the other has not been fully confirmed. The purpose of this thesis is to evaluate the hedging effectiveness of various hedging approaches when applied on interest rate futures. While examining both constant minimum variance and time-varying hedge ratios from various econometric models the aim is to identify which method provides the best hedging performance.

Hypotheses

1. Advanced econometric methods provide a greater risk reduction over simple constant hedges.
2. The minimum-variance hedge ratio and duration-based hedge ratio lead to a similar hedging performance.

Methodology

The data employed in the analysis are daily prices of German government bonds and mid- and long-term interest rate futures contracts traded on Eurex Exchange. More specifically, Euro-Bund futures and Euro-Bobl futures are chosen for hedging because of their liquidity characteristics.

To address the hypotheses the first part of the analysis is focused on the estimation of hedge ratios. While employing several methodologies, hedge ratio is evaluated by both standard OLS regression and more advanced time-series econometric methods.

The second part is dedicated to evaluating hedging effectiveness of aforementioned methods and subsequent comparison. It is assumed that an investor holds a particular amount of a bond that he or she wishes to hedge with an interest rate futures contract over a specific time period. Both in-sample and out-of-sample performance of the hedge is evaluated to also see the hedging performance in a future time period.

Following Ederington (1979) the hedging effectiveness is calculated as the percentage reduction in variance of the hedged position relative to the unhedged portfolio. Furthermore, the return of the hedged portfolio in terms of its mean and variance, and MAE of forecast errors is evaluated for better comparison of the methods.

Expected Contribution

The analysis is conducted to evaluate the effectiveness of several methodologies in terms of their hedging performance. While there is relatively abundant literature on stock and FX markets in recent years, the efficiency of methods is determined in a different product framework and as such the thesis contributes to rather sparse literature on interest rate futures hedging. The aim is to determine whether advanced models with time-varying properties are superior to the constant approaches when hedging a fixed-income portfolio. With respect to the existing research on hedge ratio estimation, the study provides empirical results of the proposed models on data gathered outside of the US and it covers the recent years characterized by an increased interest rate volatility.

Outline

1. Introduction
2. Literature review
3. Hedging interest rate risk
4. Methodology
5. Data description
6. Empirical results
7. Conclusion

Core Bibliography

Caporin, M., Jimenez-Martin, J.-A. & Gonzalez-Serrano, L. (2014). Currency hedging strategies in strategic benchmarks and the global and euro sovereign financial crises. *Journal of International Financial Markets, Institutions and Money*, *31*, 159–177.

Ederington, L. H. (1979). The hedging performance of the new futures markets. *The Journal of Finance*, *34*(1), 157–170.

Hatemi-J, A. & El-Khatib, Y. (2012). Stochastic optimal hedge ratio: Theory and evidence. *Applied Economics Letters*, *19*(8), 699–703.

Kenourgios, D., Samitas, A. & Drosos, P. (2008). Hedge ratio estimation and hedging effectiveness: The case of the s&p 500 stock index futures contract. *International Journal of Risk assessment and management*, *9*(1-2), 121–134.

Li, M.-Y. L. (2010). Dynamic hedge ratio for stock index futures: Application of threshold vecm. *Applied Economics*, *42*(11), 1403–1417.

Nguyen, P., Kim, J. H. & Henry, D. (2017). *Estimation of optimal hedge ratio: A wild bootstrap approach*. La Trobe University, Department of Economics and Finance.

Park, S. Y. & Jei, S. Y. (2010). Estimation and hedging effectiveness of time-varying hedge ratio: Flexible bivariate garch approaches. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *30*(1), 71–99.

Pepić, M. (2014). Managing interest rate risk with interest rate futures. *Ekonomika preduzeća*, *62*(3-4), 201–209.

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Acronyms

ADF augmented Dickey-Fuller

ARCH autoregressive conditional heteroskedastic

ARIMA autoregressive integrated moving average

BEKK Baba, Engle, Kraft and Kroner

CAC Cotation Assistée en Continu

CTD cheapest to deliver

DAX Deutscher Aktienindex

ECM error correction model

FTSE Financial Times Stock Exchange

GARCH generalized autoregressive conditional heteroskedastic

KPSS Kwiatkowski-Phillips-Schmidt-Shin

MMI NYSE Arca Major Market Index

MRS Markov regime-switching

NIKKEI Nikkei heikin kabuka (Nikkei Stock Average)

OLS ordinary least squares

S&P Standard and Poor's

SD standard deviation

VAR vector autoregression

VECM vector error correction model

1 Introduction

With changes in interest rates affecting the value of a fixed income portfolio, bond market participants seek risk minimization techniques to protect their portfolios from interest rate risk. Rapid expansion of financial futures markets in 1970s, gave rise to bond futures which, because of their liquidity, speed and lower transaction costs, became popular instruments for hedging interest rate risk. When combining cash assets with futures contracts, in an attempt to minimize net changes in the portfolio value, it is of significance to determine a suitable number of futures that should be held for each unit of a fixed-income portfolio, i.e. it is crucial to identify an appropriate hedge ratio.

Based on the portfolio theory, the early research of Johnson (1960), Stein (1961), and Ederington (1979) derives the minimum variance hedge ratio that minimizes the variance of an investor's portfolio. As that could be estimated by standard OLS regression, several studies question the appropriateness of the method, highlighting the problems with serial correlation and heteroskedasticity (Herbst, Kare & Caples 1989; Herbst, Kare & Marshall 1993; H. Y. Park & Bera 1987). Time-series econometric methods offer alternative ways to derive hedge ratios, while not only correcting for the drawbacks of the OLS but also bringing new aspects to consider. Recognizing the cointegrating relationship between futures and spot prices, a number of authors (Ghosh & Clayton 1996; Kenourgios, Samitas & Drosos 2008; Li 2010) incorporate long-run equilibrium and short-run dynamics with the error correction model (ECM) and report improved hedging results with the derived hedge ratio. Some argue that constant hedge ratios are in contradiction with the changeable nature of the futures and spot relationship, and rather suggest time-varying hedge ratios that account for the arrival of new information during the life of the hedge. Several studies (Lien & Tse 1999; Lypny & Powalla 1998; Yang & Allen 2005) therefore employ this dynamic hedging approach with GARCH-type models to account for heteroskedasticity in the regression residuals, in an attempt to produce superior hedge ratios. Outside of the econometrics world and specific for fixed-income only, the hedge ratios for bond portfolios can also be calculated with modified durations and conversion factors.

With several approaches to estimate hedge ratio, the superiority of one over the other has not been fully confirmed while it has equally not been established whether the advanced time-varying hedge ratios provide greater hedging effectiveness. Moreover, many of the results are concentrated in the US environment and focus on stock index and FX markets.

This work is centred around hedging a bond portfolio with interest rate futures while focusing on German bonds and their highly liquid futures market. In particular, Euro-Bund and Euro-Bobl futures traded on the Eurex Exchange are chosen because they are characteristically liquid. The analysis is conducted over the years of the global financial crisis and the aftermaths of the European sovereign debt crisis that are characterized by low interest rates.

Hedge ratio is estimated with several methodologies, including OLS, VAR, VECM, and GARCH models, as well as the duration method. Their hedging effectiveness is subsequently measured in terms of portfolio variance reduction relative to an unhedged portfolio that achieved with a particular hedging strategy while also considering risk-return trade-off since it is assumed that investors want to maximize their expected utility rather than being pure risk minimizers. The aim of the work is to investigate whether advanced models with dynamic properties provide superior performance in terms of decreasing basis risk, relative to constant approaches when hedging a fixed-income portfolio.

The rest of the thesis is organized as follows. The second chapter presents the literature dealing with the hedge ratio estimation and provides the findings on the effectiveness of hedging strategies. The third chapter introduces the theoretical background of interest rate risk and highlights the main considerations when hedging with interest rate futures. The fourth chapter provides the methodology used for the analysis while the fifth chapter describes the dataset and the economic situation during the studied periods.

Chapter six presents the results including the estimated hedge ratios and evaluation of their hedging effectiveness. The final chapter concludes the thesis with the key findings and Appendix A and B cover tables and figures that were not included in the text.

2 Literature review

Hedging as a risk minimization technique has been one of the most important applications of futures markets and as such it has been a point of interest for many academics and practitioners to determine the right volume of futures contracts needed to secure a cash market position. This section reviews the extensive literature on hedge ratio estimation and presents associated findings and comparisons.

The foundations of the standard approach for hedge ratio estimation date back to the 1960s. Johnson (1960) and Stein (1961) reformulate the hedging strategies prevailing at that time while incorporating the portfolio theory. The two main hedging concepts the papers base on are the traditional hedging theory and Holbrook Working's approach (Working 1953). The traditional theory postulates a hedger primarily seeks a strategy to avoid risk. While assuming spot and futures prices move together it is suggested that a spot market position can be covered by a futures contract of the same size and the opposite direction[1]. This argument is, however, unjustified since spot and futures price changes are not necessarily equal. Working (1953) on the other hand argues that the risk avoidance is not a sole impetus and integrates an expected profit maximization element. He claims that an individual hedges in the interest of an expected return that results from anticipations of favourable relative movements in spot and futures prices by conducting an arbitrage. As per Johnson (1960) neither of the theories precisely corresponded to market phenomena. Johnson (1960) and Stein (1961) eventually apply the basic portfolio theory while incorporating the risk avoidance of the traditional theory and the expected profit maximization suggested by Working (1953). This portfolio model of hedging proposes that hedgers allocate their holdings between hedged and unhedged cash instruments based on their degree of risk aversion to reach an optimal risk-return trade-off.

The application of the portfolio theory to hedging was further studied

[1]For further details, see Ederington (1979)[159-160]

and in some cases enhanced. Some researchers focus on utility maximization function (Anderson & Danthine 1980; Benninga, Eldor & Zilcha 1984), some examine the intertemporal structure of optimal hedging (Adler & Detemple 1988; Briys, Crouhy & Schlesinger 1990), others consider the effects of duration and hedging horizon (Figlewski 1984; Hill & Schneeweis 1982; Junkus & Lee 1984; Lindahl 1992; Viswanath 1993).

Nonetheless, one of the most influential augmentations of Johnson (1960) and Stein (1961) is presented by Ederington (1979). While evaluating Government National Mortgage Association (GNMA) and T-Bill futures as instruments for hedging, Ederington (1979) applies the Markowitz portfolio theory and integrates price change expectations to derive a hedge ratio that maximizes profit conditional on some risk averse weighted variance.

Ederington (1979) considers a hedger with fixed spot market holdings X_s deciding about how much of the assets to hedge. Following the Markowitz portfolio theory the expected return on an unhedged position $E(U)$ and the variance of this return $Var(U)$ are then as follows:

$$E(U) = X_s E[P_s^2 - P_s^1] \tag{2.1}$$

$$Var(U) = X_s^2 \sigma_s^2, \tag{2.2}$$

where P_s^1 and P_s^2 are spot prices in periods 1 and 2, respectively, and σ_s represents the variance of spot price changes.

When including the futures market and the individual's futures holdings X_f, the expected return on the portfolio $E(R)$ and the corresponding variance $Var(R)$ are determined as

$$E(R) = X_s E[P_s^2 - P_s^1] - X_f E[P_f^2 - P_f^1] - K(X_f) \tag{2.3}$$

$$Var(R) = X_s^2 \sigma_s^2 + X_f^2 \sigma_f^2 - 2 X_s X_f \sigma_{sf}, \tag{2.4}$$

where similarly P_f^1 and P_f^2 are futures prices in periods 1 and 2, respectively, σ_f is the variance of futures price changes, σ_{sf} denotes the covariance of the potential price changes from period 1 to period 2, and $K(X_f)$ are brokerage and other costs accompanying the futures transaction including the cost of margin provision.

Since there is no presumption of hedging the entire spot market position, as is the case in the traditional theory, let b constitute the proportion of the spot market holdings being hedged:

$$b = \frac{X_f}{X_s}. \tag{2.5}$$

After expressing X_f from the equation (2.5) and substituting it into the equation (2.4), $Var(R)$ can be expressed as follows:

$$Var(R) = X_s^2[\sigma_s^2 + b^2\sigma_f^2 - 2b\sigma_{sf}]. \tag{2.6}$$

Taking the first derivative of the equation (2.6) and setting it to zero yield the following from which the minimum variance hedge ratio b^* can be further expressed:

$$\frac{\partial Var(R)}{\partial b} = X_s^2[2b\sigma_f^2 - 2\sigma_{sf}] \tag{2.7}$$

$$b^* = \frac{\sigma_{sf}}{\sigma_f^2}. \tag{2.8}$$

b^* is the portfolio model's optimal risk minimizing hedge ratio and is expressed as the ratio of the covariance between spot and futures prices to the variance of futures prices. This hedge ratio formula mirrors the OLS slope estimator from a traditional regression framework when spot price data are regressed on futures price data[2]. Ederington (1979) shows in his results that the optimal hedge ratio is on many occasions less than 1 suggesting that even for risk minimizers only it might be optimal to hedge just a proportion of their entire portfolio.

While the approach had generally been accepted the question arose around the estimation technicalities. Some papers estimate the hedge ratio with price levels (Witt, Schroeder & Hayenga 1987; Young, Hogan & Batten 2004), others incorporate first differences (Carter & Loyns 1985; Hill & Schneeweis 1982; Kenourgios et al. 2008; Myers & Thompson 1989) or percentage price changes (Brown 1985; Lypny & Powalla 1998). Young et al. (2004) argues that the application should depend on the hedging horizon and the variability of returns in the cash and futures markets. Focused on the returns' variability

[2]OLS methodology is further defined in section 4.1

while targeting longer time periods and using weekly or monthly data it is suggested to employ the first differences or the percentage price changes. This is supported by previous conclusions of Brown (1985). However, for shorter hedging horizons when using daily or more frequent data the price levels seem to provide more reliable results given the nonsynchronous trading between cash and futures markets.

When hedging interest rate risk duration-based approach is another technique to obtain a constant hedge ratio estimate. Gay, Kolb & Chiang (1983) and Landes, Stoffels & Seifert (1985) were among the first to consider the relative durations of cash bond and futures contract to minimize the net price change. A few studies concentrate on the comparison of the duration method with the OLS. Taevs & Jacob (1986) propose that the two methods are equivalent if the regression uses forecasted values. When hedging the long-term Bellwether bond and the two-year T-note Daigler (1998) questions this finding by demonstrating that the two methods can produce significantly different results for specific quarters. Further he shows in his results that while both being more efficient than a no-hedge and two naïve hedges neither of the methods is consistently superior over the other.

Over the years some problems with the simple regression model became apparent. Herbst et al. (1989) were among the first to stress the presence of serial correlation in the OLS residuals. To correct for the problem they employ a Box-Jerkins autoregressive integrated moving average (ARIMA) method and report better hedging results. Herbst et al. (1993) confirm the problem of serial correlation and heteroskedasticity and further emphasize that the OLS method fails to consider the effect of carrying costs and assumes a constant basis. They consider that in reality the basis declines when approaching a contract's maturity and by incorporating the time variable they reach an improved hedging performance. Recognizing the convergence effect other empirical studies deliver similar results (Castelino 1990; 1992; Leistikow 1993; Lindahl 1992; Viswanath 1993). More specifically, Viswanath (1993) includes the basis as an independent variable in the regression analysis and proves the corrected method leads to smaller portfolio variances. While examining MMI and S&P 500 stock index futures Lindahl (1992) suggests that hedging should be perceived as a dynamic technique rather than maintaining a constant measure, i.e. the hedge ratio should be adjusted when approaching a contract's expiration. This notion is supported by previous evidences by Lo & MacKinlay (1988) and Malliaris & Urrutia (1991) who show that the hedge ratios and measures of hedging effectiveness are non-stationary.

The shortcomings of the OLS estimation and the development of time-series econometric methods soon gave rise to alternative approaches in hedge ratio estimation. Granger (1981) and Engle & Granger (1987) introduce the concept of cointegration. When testing market efficiency many papers (Barnhart & Szakmary 1991; Krehbiel & Adkins 1993; Lai & Lai 1991; Wahab & Lashgari 1993) afterwards provided empirical evidence that spot and futures prices are cointegrated. Ghosh (1993b), Ghosh (1993a), and Ghosh & Clayton (1996) were first to integrate this long-term equilibrium relationship and short-run dynamics of cash and futures prices while employing error correction model (ECM) for hedging. They claim that when one does not make account for the cointegrating relationship and does not include the error-correction term in the regression the minimum variance hedge ratio is then biased downwards resulting in a smaller than optimal position taken [3]. Ghosh & Clayton (1996) examine stock index futures for CAC 40, FTSE 100, DAX, and NIKKEI, and show that the hedge ratio derived from the ECM is superior to the one estimated by the OLS in terms of both portfolio risk reduction and higher mean holding-period returns. Further studies also demonstrate poor hedging performance when omitting the cointegrating relationship (Chou, Denis & Lee 1996; Ghosh 1995; Kenourgios et al. 2008; Lien 1996).

Heaney (1998) suggests one should also consider cost-of-carry terms and includes interest rate and stock level in the cointegrating vector to avoid misspecification. Yet Ferguson & Leistikow (1999) later conclude the difference

[3] Lien (1996) provides a mathematical proof supporting conclusions of Ghosh (1993a).

between the hedging performance of this modified method and the ECM is of neither economic nor statistical significance. Additional applications are presented in Kroner & Sultan (1993) who employs vector error correction model (VECM) and Li (2010) where threshold VECM is used for testing the impact of arbitrage threshold behaviours on futures hedging effectiveness. The results generally support the usage of ECM/VECM.

The development of the autoregressive conditional heteroskedastic (ARCH) model by Engle (1982) and the generalized autoregressive conditional heteroskedastic (GARCH) model by Bollerslev (1986) allowed for another alternative approach to the regression method in hedge ratio estimation. Since the evidence of the time variation in stock return distributions (Baillie & DeGennaro 1990; Bollerslev 1987) the suitability of the constant hedge ratios have been questioned whilst it is argued the joint distribution of cash and futures prices is changing throughout time. E.g. Hatemi-J & El-Khatib (2012) show in their simulation and empirical analysis that the optimal hedge ratio is stochastic and changes significantly with time. They suggest investors should employ a time-dependent conditional variance model and consecutively rebalance the underlying portfolio.

The aim of many researchers was to correct for the problems encountered in the OLS estimation. H. Y. Park & Bera (1987) and Bera, Bubnys & Park (1993) use the ARCH model to account for the heteroskedasticity and non-normality present in the disturbances of the OLS regression. Their dynamic hedge ratios lead to an improved performance of both direct and cross hedging. Similar results are delivered by Cecchetti, Cumby & Figlewski (1988) who apply bivariate ARCH specification to allow for the time-varying distribution of prices while also focusing on maximizing an investor's expected utility.

The GARCH-type models received fairly wide attention. Using data on commodities and stock indices, respectively, Baillie & Myers (1991) and T. H. Park & Switzer (1995a) apply bivariate GARCH and demonstrate an improved hedging efficiency over both the OLS and ECM models. T. H.

Park & Switzer (1995b) present equivalent findings while incorporating the mean-variance expected utility function. Additionally, they consider transaction costs incurred due to portfolio rebalancing and conclude the dynamic hedging is more efficient than maintaining a constant hedge even after accounting for transaction costs. Focused on the German market specifically, Lypny & Powalla (1998) use GARCH(1,1) combined with an error correction of the mean returns and document significant improvements over the OLS and ECM models when hedging stock index futures on the DAX index. Lien & Tse (1999) tests hedging effectiveness of the fractionally integrated ECM, the standard ECM, the OLS, the VAR, and the GARCH. Their results show that the ECM incorporating conditional heteroskedasticity is the dominant strategy while the OLS is outperformed by all other strategies. Comparable findings in favour of GARCH dynamic hedging are presented in Sephton (1993), Gagnon & Lypny (1997), and Yang & Allen (2005).

While the evidence suggests the hedging position should be adjusted conditionally on new market information some papers contradict the proposition while demonstrating that advanced methods do not always bring huge benefits. In the application on commodity prices Myers (1991) determines that the GARCH performs only marginally better than the regression method and concludes the latter is sufficient unless there are some considerable structural breaks in the price series characterized by increased volatility. This view is supported by a later study by Caporin, Jimenez-Martin & Gonzalez-Serrano (2014) comparing the exponential weighted moving average (EWMA) filter technique and several multivariate GARCH models with the constant strategies during the financial turmoil in 2007-2008 and the subsequent Euro sovereign debt crisis. They report an improved hedging effectiveness of advanced models during the crisis periods while the opposite holds in non-crisis times and after the intervention of the European Central Bank. Kroner & Sultan (1993) assert that time-varying ARCH hedge ratios are highly unstable and make statistical testing impossible. Comparing with the naïve hedge and the traditional OLS they evaluate the VECM approach as the best strategy for currency hedging. Similarly Chakraborty & Barkoulas (1999) analysed hedging performance with five currencies and only in one case the time-varying strategy outperforms the constant hedge. Multiple studies follow with questioning the strength of the time-varying specification (Czekierda & Zhang 2010; Lien & Luo 1994; Lombana Betancourt & Al Azzawi 2013; Thomas & Brooks 2001; Wilkinson, Rose & Young 1999; Ye & Chen 2006) while some even report better results under traditional regression approach (Holmes 1996; Lien 2009; Lien & Shrestha 2008).

As Lien (2009) points out the results may stem from the fact that the hedging effectiveness is measured based on the unconditional variance and the traditional OLS hedge ratio actually minimizes the unconditional variance while the conditional hedge ratio intends to minimize the conditional variance. Power, Vedenov, Anderson & Klose (2013) employs a nonparametric copula-based GARCH (NPC-GARCH) model. The results do not indicate a significant portfolio variance reduction over the OLS yet the model performs better when assessed in terms of the expected short fall (lower tail risk).

Over the past two decades researchers have attempted to find merit in even more complex models. Many studies focus on the comparison of the dynamic conditional correlation GARCH (DCC-GARCH) model introduced by Engle (2002) with the traditional multivariate constant conditional correlation GARCH (CCC-GARCH) by Bollerslev (1990). Ku, Chen & Chen (2007) show in their results that the DCC-GARCH followed by the OLS lead to the greatest hedging effectiveness while the CCC-GARCH model performs the worst. S. Y. Park & Jei (2010) extend the DCC-GARCH by incorporating an asymmetric and flexible distribution specification. While the augmentation leads to a better goodness-of-fit of the estimated models the authors conclude a higher variance reduction compared to the OLS is not guaranteed. Comparable results are presented by Lien, Tse & Tsui (2002) and Fan, Roca & Akimov (2010) which possibly indicates that the forecasts generated by more advanced models might be too variable. Additionally, C. Brooks, Černỳ & Miffre (2012) determine optimal hedge ratios in a utility-based framework allowing for the effect of higher moments of the hedging decision. They similarly conclude the enhanced models provided at best very modest improvement over the constant OLS hedge.

Markov regime-switching (MRS) models and their extensions constitute another popular methods for hedge ratio estimation. As Alizadeh & Nomikos (2004) explain the dynamic relationship between spot and futures prices may be featured by regime shifts whereby the hedge ratio is allowed to be dependent upon the prevailing market conditions. While investigating S&P 500 and FTSE 100 markets Alizadeh & Nomikos (2004) report the MRS method outperforms the OLS and GARCH models within sample, however, the out-of-sample analysis produces mixed results. H.-T. Lee & Yoder (2007) and Su & Wu (2014) combine the MRS technique with other GARCH-type models - BEKK and DCC, respectively, whereby they allow for the time-varying feature and also a state-dependence. Similarly as in Alizadeh & Nomikos (2004), the in-sample results are in favour of the aforementioned combinations, while the out-of-sample performance is shown to be only marginally better. H.-T. Lee & Yoder (2007) additionally perform the data-snooping reality check proposed by White (2000) to test the forecasting superiority of their RS-BEKK-GARCH model. They demonstrate that the difference in variance reduction between the RS-BEKK-GARCH and other benchmark models is not of statistical significance. Further enhancements of the MRS are presented in H.-T. Lee (2009) and H.-T. Lee (2010) who develop a regime-switching Gumbel-Clayton (RSGC) copula GARCH and an independent switching dynamic conditional correlation (IS-DCC) GARCH, respectively. Both models outperform GARCH and other benchmark hedges and hence constitute prospective augmentations of the MRS.

An interesting technique for hedge ratio estimation is presented in Nguyen, Kim & Henry (2017). Rather than obtaining a single point estimate the authors use the wild bootstrap to estimate a confidence interval and percentiles for the optimal hedge ratio. The interval estimator is said to be more informative and provides a number of possible hedging strategies.

Moreover, it tackles the issues with non-normality, unknown forms of heteroskedasticity, and influential outliers. Nguyen et al. (2017) show that the wild bootstrap percentile-based hedging performs better than that of the naïve and DCC-GARCH models.

There has been a variety of results in the literature on the hedge ratio estimation. While the OLS has been deemed a standard strategy, time-series econometrics and many other more sophisticated models challenged its efficiency and highlighted its problems. Nonetheless, the overall superiority of advanced models has not been proven (S.-S. Chen, Lee & Shrestha 2013; Y.-T. Chen, Ho & Tzeng 2014). Moosa (2003) argues the model specification used has a very small effect on the outcome and suggests the correlation between the spot prices and prices of the hedging instrument is of the greatest importance. Furthermore, Lien & Shrestha (2008) claims the differing results stem from the estimation error.

While most of the aforementioned studies focus on hedging stock indices, currencies, and commodities, the thesis both concentrates and contributes to literature on the fixed-income market and the efficiency of German interest rate futures.

3 Theoretical background

3.1 Interest rate risk

Of major concern to investors in the bond market is interest rate risk. As Markellos & Psychoyios (2013) point out the market participants are particularly concerned about interest rates' future development and volatility since market interest rate changes consequently have an effect on the price of a bond held in a portfolio and its yield. If the market interest rates rise above the bond's yield, newly issued investments, that are similar to the bond held in a portfolio, will offer higher payments. To make the bond equally attractive, the relatively lower yield has to be compensated by a lower bond's price. If the current market interest rates fall, the bond held is more appealing to investors since it offers better coupon payments and consequently its market price goes up. The relationship between interest rates and bond prices is therefore inverse, i.e. a bond's price declines when interest rates increase and it goes up when interest rates decrease[4]. Interest rate risk is particularly important for the bonds with a fixed-rate coupon and affects also the bonds issued by a government. While the coupon payments and the principal are guaranteed by the issuer, the market price of the bond changes over the bond's life, mainly because of market interest rate changes, creating a possibility of lower profit when the bond is sold before it matures.

An important measure of a bond's price sensitivity to interest rate changes is duration. It indicates how long on average a bond's holder has to wait before receiving cash payments stemming from the bond. Following Hull (2012, pp. 89–92) a bond's price, B, is determined as the present value of all the future cash flows:

$$B = \sum_{i=1}^{n} CF_i e^{-yt_i}, \tag{3.1}$$

where CF_i is a cash flow at time t_i ($1 \leq i \leq n$), y is the bond's yield

[4]The relationship can be directly seen in a bond's pricing formula. For empirical evidence see e.g. Kluza & Sławiński (2004).

(continuously compounded), and n is the number of periods (years). The Macaulay duration of the bond, D, is then defined as follows:

$$D = \sum_{i=1}^{n} t_i \left[\frac{CF_i e^{-yt_i}}{B} \right]. \tag{3.2}$$

The duration is thus a weighted average of the maturities of the cash flows with weights being equal to the proportion of the bond's total present value given by the cash payment at time t_i. Considering a small change in the bond's yield, Δy, an approximate relationship between a percentage change in the bond's price and a change in its yield is suggested to be as such

$$\frac{\Delta B}{B} = -D\Delta y. \tag{3.3}$$

When a bond's yield is expressed with a compounding frequency of m times per year the modified duration poses an alternative in the following form:

$$MD = \frac{D}{1 + y/m} \tag{3.4}$$

The relationship between a bond's price and its yield then becomes as follows:

$$\frac{\Delta B}{B} = -MD\Delta y. \tag{3.5}$$

From equations 3.3 and 3.5 it is obvious that the greater the duration of a bond the greater the percentage change in the bond's price for a given interest rate change. Hence a greater duration of a bond leads to its higher interest rate risk. As Čerović, Pepić, Čerović & Čerović (2014) emphasize to have a complete view of the bond's price sensitivity one should also consider the convexity of the bond. Nevertheless, for the purpose of hedge ratio estimation this can remain undefined.

3.2 Interest rate futures hedging

Next to other derivative products (swaps, options, caps and floors) interest rate futures have been popular instruments for hedging interest rate risk. As per Fabozzi (2007, p. 360) they offer several advantages over the cash market in terms of cost, speed and liquidity. Nowadays this is supported by constant technology enhancements at leading derivatives exchanges[5]. The fast trading infrastructure attracts many market makers and high frequency traders which in turn provide greater liquidity. Moreover, leveraged positions using interest rate futures can be easily created because of lower margins that are typically less costly than bond issues or OTC derivatives margin requirements.

Hedging itself aims at reducing the risk inherent in the underlying market and transfers it into the so-called basis risk (Kolb & Overdahl 2003, p. 70). Basis is defined as the difference between spot price of a hedged bond and futures delivery price and converges to zero as the futures contract approaches its maturity. Because of this convergence concept there is no uncertainty when the hedge is held until maturity. However, insecurity arises when the hedge is lifted beforehand since the basis generally varies before the futures expiration date (Czekierda & Zhang 2010).

Moreover, as Young et al. (2004) remarks, due to liquidity considerations but also because of the fact that a corresponding futures contract for the bond in question simply does not exist the hedging strategy is often performed with mismatched contracts. Cross-hedging may stem from both differing underlying asset and differing maturity specifications. When the characteristics between the hedged and hedging instrument do not match the instrument's prices might then not move by the same amounts in response to changes in the underlying interest rates.

Since the interest rate sensitivity of a fixed income future closely relates to the CTD (cheapest to deliver) bond, Fabozzi (2007, p. 632) additionally suggests the CTD issue should be included in considerations to enhance the performance of the hedge. More specifically, the reliability of the hedge falls as the correlation between the bond to be hedged and the CTD issue decreases (Eurex 2007, p. 42).

Because of the aforementioned points it is important for the hedger to consider the correlation between the interest rates associated with cash and

[5]Eurex Exchange implements major releases of its T7 technology almost every year. For more information see https://www.eurexchange.com/exchange-en/resources/initiatives.

futures positions, respectively, and determine an appropriate method for hedge ratio estimation to mitigate the materialization of the basis risk.

4 Methodology

The first part of the analysis is dedicated to the estimation of hedge ratios using several methodologies. The second part focuses on hedging effectiveness whereby the employed techniques are assessed and compared based on their hedging performance. Data processing is implemented in R using RStudio IDE.

4.1 Hedge ratio estimation

4.1.1 Duration model

To minimize the risk of a fixed-income portfolio the duration-based approach considers relative durations of the cash bond and the futures contract. The modified duration representing the percentage change in the bond's price for one percent change in the yield is used for the hedge ratio calculation. It is assumed that the futures contract is highly influenced by the CTD (cheapest-to-deliver) bond, i.e. the deliverable bond with the highest implied repo rate. Based on Pepić (2014), the modified duration hedge ratio h_d is then obtained as follows

$$h_d = \frac{S_t}{F_t} \times \frac{MD_s}{MD_{ctd}} \times cf_{ctd} \tag{4.1}$$

where S_t is the price of the bond, F_t is the futures price, MD_s and MD_{ctd} are modified durations of the cash asset and the CTD, respectively, and cf_{ctd} is the conversion factor of the CTD bond. It is assumed that there is no change in the CTD bond during the life of the hedge. If that is not the case the hedge needs to be adjusted.

4.1.2 OLS model

As previously outlined the minimum variance hedge ratio can be estimated by the conventional OLS regression. The model involves regressing the spot price series on futures price series. Following Young et al. (2004) the hedge ratio for a short hedging horizon with daily data is estimated using price levels as follows:

$$\ln S_t = \alpha + \beta \ln F_t + \epsilon_t \tag{4.2}$$

where α is the constant term, $\ln S_t$ and $\ln F_t$ represent logarithms of spot and futures prices, respectively, and ϵ_t is the error term. An alternative is to carry out the estimation using price changes:

$$\Delta \ln S_t = a + b\Delta \ln F_t + u_t \tag{4.3}$$

where, given the natural logarithms of S_t and F_t, $\Delta \ln S_t$ and $\Delta \ln F_t$ signify spot and futures percentage price changes, respectively, a is a constant, and u_t denotes the error term. The estimated coefficients of the independent variable in both equations, β and b, are then one-period optimal hedge ratios. The OLS estimator corresponds to the previously derived relationship in the equation 2.8 and assumes constant variances and covariances of spot and futures prices.

Serial correlation

The presence of serial correlation in the error term is examined by the Box-Pierce test developed by Box & Pierce (1970) that tests the joint hypothesis that autocorrelation coefficients are not significantly different from zero with the below statistic

$$Q = T \sum_{k=1}^{m} \hat{\tau}_k^2 \overset{a}{\sim} \chi_m^2 \tag{4.4}$$

where $\hat{\tau}_k$, $k = 1, ..., m$, are autocorrelation coefficients, m is the maximum number of lags, and T is the sample size. The test formulated by Ljung & Box (1978) supplements the previous test with better small sample properties. The test uses the statistic that asymptotically mirrors the Box-Pierce test

$$Q^* = T(T+2) \sum_{k=1}^{m} \frac{\hat{\tau}_k^2}{T-k} \overset{a}{\sim} \chi_m^2. \tag{4.5}$$

4.1.3 VAR model

When present, the problem with serial correlation can be corrected by the vector autoregressive model (VAR), as was implemented in e.g. Lien & Tse

(1999). The VAR model separately regresses the two variables from the OLS equation on the lagged values of both variables. The bivariate VAR model for log-transformed spot and futures price series, $\ln S_t$ and $\ln F_t$, is then as follows:

$$\begin{aligned} \Delta \ln S_t &= c_s + \sum_{i=1}^{k} \beta_{si} \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{si} \Delta \ln F_{t-i} + \epsilon_{st} \\ \Delta \ln F_t &= c_f + \sum_{i=1}^{k} \beta_{fi} \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{fi} \Delta \ln F_{t-i} + \epsilon_{ft} \end{aligned} \tag{4.6}$$

where c_s and c_f are constants, β_{si}, β_{fi}, γ_{si}, and γ_{fi}, for $i = 1, ..., k$, are positive parameters, and ϵ_{st} and ϵ_{ft} denote i.i.d. error terms with zero mean. The number of lags k can be identified with the likelihood ratio test and information criteria. Following C. Brooks (2014), the likelihood ratio test is given by the following test statistic

$$LR = T[log|\hat{\Sigma}_r| - log|\hat{\Sigma}_u|] \tag{4.7}$$

where $\hat{\Sigma}$ is the variance-covariance matrix of residuals, $|\hat{\Sigma}_r|$ is the determinant of this matrix in the restricted model, $|\hat{\Sigma}_u|$ is the determinant of the matrix in the unrestricted model, and T is the sample size. Under the null hypothesis $LR \overset{a}{\sim} \chi^2_q$, where q is the total number of restrictions.

Another method to determine an appropriate number of lags in the VAR model is the use of information criteria. Given that it is preferable to have the same number of lags in each equation, the multivariate versions of the Akaike information criterion (MAIC) and Schwartz Bayesian information criterion (MSBIC) that are expressed as follows:

$$MAIC = ln|\hat{\Sigma}| + \frac{2k'}{T} \tag{4.8}$$

$$MSBIC = ln|\hat{\Sigma}| + \frac{k'}{T} lnT \tag{4.9}$$

where $\hat{\Sigma}$ is the variance-covariance matrix of the residuals, T is the sample size, and k' is the total number of regressors in all equations

To determine the hedge ratio let σ_{ff} denote the variance of ϵ_{ft} and σ_{sf} denote the covariance of ϵ_{st} and ϵ_{ft}. The minimum variance hedge ratio from the VAR model is then

$$h = \frac{\sigma_{sf}}{\sigma_{ff}}. \tag{4.10}$$

4.1.4 VECM model

As Lien (1996) proved, if the cointegrating relationship between variables is not taken into account the hedge ratio is less than optimal. It is suggested that if the series are cointegrated one should incorporate an error correction term in the VAR model to account for this long-run relationship. Therefore the time series properties, more specifically the presence of unit roots and cointegration, are examined first before further modelling.

Unit root

The augmented Dickey-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are performed to test for unit root and stationarity of the individual price series. The tests are of importance to avoid problems with spurious regression and statistical inference. Moreover, they are needed for the examination of a cointegration relationship which in turn justifies the model selection.

The ADF test is an extension of the Dickey-Fuller (DF) test (Dickey & Fuller 1979) that accounts for the serial correlation in the error term. As with the DF test, the model can take the form with an intercept or with an intercept and a deterministic trend. The test regression is then as follows:

$$\Delta y_t = \mu + \lambda t + \theta y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t, \quad t = 1, 2, ..., \tag{4.11}$$

where μ is a constant, λt denotes a deterministic trend, y_t is the series of interest, and p is the number of lagged changes of the dependent variable which can be determined according to the Akaike Information Criterion (AIC). The null hypothesis of a unit root $H_0 : \theta = 0$ is tested against the one-sided alternative $H_A : \theta < 0$ of no unit root. The critical values for the DF test are suggested by e.g. Banerjee, Dolado, Galbraith & Hendry (1993).

While the ADF test detects a potential unit root the rejection of the null hypothesis does not necessarily imply a stationary series.

As Kočenda & Černý (2015) recommends the KPSS test should be carried out along the ADF test since the former focuses directly on the stationarity. As per Kwiatkowski, Phillips, Schmidt & Shin (1992) it is assumed that the time series being checked for level stationarity y_t can be decomposed into a random walk r_t and a stationary error ϵ_t. When testing for the trend stationarity, this further includes a deterministic trend βt as follows:

$$y_t = \beta t + r_t + \epsilon_t, \;\; r_t = r_{t-1} + u_t \tag{4.12}$$

where u_t are normal i.i.d. errors with a zero mean and variance σ_u^2. It is further assumed the initial value r_0 is fixed and serves as an intercept. The null hypothesis of the stationarity corresponds to the situation when $\sigma_u^2 = 0$ and $r_t = r_0$ for all t, i.e. the random walk has zero variance.

Depending on whether the level or trend stationarity is tested, the regression equations $y_t = \alpha_0 + e_t$ or $y_t = \alpha_0 + \beta t + e_t$ are estimated by the OLS and the estimated rediduals $\hat{e}_t$ are used for the calculation of LM statistics for level and trend stationarity, η_μ and η_τ, respectively:

$$\eta_{\mu/\tau} = T^{-2} \frac{1}{s^2(l)} \sum_{t=1}^{T} S_t^2, \;\; \text{where} \;\; S_t = \sum_{i=1}^{t} e_i, \;\; t = 1, ..., T \tag{4.13}$$

where S_t is the partial sum of the residuals e_t from the estimated equations and $s^2(l)$ is the estimator of their long-run variance[6].

Cointegration

As discussed before the concept of stationarity is important for the validity of classical regression properties and statistical inference. However, a regression involving $I(1)$ variables, i.e. the variables integrated of order 1, is meaningful when the series are cointegrated. Cointegration arises when a linear combination of the non-stationary variables forms a stationary $I(0)$ process which in turn implies the long-run equilibrium relationship between the variables. The formal definition provided by Engle & Granger (1987) is as follows:

[6]For more details, see Kwiatkowski et al. (1992).

"the components of the vector x_t are said to be cointegrated of order d, b, denoted $x_t \sim CI(d,b)$, if all components of x_t are $I(d)$ and if there exists a nonzero vector α so that the linear combination of the components of x_t, i.e. $z_t = \alpha' x_t$, is $I(d-b), \;\; b > 0$"

where the vector α denotes the cointegrating vector.

Following Engle & Granger (1987) the cointegration of two $I(1)$ variables is examined with the residual-based approach. The OLS residuals from the estimated equation are tested for stationarity by the ADF test with the following test regression:

$$\Delta \hat{u}_t = \theta \hat{u}_{t-1} + \nu_t \tag{4.14}$$

where ν_t is an *i.i.d.* error term. Given the test is conducted on the residuals of the estimated model rather than on raw data the distribution of the test statistic is different and therefore the inference should be carried out with different critical values, e.g. the ones provided by Davidson & MacKinnon (1993). The ADF without an intercept and a trend, and the KPSS tests are performed as well to supply more evidence on the stationarity of the residuals. If the residuals resemble stationary $I(0)$ process the tested variables are said to be cointegrated.

VECM

Under the condition that the time series are cointegrated Engle & Granger (1987) presents the error correction model (ECM) that commingles the short-run dynamics of the variables with long-run equilibrium. The vector error correction model (VECM) has then the form of a bivariate VAR model with first differenced logarithms that additionally incorporates the error correction term as follows:

$$\begin{aligned} \Delta \ln S_t &= c_s + \sum_{i=1}^{k} \beta_{si} \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{si} \Delta \ln F_{t-i} + \delta_s E_{t-1} + \epsilon_{st} \\ \Delta \ln F_t &= c_f + \sum_{i=1}^{k} \beta_{fi} \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{fi} \Delta \ln F_{t-i} - \delta_f E_{t-1} + \epsilon_{ft} \end{aligned} \tag{4.15}$$

where again c_s and c_f are intercepts, β_{si}, β_{fi}, γ_{si}, and γ_{fi}, for $i = 1, ..., k$, are positive parameters, ϵ_{st} and ϵ_{ft} denote i.i.d. errors with zero mean. E_{t-1} is the so-called error correction term that is given by

$$E_{t-1} = \ln S_{t-1} - \alpha \ln F_{t-1} \tag{4.16}$$

where α is the cointegrating vector. E_{t-1} is therefore defined by the lagged residuals from the cointegrating regression. The error correction term identifies how the dependent variable adjusts to the deviation from the long-run equilibrium that occurred in the previous period. The coefficients on the error correction term in the two equations, δ_s and δ_f, then give the speed of adjustment, i.e. at what speed the markets respond to the deviation from the long-term equilibrium.

The residuals from the VECM model are used to calculate unconditional variances of spot and futures prices, σ_{ss} and σ_{ff}, respectively, and the covariance σ_{sf}, which further define the hedge ratio as per the equation 4.10.

4.1.5 GARCH model

Given that the arrival of a new information can have a great influence on the risk of assets the variances and covariances of the price series might be changing over time. Since the contribution of Engle (1982) and Bollerslev (1986) on the formulation of second moments of time series it has been suggested by many researchers that if the joint distribution of spot and futures prices is time-varying the hedge ratio should be built based on a time-dependent conditional variance model.

Engle (1982) introduced the autoregressive conditional heteroskedastic (ARCH) model characterized by the conditional variance that is allowed to vary over time as a function of past errors while leaving the unconditional variance, that is captured by a constant, unchanged. Bollerslev (1986) generalized the ARCH model allowing the conditional variance to depend on its own previous lags bringing a more flexible lag structure. The number of lags can be again determined by information criteria, nonetheless as per C. Brooks (2014) GARCH (1,1) is usually sufficient to model the volatility in the data. In this work the GARCH(1,1) specification is employed to model the residual series in the context of the previously introduced VAR model that is used as a mean equation.

Before fitting any model it is examined whether there indeed are any dependencies and ARCH effects in the residuals. The previously introduced Box-Pierce and Ljung-Box tests are used to test for autocorrelation. The conditional heteroskedasticity is then tested by the ARCH-LM test to confirm whether there is an ARCH pattern in the residuals. Following C. Brooks (2014) this method tests the null hypothesis that the coefficients on all q lags of the squared residuals are simultaneously equal to zero.

Considering the time-varying hedge ratio is based on the interaction between two variables the following multivariate linear version of GARCH(1,1) based on Bollerslev, Engle & Wooldridge (1988) is employed

$$H_t = \begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss,t} \\ c_{sf,t} \\ c_{ff,t} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} \epsilon^2_{s,t-1} \\ \epsilon_{s,t-1}, \epsilon_{f,t-1} \\ \epsilon^2_{f,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \times \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix} \tag{4.17}$$

where $h_{ss,t}$ and $h_{ff,t}$ are the conditional variances of the errors from the equation 4.6, $\epsilon_{s,t}$ and $\epsilon_{f,t}$, respectively, and $h_{sf,t}$ is the conditional covariance between spot and futures series. $\epsilon_t = (\epsilon_{st}, \epsilon_{ft})'$ are assumed to follow a bivariate normal distribution with zero mean and a conditional variance-covariance matrix H_t with a constant correlation coefficient ρ. It is further assumed that the matrices of coefficients, A_1 and B_1, are positive semidefinite.

Since the number of parameters to be estimated is rather high even in this simple GARCH(1,1) specification, Bollerslev et al. (1988) further proposes the diagonal vec (DVEC) version of the model that sets the off-diagonal elements of the A_1 and B_1 matrices to zero leaving the conditional variance dependent only on the lagged squared residuals and its own lagged values. The DVEC representation of the conditional variances and covariance is then as follows

$$\begin{aligned} h_{ss,t} &= c_{ss} + \alpha_{ss}\epsilon^2_{s,t-1} + \beta_{ss}h_{ss,t-1} \\ h_{sf,t} &= c_{sf} + \alpha_{sf}\epsilon_{s,t-1}\epsilon_{f,t-1} + \beta_{sf}h_{sf,t-1} \\ h_{ff,t} &= c_{ff} + \alpha_{ff}\epsilon^2_{f,t-1} + \beta_{ff}h_{ff,t-1} \end{aligned} \tag{4.18}$$

where the α and β coefficients are non-negative.

Following Kroner & Sultan (1993) the time varying hedge ratio δ_{t-1} that aims to minimize the variance of the return $S_t - \delta_{t-1}F_t$ conditional on the information set at time $t-1$ is calculated as follows

$$\delta_{t-1} = \frac{h_{sf,t-1}}{h_{ff,t-1}}. \tag{4.19}$$

4.2 Hedging effectiveness

The hedging effectiveness is measured in terms of return and portfolio variance reduction to extract the risk-return trade-off in the hedged portfolio. The performances of the particular hedge ratios are then compared against each other including the results of a no hedge and a naïve one-to-one hedge. Both in-sample and out-of-sample testing considering various hedging horizons are employed.

4.2.1 Return and percentage variance reduction

A common method to evaluate the performance of a hedging strategy is to consider whether the hedge does indeed reduce the variance of the portfolio's returns, i.e. whether the strategy decreases the portfolio's risk. The return on the unhedged cash position which corresponds to the change in the bond portfolio value can be determined with a Basis Point Value (BPV). The BPV gives the price change of the position for a change of one basis point in the yield to maturity (YTM) of the bond (Fabozzi 2007, p. 182) and is specified as follows:

$$BPV = S_t \times MD_s \times 0.0001 \tag{4.20}$$

where S_t is the cash bond price and MD_s is the modified duration of the bond. For a given change in the YTM of the bond (in basis points), the BPV is then used to calculate the profit or loss of the cash position, i.e. the return of the unhedged portfolio $r_{u,t}$, as

$$r_{u,t} = \Delta YTM \times BPV. \tag{4.21}$$

The profit or loss stemming from the futures position is determined by the change in the futures price $\Delta \ln F$ multiplied by the optimal hedge ratio h^*. The hedge ratio h^* that determines the number of futures contracts needed for the hedge is estimated separately by each method introduced in the previous section. The overall return on the hedged portfolio $r_{h,t}$ is then given as follows

$$r_{h,t} = r_{u,t} - h^* \Delta \ln F_t. \tag{4.22}$$

Following Ederington (1979) the percentage reduction in the variance of the hedge portfolio compared to the unhedged cash position is as follows

$$PVR = \frac{Var(U) - Var(H)}{Var(U)}. \tag{4.23}$$

where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.

With hedging investors aim to secure their positions and reduce risk. However, in many cases they are concerned not only with the variance reduction but also with the risk-return trade-off resulting from the hedge. For this the mean returns of the hedged portfolios achieved with the particular hedging strategies are compared.

4.2.2 In-sample and out-of-sample testing

Since the hedging effectiveness of the models may vary with the length of the hedge, the models' performance is evaluated for the hedging horizons of 5, 10, 20, and 30 days. The calculations are carried out using rolling windows whereby a function is applied to the subsets of the data that have fixed width while indexing one observation for each calculation. For each hedging horizon and for each hedging technique the compound returns from daily returns within a rolling window are obtained. Subsequently, the mean of the compound returns, variance of these returns, and percentage variance reduction to the unhedged portfolio are calculated.

The performance is tested both in sample and out of sample. While the majority of observations are used for the estimation of the hedge ratio, 30 observations are reserved for the out-of-sample testing. As suggested by

Hsu, Tseng & Wang (2008), the out-of-sample series of time-varying hedge ratios can be obtained by rolling over a particular number of samples while always forecasting the hedge ratio for the following day. The forecasted hedge ratio is then calculated as the one-period-ahead forecast of the conditional covariance divided by the one-period-ahead forecast of the conditional variance. The sample to be used for the next day is updated by taking into account a particular number of the nearest observations preceding the day for which the forecast is performed. The calculations are repeated through the end of the dataset yielding a series of one-period forecasted hedge ratios.

5 Data

The data was acquired from The Bloomberg Terminal. It comprises the daily closing prices of Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures traded on the Eurex Exchange, and the prices of corresponding CTD bonds. Furthermore, it is complemented with the daily data on the bonds' modified durations, yields to maturity and conversion factors. While there are several expiries only front-month futures contracts are considered. The CTD was chosen as the cash side to be hedged to retain the continuity in prices and enable estimation.

The analysis is conducted for two time periods. The first period runs from 2 April 2007 to 16 October 2009 comprising of 665 observations. This period captures the global financial crisis and the effects of problematic subprime mortgages, liquidity, and trust amongst banks. The second period covers the time between 9 December 2013 and 8 June 2016 with the total of 653 observations, and represents the aftermaths of the European sovereign debt crisis. Furthermore, these periods were chosen based on the advantage that they offer, compared to other periods, in terms of less changes in CTD bonds associated with the futures contracts. Frequent changes in CTD bonds might cause artificial breaks in the data as the change in the price level on the cash side may be more profound than on the futures side, potentially causing negative correlation between spot and futures. Therefore, while the usage of CTD bond ensures the consistent bond selection and the availability of data on both cash and futures side, less changes in CTD bonds during the studied periods are desired to avoid any misleading results.

As is common for bond futures contracts, the short side can actually deliver any security from the deliverable basket containing the instruments within a specific maturity range and with a specific coupon rate. The seller can therefore optimize the transaction by choosing the bond that is cheapest to deliver (CTD). The CTD bond then provides the highest implied repo rate which is the rate from the cash-and-carry transaction[7]. Consequently, the CTD bond typically shows higher trading volumes when compared to other bond issues (Kolb & Overdahl 2010, p. 137). While only a small proportion goes to delivery, bond futures contracts are rather rolled over to the next expiry month. The possibility of physical delivery, however, generally creates a close correlation between the CTD and the futures making the pricing of bond futures contracts dependent on the CTD issue.

The deliverable securities for FGBL and FGBM are medium- and long-term instruments issued by the Federal Republic of Germany. The most heavily traded securities in the German bond market are then Bund and Bobl characterized by the maturities of 10 and 5 years, respectively[8]. While being an important source of government funding, German bonds are highly accepted as collateral and serve as the benchmark in the Euro area because of their relative liquidity and credit quality. Often perceived as the safest fixed-income investment in the Eurozone, German bonds are often affected by the flight-to-safety and flight-to-liquidity phenomena which are at times more pronounced than for other sovereign bonds. Barrios, Iversen, Lewandowska & Setzer (2009) and Ejsing & Sihvonen (2009) propose that the benchmark status might also be attributed to the existence of highly liquid futures market that provides investors with hedging opportunities.

As per Barrios et al. (2009), the flight-to-safety indeed affected the German bond market during the financial crisis, with increased demand pushing the bond prices up while depressing the yields, and that more profoundly than in other countries. After September 2008, the spreads between the German Bund and other Euro-area sovereign bonds increased significantly evolving into the European sovereign debt crisis due to rising public debt. The central banks aimed at increasing liquidity and restoring economic growth. While implementing various policy measures of the European Central Bank (ECB)

[7] The cash-and-carry involves purchasing a bond with borrowed funds in the spot market while selling the corresponding future in the futures market.

[8] The original maturities of Bund are 10 and 30 years, however, only the 10-year contracts apply for the delivery against the Euro-Bund future (FGBL). Instead, Euro-Buxl futures (FGBX) offers the delivery of the securities with the remaining maturity of 24 to 35 years.

and providing assurances in the "whatever it takes" speech of the ECB President Mario Draghi in 2012, the financial stability in the Eurozone improved and interest rates were falling (Van Der Heijden, Beetsma & Romp 2018). The decline in interest rates was further intensified by the quantitative easing (QE) policy in 2015. With the QE, a central bank purchases government securities in order to increase the money supply and encourage lending and investment in an already low interest rate environment. The yields were pushed further down, with some in Germany falling below zero.

As outlined previously, the German bond market has the most liquid futures market out of the whole Eurozone, contributing to the liquidity of the cash market and enhancing the price discovery in euro-area interest rates, as per Ejsing & Sihvonen (2009). Offered at the biggest derivatives exchange in Europe[9], Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures are heavily traded fixed-income derivatives that enable hedging and speculating activities on the euro-area interest rates. The direct market participants in FGBL and FGBM comprise both brokers, market-makers, and high-frequency traders. The contract specifications are provided in Table 1. In 2013 the trading platform was launched using the T7 technology developed by Deutsche Boerse advancing electronic trading in derivatives. As with the bond market, futures prices were affected by the ECB policies and declining interest rates during the studied years.

Figure 1 shows the development of price series over the studied periods and Table 2 provides the summary statistics. It is visible that generally the spot price follows the futures price yet the relation seems to be weaker than the one usually found in the stock index markets. The reason may lie in the different nature of the markets. While the equity markets have experienced technological advances over the last decade and have been dominated by HFT firms bringing prices close together with algorithmic trading, the bond market is still marked with trading often conducted by phone. Furthermore, the price departures might also be caused by the different closing times

[9] See Statista (2019).

between cash and futures markets[10]. While acknowledging a weaker cohesion the futures and CTD prices seem to depart more significantly towards the end of period 2, with basis widening for both FGBL and FGBM contracts. This could convey a smaller ability of the futures contract to hedge the underlying asset.

The CTD prices vary around the par value of 100 while rising in the second half of period 1. In period 2 the prices are at a higher level reaching the values over 115 while the variance lowered. The futures are trading above the CTD bonds with FGBL hitting 160 and FGBM trading over 130 towards the end of period 2. Both futures and bond prices rose significantly between the two periods reflecting the economic situation and a drop in interest rates. All series seem to embody a non-stationary process which is further tested in the next section.

Table 1: Contract specifications

Contract name	Euro-Bund Futures	Euro-Bobl Futures
Product ID	FGBL	FGBM
Contract standard	Notional long- and medium-term debt securities issued by the Federal Republic of Germany with a 6 percent coupon.	
Remaining term (in years)	8.5 to 10.5	4.5 to 5.5
Contract value	EUR 100,000	
Settlement	Physical settlement with debt securities of a minimum issue amount of EUR 5 billion.	
Price Quotation	In percent of the par value.	
Tick & tick value	0.01 percent equivalent to EUR 10.	
Contract months	Up to 9 months: The three nearest quarterly months of the March, June, September and December cycle.	
Trading hours	01:00-22:00 CET, on the Last Trading Day 01:00-12:30 CET	

Source: Eurex (2019)

[10] While the Boerse Frankfurt closes the market at 8:00 p.m. CET, Eurex allows trading till 10:00 p.m. CET.

Figure 1: Euro-Bund and Euro-Bobl futures, and CTD prices

Table 2: Summary statistics

Period 1	FGBL		CTD		FGBM		CTD	
	Min.	109.8	Min.	93.17	Min.	105.2	Min.	94.6
	1st Qu.	113.3	1st Qu.	97.56	1st Qu.	107.8	1st Qu.	100.60
	Median	115.3	Median	99.91	Median	109.9	Median	103.56
	Mean	116.9	Mean	101.51	Mean	111.0	Mean	103.44
	3rd Qu.	121.4	3rd Qu.	106.77	3rd Qu.	115.4	3rd Qu.	107.47
	Max.	126.4	Max.	109.61	Max.	117.9	Max.	109.32
	Std.	4.51	Std.	4.84	Std.	3.81	Std.	3.89
Period 2	**FGBL**		**CTD**		**FGBM**		**CTD**	
	Min.	139.0	Min.	96.91	Min.	124.2	Min.	112.6
	1st Qu.	148.0	1st Qu.	102.77	1st Qu.	127.9	1st Qu.	114.2
	Median	154.0	Median	105.97	Median	129.2	Median	115.0
	Mean	153.4	Mean	106.33	Mean	129.0	Mean	115.0
	3rd Qu.	158.7	3rd Qu.	110.69	3rd Qu.	130.6	3rd Qu.	115.9
	Max.	166.6	Max.	115.51	Max.	133.3	Max.	116.6
	Std.	6.98	Std.	4.78	Std.	2.05	Std.	0.99

Note: The summary statistics is calculated for Euro-Bund (FGBL) and Euro-Bobl (FGBM) prices and their corresponding cheapest-to-deliver (CTD) bonds during 2 April 2007 to 16 October 2009 (Period 1) and 9 December 2013 to 8 June 2016 (Period 2).

6 Empirical results

The first part of this section presents the model results as well as the auxiliary tests to determine the properties of the data. The optimal hedge ratio is estimated with the following models: OLS with levels, OLS with percentage changes, VAR, VECM, GARCH, and the duration method. The second part provides the hedging effectiveness of the aforementioned methods in terms of risk-return trade-off, both in- and out-of-sample. Lastly, robustness testing is conducted to check the consistency of the results.

6.1 Hedge ratio estimation

6.1.1 Unit root

Before the estimation of the models the price series are transformed to natural logarithms and tested for unit root and level stationarity to determine their order of integration. Table 3 provides ADF and KPSS statistics. For all series in both periods, the null hypothesis of unit root is not rejected when the ADF is executed on levels while the null is rejected at 1% level of significance when the series are first-differenced. The results are supported by the KPSS test that rejects the null of stationarity at 1% significance level for level variables and fails to reject the null of stationarity at 5 % level of significance after first-differencing. Both spot and futures series are therefore $I(1)$, i.e. integrated of order one. When integrated of the same order the prices meet a prerequisite for the testing of cointegration relationship between the series as is required by Engle & Granger (1987).

6.1.2 OLS

To determine Ederington's minimum variance hedge ratio (Ederington 1979) and further proceed with the cointegration testing the standard OLS model with series in levels is estimated as per the equation 4.2. The results for both FGBL and FGBM in the two periods are given in Table 4. In period 1, the coefficients on the futures series and therefore the estimated hedge ratios are statistically significant and high in value, with the hedge ratio for

Table 3: ADF and KPSS tests

(a) FGBL

			ADF	KPSS
Period 1:	lnFGBL	level	-2.1099	6.0431***
		Δ	-8.9641***	0.1001*
	lnCTD	level	-2.3441	7.5975***
		Δ	-9.2106***	0.0469*
	$\hat{\epsilon}_t$	level	-2.5207	2.1504***
Period 2:	lnFGBL	level	-2.2081	7.2340***
		Δ	-8.0729***	0.0768
	lnCTD	level	-1.6941	2.0964***
		Δ	-7.2045***	0.5002*
	$\hat{\epsilon}_t$	level	-1.67634	2.5146***

(b) FGBM

			ADF	KPSS
Period 1:	lnFGBM	level	-1.8862	6.8574***
		Δ	-8.9657***	0.1309*
	lnCTD	level	-2.0951	3.6010***
		Δ	-7.8552***	0.1648*
	$\hat{\epsilon}_t$	level	-2.5692	1.7696***
Period 2:	lnFGBM	level	-2.8132	6.5855***
		Δ	-8.6161***	0.0543
	lnCTD	level	-2.4942	3.8134***
		Δ	-8.1071***	0.1677
	$\hat{\epsilon}_t$	level	-2.4883	2.7357***

Note: lnFGBL and lnFGBM denote logarithms of Euro-Bund and Euro-Bobl futures prices, respectively, and lnCTD is the logarithm of their respective cheapest-to-deliver bonds. $\hat{\epsilon}_t$ are residuals from the OLS equation $\ln S_t = \alpha + \beta \ln F_t + \epsilon_t$ for FGBL and FGBM, and their CTDs. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

FGBL greater than 1. In period 2, the hedge ratios are estimated to be much smaller with coefficients 0.41 and -0.10 for FGBL and FGBM, respectively. In case of FGBM the coefficient is negative and insignificant presaging a potential inappropriateness of the model.

It is reasonable to believe that there is a long-run relationship between the spot and futures prices given the close relationship between the CTD and the bond futures. Since both series are non-stationary $I(1)$ processes, the prices are further tested for cointegration. After the estimation of the equation 4.2 the residuals are saved and tested for unit root and stationarity to determine whether the spot and futures series could result in a stationary process when combined. As is shown in Table 3 the ADF test does not reject the null hypothesis of unit root while the null of stationarity is rejected by the KPSS test at 1% significance level. The results, consistent across both futures contracts and both studied periods, indicate the non-stationarity of residuals and therefore the cointegration relationship has not been proven.

In the absence of a long-run cointegrating relationship between spot and futures the series are differenced resulting in price percentage changes and estimated by the OLS as given in the equation 4.3. The estimation results are provided in Table 5. The coefficients of the independent variable are now all significant and positive. The hedge ratios in period 1 are estimated to be around 0.9 for both FGBL and FGBM, while period 2 resulted in the hedge ratios of 0.79 and 0.43 for FGBL and FGBM, respectively, being greater than the values estimated with the level variables.

The residuals of the OLS equations are further tested for serial correlation. The autocorrelation in the residuals violates classical linear model assumptions and if present it should be treated appropriately. The residuals are examined with Box-Pierce and Ljung-Box tests with results provided in Table A1 in Appendix A. There is a strong evidence of serial correlation as the null hypothesis of no autocorrelation is rejected at 1% significance level in almost all cases. There is no evidence of serial correlation in the residuals of the OLS model with percentage changes in case of FGBM in period 2.

Table 4: OLS levels

(a) FGBL - Period 1

	Dependent variable:
	$\log(\mathrm{CTD1}_{\mathrm{FGBL}})$
Constant	-0.7665^{**}
	(0.3722)
log(FGBL1)	1.1311^{***}
	(0.0777)
Observations	635
R^2	0.868
Adjusted R^2	0.868
Residual Std. Error	0.017 (df = 633)
F Statistic	$4{,}164.357^{***}$ (df = 1; 633)

(b) FGBM - Period 1

	Dependent variable:
	$\log(\mathrm{CTD1}_{\mathrm{FGBM}})$
Constant	0.1526
	(0.4083)
log(FGBM1)	0.9524^{***}
	(0.0857)
Observations	635
R^2	0.741
Adjusted R^2	0.741
Residual Std. Error	0.019 (df = 633)
F Statistic	$1{,}815.011^{***}$ (df = 1; 633)

(c) FGBL - Period 2

	Dependent variable:
	$\log(\mathrm{CTD2}_{\mathrm{FGBL}})$
Constant	2.6246^{***}
	(0.9912)
log(FGBL2)	0.4060^{**}
	(0.1977)
Observations	623
R^2	0.154
Adjusted R^2	0.153
Residual Std. Error	0.042 (df = 621)
F Statistic	113.011^{***} (df = 1; 621)

(d) FGBM - Period 2

	Dependent variable:
	$\log(\mathrm{CTD2}_{\mathrm{FGBM}})$
Constant	5.2300***
	(0.7863)
log(FGBM2)	-0.1000
	(0.1623)
Observations	623
R^2	0.035
Adjusted R^2	0.033
Residual Std. Error	0.008 (df = 621)
F Statistic	22.533^{***} (df = 1; 621)

Note: The results are obtained from the OLS estimation of $\ln S_t = \alpha + \beta \ln F_t + \epsilon_t$ for FGBL and FGBM, and their respective CTDs. Figures in parentheses are HAC standard errors.*, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 5: OLS percentage changes

(a) FGBL - Period 1

	Dependent variable:
	diff(log($CTD1_{FGBL}$))
Constant	0.0001***
	(0.0000)
diff(log(FGBL1))	0.9332***
	(0.0226)
Observations	634
R^2	0.807
Adjusted R^2	0.806
Residual Std. Error	0.002 (df = 632)
F Statistic	2,635.152*** (df = 1; 632)

(b) FGBM - Period 1

	Dependent variable:
	diff(log($CTD1_{FGBM}$))
Constant	−0.0001
	(0.0001)
diff(log(FGBM1))	0.9538***
	(0.0325)
Observations	634
R^2	0.649
Adjusted R^2	0.648
Residual Std. Error	0.002 (df = 632)
F Statistic	1,166.284*** (df = 1; 632)

(c) FGBL - Period 2

	Dependent variable:
	diff(log($CTD2_{FGBL}$))
Constant	−0.0001
	(0.0001)
diff(log(FGBL2))	0.7880***
	(0.0720)
Observations	622
R^2	0.589
Adjusted R^2	0.588
Residual Std. Error	0.002 (df = 620)
F Statistic	887.437*** (df = 1; 620)

(d) FGBM - Period 2

	Dependent variable:
	diff(log($CTD2_{FGBM}$))
Constant	−0.00005
	(0.00004)
diff(log(FGBM2))	0.4328***
	(0.0811)
Observations	622
R^2	0.404
Adjusted R^2	0.403
Residual Std. Error	0.001 (df = 620)
F Statistic	419.650*** (df = 1; 620)

Note: The results are obtained from the OLS estimation of $\Delta \ln S_t = a + b\Delta \ln F_t + u_t$ for FGBL and FGBM, and their respective CTDs. Figures in parentheses are HAC standard errors.*, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively. respectively.

6.1.3 VAR and VECM

To account for the serial correlation in the residuals the spot and futures series are modelled with the VAR model given in the equation 4.6. To select the appropriate number of lags to eliminate the autocorrelation the Akaike and Schwartz information criteria are used. In the majority of cases a VAR of the second order is estimated, while the optimal number of three lags is identified for FGBM in period 1. The results of the VAR models for period 1 and 2 are available in Tables A3 and A5 in Appendix A. The coefficients on the lags of differenced logarithms of prices are in many cases statistically insignificant which suggests the hedge ratio estimated by the VAR might not be that much different from the one estimated by the OLS.

While the spot and futures prices were not found to be cointegrated, the VECM model might still potentially offer some meaningful results. As per Lien & Tse (1999) the error correction term could substitute the effect of the no-arbitrage relationship between spot and futures while also account for the effect of the basis on the price movements as the basis declines towards the maturity of the futures contract. The VAR model is therefore amended with the error correction term according to the equation 4.15 forming the VECM model. The results for the two periods are provided in Tables A7 and A9 in Appendix A. After the inclusion of the error correction term more coefficients on the differenced logarithms of prices become statistically significant. The coefficient on the error correction term (ECT) itself is significant for both FGBL and FGBM in both periods, while being mostly negative. The positive coefficient on the ECT is identified for FGBL in period 1 which signals the lack of convergence. In all cases the ECT's coefficient from the equation with futures series as the dependent variable is greater than the one from the equation for spot series which means that the futures prices have greater speed of adjustment to the last period's deviation from a long-run equilibrium than the spot prices.

The hedge ratios from VAR and VECM models are then calculated as the ratios of the covariance between the error terms ϵ_{st} and ϵ_{ft} to the variance of ϵ_{ft} from the VAR and VECM equations, respectively (see the equation 4.10). The variance-covariance matrices and the estimated hedge ratios are given in Tables 6 and 7 for FGBL and FGBM, respectively. The hedge ratios are generally only slightly different from the OLS hedge ratio, see Table 8 providing the summary of constant hedge ratios[11]. While there are only slight differences for FGBL in the first period, for FGBM the VECM hedge ratio is greater than both the VAR and OLS estimates. In the second period however the VECM hedge ratios are smaller than the other two. Therefore, there is not much consistency other than the hedge ratios estimated in period 2 are much smaller than the ones estimated for period 1. Seemingly weakened relationship between CTD bonds and their futures might be a consequence of the yields being kept low and relatively smaller variance of CTDs and FGBM futures in the second period.

Table 6: FGBL Variance-covariance matrices

	VAR		VECM	
Period 1	σ_{ss}	σ_{ff}	σ_{ss}	σ_{ff}
σ_{ss}	1.6894e-05	1.4928e-05	1.4743e-05	1.4743e-05
σ_{ff}	1.4928e-05	1.6087e-05	1.4743e-05	1.5975e-05
σ_{sf}/σ_{ff}	0.9280		0.9229	
Period 2	σ_{ss}	σ_{ff}	σ_{ss}	σ_{ff}
σ_{ss}	1.2426e-05	9.6757e-06	1.2799e-05	9.4158e-06
σ_{ff}	9.6757e-06	1.2216e-05	9.4158e-06	1.2227e-05
σ_{sf}/σ_{ff}	0.7920		0.7701	

Note: σ_{ff} and σ_{ss} denote the variance of ϵ_{ft} and ϵ_{st}, respectively, and σ_{sf} indicates the covariance of ϵ_{st} and ϵ_{ft} from the VAR equation 4.6 and the VECM equation 4.15 for FGBL and the corresponding CTD.

[11]The OLS model estimated with percentage changes is considered for further analysis. The OLS model will levels is not employed in the upcoming analyses.

Table 7: FGBM Variance-covariance matrices

	VAR		VECM	
Period 1	σ_{ss}	σ_{ff}	σ_{ss}	σ_{ff}
σ_{ss}	8.9664e-06	6.8383e-06	9.9358e-06	7.2520e-06
σ_{ff}	6.8383e-06	7.3585e-06	7.2520e-06	7.5362e-06
σ_{sf}/σ_{ff}	0.9293		0.9623	
Period 2	σ_{ss}	σ_{ff}	σ_{ss}	σ_{ff}
σ_{ss}	1.6367e-06	1.5278e-06	1.6886e-06	1.3965e-06
σ_{ff}	1.5278e-06	3.4866e-06	1.3965e-06	3.6738e-06
σ_{sf}/σ_{ff}	0.4382		0.3801	

Note: σ_{ff} and σ_{ss} denote the variance of ϵ_{ft} and ϵ_{st}, respectively, and σ_{sf} indicates the covariance of ϵ_{st} and ϵ_{ft} from the VAR equation 4.6 and the VECM equation 4.15 for FGBM and the corresponding CTD.

Table 8: Constant hedge ratios

	Period 1		Period 2	
	FGBL	FGBM	FGBL	FGBM
OLS	0.9332	0.9538	0.7880	0.4328
VAR	0.9280	0.9293	0.7920	0.4382
VECM	0.9229	0.9623	0.7701	0.3801

Note: The OLS hedge ratio is obtained from estimating the equation $\Delta \ln S_t = a + b\Delta \ln F_t + u_t$.

6.1.4 GARCH

The VAR model can be further extended by modelling the features in the residuals with the multivariate GARCH model. The GARCH model postulates that the current fitted variance comprises the long-term average variance and the previous period's actual volatility and the predicted variance from the model. The GARCH model can therefore eliminate conditional heteroskedasticity in the residuals whereby creating conditions for time-varying hedge ratios.

Before employing the GARCH model the residuals are tested for the presence of ARCH effects. The Box-Pierce test and the Ljung-Box test are used to determine the extent to which the residuals are autocorrelated whilst the ARCH-LM test is employed to test for conditional heteroskedasticity. Table A11 in Appendix A provides the results for the residuals from both equations of the VAR model. While in some cases the Q-statistics for Box-Pierce and Ljung-Box tests were not significant to reject the null of no autocorrelation, in every period for both FGBL and FGBM the Q-statistic is significant in at least one of the two VAR equations. The same holds for the ARCH-LM test whereby the null hypothesis of no conditional heteroskedasticity is rejected for at least one set of residuals in each case. Overall the results suggest there are ARCH effects present in the residuals of the VAR's futures equations in period 1, while there are more dependencies in the spot equations in period 2. Figures A1, A2, A3, and A4 in Appendix A depict the ACF and PACF of squared residuals from the VAR model. Along with the previous results these graphs show there are dependencies in the data that could be modelled with ARCH-type models.

Based on previous studies a multivariate GARCH(1,1) model is employed to model the residuals of the previously estimated VAR model[12]. The parameters of the model are estimated by the maximum likelihood estimation method. The complete results of the GARCH model for the two periods are given in Tables A13 and A15 in Appendix A. Omega which denotes a constant and represents the long-run average variance in the model is small in value and statistically insignificant. The parameter alpha (ARCH effect) representing the lagged variance is mostly significant with generally low values apart from the parameter in the spot variance equation for FGBL in period 2. The low significant values suggest the volatility is influenced by the past shocks though the markets are relatively calm. The parameter beta (GARCH effect) that is the coefficient on the lagged squared residuals is statistically significant in all cases with mostly high values suggesting the

[12]While both VAR and VECM models were estimated previously, only VAR is considered for GARCH modelling because of computation obstacles with error correction term.

persistence of volatility[13].

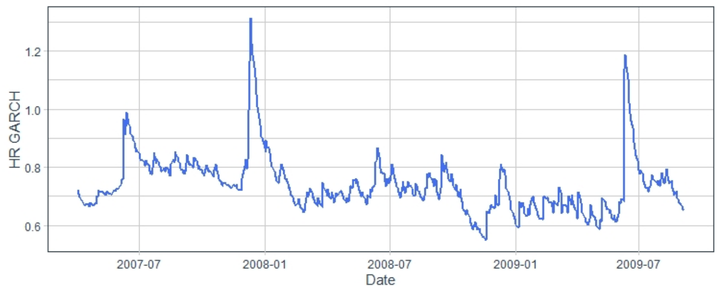
The GARCH hedge ratios are calculated similarly as with the VAR model, i.e. as the ratio of the covariance between the residuals from the spot and futures equations and the variance of futures residuals as per the equation 4.19. Figure 2 shows the estimated hedge ratios for FGBL and FGBM in periods 1 and 2, and Table 9 provides the summary statistics. It is visible that the time-varying hedge ratios are substantially volatile with wide range of values. For FGBL and FGBL in the first period, and for FGBL in the second period the hedge ratios are much higher than the constant counterparts, in some cases reaching the values over 1.4 per spot contract. The graphs exhibit several spikes reflecting increased volatility in the prices. As with constant hedge ratios, the estimates from the second period are generally lower than the ones from the first period.

Table 9: GARCH Hedge Ratio: summary statistics

Period 1				Period 2			
FGBL		FGBM		FGBL		FGBM	
Min.	0.5486	Min.	0.6720	Min.	0.4157	Min.	0.4261
1st Qu.	0.6775	1st Qu.	0.8378	1st Qu.	0.5604	1st Qu.	0.4711
Median	0.7242	Median	0.9242	Median	0.6182	Median	0.5330
Mean	0.7394	Mean	0.9374	Mean	0.6481	Mean	0.5470
3rd Qu.	0.7820	3rd Qu.	1.0254	3rd Qu.	0.6882	3rd Qu.	0.5894
Max.	1.3127	Max.	1.4280	Max	1.4178	Max.	0.9454

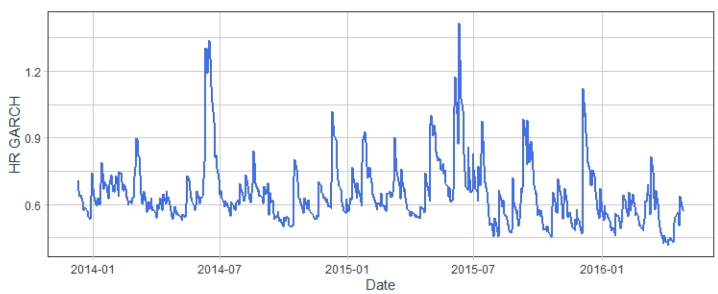
Note: The summary statistics is calculated for the time-varying hedge ratios estimated by the GARCH for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures.

[13] As per Alexander (2008) the ARCH effect usually fluctuates between 0.05 for a relatively stable market to 0.1 signalling a nervous market. With respect to the GARCH effect, the values usually fluctuate between 0.85 to 0.98 with higher values signalling a higher volatility persistence.



(a) FGBL - Period 1

(b) FGBM - Period 1



(c) FGBL - Period 2

(d) FGBM - Period 2

Figure 2: GARCH Hedge Ratios

6.1.5 Duration model

The duration hedge ratio is calculated at the beginning of the hedging period and held constant for the entire horizon of the hedge. Since the cash asset to be hedged is a CTD bond the equation 4.1 further simplifies to the following formula

$$h_d = \frac{S_t}{F_t} \times cf_{ctd}. \tag{6.1}$$

When testing the hedging effectiveness the hedge ratio is recalculated for each hedging window.

6.2 Hedging effectiveness

6.2.1 In-sample testing

The hedge ratios estimated in the previous section are further examined in terms of their hedging effectiveness. The hedging effectiveness is tested for 5-, 10-, 20- and 30-day hedging horizons while applying rolling windows over the sample. The mean and variance of returns as well as percentage variance reduction to the unhedged position is calculated for each hedging strategy and each hedging horizon.

The results for hedging with FGBL and FGBM in the first period are provided in Tables 10 and 11, respectively. The hedging strategies seem to work very well though at the cost of lower return as the un-hedged position yields highest mean return. The second highest returns are achieved with the GARCH time-varying hedge ratios while the duration method results in negative returns. When taking a 10-day hedge for FGBL as an example, the mean return from the GARCH is 0.04% whilst the duration yields a mean return of -0.2%. As one would expect the minimum returns are limited when the position is hedged as the unhedged position has lowest minimum return. In period 1 the variances of returns are generally very low. The greatest variance reduction is achieved with the duration method and the one-to-one hedge. The constant hedge ratios OLS, VAR, and VECM perform in a similar way since the hedge ratios were estimated to be very close. For FGBM the VECM hedge ratio yields a higher return than the other two. The GARCH model gives a lowest percentage variance reduction, however, it is still over 88% and 95% for FGBL and FGBM, respectively, while at the same time it offers a higher return than other strategies. The hedging performance is consistent across hedging horizons, there are no significant differences.

The results for the second period are presented in Tables 12, and 13. As in period 1 the highest mean return is given by the unhedged position. However, while for FGBL the GARCH model offers the highest return compared with other hedging strategies, for FGBM the GARCH mean return is lower than the one provided by OLS, VAR, and VECM, with the VECM being the highest. While for FGBL the minimum return is limited, the hedging strategies with FGBM sometimes resulted is lower minimum return than the one of unhedged position. Similarly, the maximum return is sometimes higher after hedging compared to the cash only position for both FGBL and FGBM. The variances of returns are higher than in period 1 and as expected, given a seemingly lower connection between CTDs and futures in this period, the percentage variance reductions are smaller. The duration method still offers the highest variance reduction and negative or small returns. For FGBL, the OLS, VAR, and VECM reduce the variance by roughly 75-77% in all hedging horizons, with the VECM giving the smallest reduction. The GARCH still reduces the variance of returns by at least 66% with the highest return among all hedging strategies. The one-to-one strategy with FGBM massively increases variance of returns, by at least 70% for each hedging horizon. The OLS, VAR, and VECM reduce the variance at around 25% rate for 5-day hedges, while the performance decreases with hedging horizon. Contrary to FGBL, the VECM hedge ratio now provides greater variance reduction than the OLS and VAR. The GARCH has been found to reduce the variance by only 5.5% for the 5-day hedge while increasing variance for other hedging horizons. Coupled with lower return, for FGBM the GARCH model did not perform well.

In summary, hedging strategies are, in almost all cases, effective in percentage variance reduction and there is an apparent risk-return trade-off. The hedging effectiveness is stronger in period 1 given there is a closer relation of CTDs and futures prices and higher variance of price series in that period. The duration method reduces the variance most while providing low or negative returns. A similar performance is observed with the one-to-one hedge though on some occasions the variance rather increased therefore this strategy could be perceived as unreliable. The constant hedge ratios - OLS, VAR, and VECM - performed similarly, with an occasional dominance of the VECM which might be given by the significance and size of the error correction term for that particular asset and period. Apart from the FGBM in period 2, the GARCH model performed well providing high returns and still relatively good percentage variance reduction. Generally, there seems to be no differences in hedging effectiveness among the hedging horizons.

Table 10: FGBL In-sample testing: Period 1

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00064	-0.02626	0.03028	0.000090	-
	1-to-1	1	0.00009	-0.00630	0.01108	0.000002	97.54%
	Duration	0.7517*	-0.00026	-0.00037	-0.00019	0.000000	99.99%
	OLS	0.9332	0.00013	-0.00722	0.01062	0.000003	96.90%
	VAR	0.9280	0.00013	-0.00731	0.01058	0.000003	96.81%
	VECM	0.9229	0.00014	-0.00741	0.01057	0.000003	96.72%
	GARCH	0.7394*	0.00020	-0.01102	0.01152	0.000010	88.39%
10-day	Cash	-	0.00130	-0.02993	0.04259	0.000164	-
	1-to-1	1	0.00019	-0.00563	0.01043	0.000004	97.78%
	Duration	0.7517*	-0.00183	-0.00238	-0.00148	0.000000	99.96%
	OLS	0.9332	0.00027	-0.00565	0.01016	0.000005	97.21%
	VAR	0.9280	0.00027	-0.00579	0.01015	0.000005	97.13%
	VECM	0.9229	0.00028	-0.00592	0.01013	0.000005	97.05%
	GARCH	0.7394*	0.00041	-0.01301	0.01788	0.000018	88.85%
20-day	Cash	-	0.00259	-0.03287	0.06222	0.000319	-
	1-to-1	1	0.00039	-0.00474	0.01264	0.000006	98.03%
	Duration	0.7517*	-0.00123	-0.00157	-0.00100	0.000000	99.99%
	OLS	0.9332	0.00054	-0.00658	0.01450	0.000008	97.55%
	VAR	0.9280	0.00055	-0.00672	0.01464	0.000008	97.48%
	VECM	0.9229	0.00056	-0.00686	0.01479	0.000008	97.40%
	GARCH	0.7394*	0.00083	-0.01544	0.02633	0.000034	89.24%
30-day	Cash	-	0.00390	-0.04234	0.07418	0.000497	-
	1-to-1	1	0.00062	-0.00515	0.01195	0.000009	98.17 %
	Duration	0.7517*	-0.00318	-0.00393	-0.00267	0.000000	99.97%
	OLS	0.9332	0.00084	-0.00639	0.01326	0.000011	97.72%
	VAR	0.9280	0.00085	-0.00656	0.01336	0.000012	97.64%
	VECM	0.9229	0.00087	-0.00672	0.01346	0.000012	97.57%
	GARCH	0.7394*	0.00127	-0.01653	0.02939	0.000056	88.64%

Note: The hedging effectiveness of strategies is evaluated for Euro-Bund (FGBL) future during 2 April 2007 to 4 September 2009 in the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 11: FGBM In-sample testing: Period 1

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00059	-0.01640	0.01941	0.000038	-
	1-to-1	1	-0.00001	-0.00374	0.00386	0.000001	98.51%
	Duration	0.8667*	0.00005	0.00002	0.00008	0.000000	99.99%
	OLS	0.9538	0.00002	-0.00357	0.00358	0.000001	98.14%
	VAR	0.9293	0.00003	-0.00390	0.00347	0.000001	97.78%
	VECM	0.9623	0.00001	-0.00346	0.00362	0.000001	98.24%
	GARCH	0.9374*	0.00007	-0.00512	0.00684	0.000002	95.69%
10-day	Cash	-	0.00118	-0.01821	0.03116	0.000074	-
	1-to-1	1	-0.00002	-0.00419	0.00388	0.000001	98.75%
	Duration	0.8667*	-0.00062	-0.00089	-0.00034	0.000000	99.96%
	OLS	0.9538	0.00003	-0.00439	0.00377	0.000001	98.30%
	VAR	0.9293	0.00006	-0.00450	0.00394	0.000002	97.90%
	VECM	0.9623	0.00002	-0.00436	0.00379	0.000001	98.41%
	GARCH	0.9374*	0.00015	-0.00533	0.01015	0.000003	95.65%
20-day	Cash	-	0.00235	-0.02921	0.03658	0.000154	-
	1-to-1	1	-0.00003	-0.00434	0.00467	0.000002	98.87%
	Duration	0.8667*	-0.00079	-0.00099	-0.00059	0.000000	99.99%
	OLS	0.9538	0.00008	-0.00518	0.00510	0.000003	98.35%
	VAR	0.9293	0.00014	-0.00571	0.00532	0.000003	97.92%
	VECM	0.9623	0.00006	-0.00501	0.00502	0.000002	98.48%
	GARCH	0.9374*	0.00031	-0.00773	0.00989	0.000007	95.66%
30-day	Cash	-	0.00358	-0.03741	0.04562	0.000255	-
	1-to-1	1	-0.00003	-0.00567	0.00446	0.000003	98.89%
	Duration	0.8667*	-0.00167	-0.00211	-0.00121	0.000000	99.97%
	OLS	0.9538	0.00014	-0.00658	0.00563	0.000004	98.32%
	VAR	0.9293	0.00023	-0.00706	0.00650	0.000005	97.86%
	VECM	0.9623	0.00011	-0.00641	0.00533	0.000004	98.45%
	GARCH	0.9374*	0.00048	-0.00886	0.01404	0.000012	95.32%

Note: The hedging effectiveness of strategies is evaluated for Euro-Bobl (FGBM) future during 2 April 2007 to 4 September 2009 in the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 12: FGBL In-sample testing: Period 2

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00118	-0.03090	0.01633	0.000050	-
	1-to-1	1	0.00001	-0.02023	0.01840	0.000013	74.50%
	Duration	0.4858*	-0.00005	-0.00012	0.00005	0.000000	99.99%
	OLS	0.7880	0.00026	-0.02010	0.01698	0.000012	76.52%
	VAR	0.7920	0.00025	-0.02010	0.01701	0.000012	76.57%
	VECM	0.7701	0.00028	-0.02012	0.01686	0.000012	76.25%
	GARCH	0.6481*	0.00048	-0.02663	0.01602	0.000016	68.56%
10-day	Cash	-	0.00247	-0.04306	0.02266	0.000097	-
	1-to-1	1	0.00002	-0.02053	0.01941	0.000024	75.18%
	Duration	0.4858*	-0.00203	-0.00225	-0.00189	0.000000	99.99%
	OLS	0.7880	0.00054	-0.02198	0.01904	0.000022	77.11%
	VAR	0.7920	0.00053	-0.02196	0.01905	0.000022	77.16%
	VECM	0.7701	0.00059	-0.02211	0.01901	0.000022	76.83%
	GARCH	0.6481*	0.00102	-0.02967	0.01854	0.000030	68.74%
20-day	Cash	-	0.00524	-0.05226	0.03485	0.000184	-
	1-to-1	1	0.00007	-0.02078	0.01861	0.000047	74.20%
	Duration	0.4858*	-0.00324	-0.00361	-0.00302	0.000000	99.99%
	OLS	0.7880	0.00117	-0.02144	0.01823	0.000045	75.55%
	VAR	0.7920	0.00115	-0.02143	0.01823	0.000045	75.61%
	VECM	0.7701	0.00126	-0.02150	0.01819	0.000046	75.25%
	GARCH	0.6481*	0.00219	-0.02876	0.01799	0.000062	66.25%
30-day	Cash	-	0.00791	-0.06808	0.04620	0.000280	-
	1-to-1	1	0.00013	-0.02143	0.02056	0.000072	74.34%
	Duration	0.4858*	0.00315	0.00258	0.00415	0.000000	99.93%
	OLS	0.7880	0.00179	-0.02078	0.02227	0.000069	75.29%
	VAR	0.7920	0.00175	-0.02078	0.02223	0.000069	75.36%
	VECM	0.7701	0.00193	-0.02080	0.02241	0.000070	74.96%
	GARCH	0.6481*	0.00332	-0.02751	0.02434	0.000092	66.93%

Note: The hedging effectiveness of strategies is evaluated for Euro-Bund (FGBL) future during 9 December 2013 to 27 April 2016 in the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 13: FGBM In-sample testing: Period 2

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00048	-0.01041	0.00700	0.000007	-
	1-to-1	1	0.00011	-0.01419	0.01517	0.000013	-71.26%
	Duration	0.7856*	-0.00050	-0.00065	-0.00044	0.000000	99.96%
	OLS	0.4328	0.00032	-0.01122	0.00961	0.000006	24.74%
	VAR	0.4382	0.00032	-0.01124	0.00966	0.000006	24.52%
	VECM	0.3801	0.00034	-0.01112	0.00909	0.000005	26.27%
	GARCH	0.5470*	0.00022	-0.01663	0.01146	0.000007	5.49%
10-day	Cash	-	0.00101	-0.01382	0.01058	0.000014	-
	1-to-1	1	0.00022	-0.01443	0.01454	0.000025	-84.13%
	Duration	0.7856*	-0.00156	-0.00197	-0.00138	0.000000	99.82%
	OLS	0.4328	0.00067	-0.01346	0.00912	0.000011	20.12%
	VAR	0.4382	0.00067	-0.01346	0.00917	0.000011	19.82%
	VECM	0.3801	0.00071	-0.01351	0.00865	0.000011	22.29%
	GARCH	0.5470*	0.00047	-0.01720	0.01061	0.000014	-5.12%
20-day	Cash	-	0.00215	-0.01307	0.01951	0.000026	-
	1-to-1	1	0.00047	-0.01433	0.01458	0.000050	-96.58%
	Duration	0.7856*	-0.00190	-0.00234	-0.00171	0.000000	99.89%
	OLS	0.4328	0.00143	-0.01306	0.01155	0.000021	18.04%
	VAR	0.4382	0.00142	-0.01307	0.01145	0.000021	17.69%
	VECM	0.3801	0.00151	-0.01299	0.01252	0.000020	20.74%
	GARCH	0.5470*	0.00101	-0.01668	0.01143	0.000028	-10.32%
30-day	Cash	-	0.00322	-0.02086	0.02066	0.000035	-
	1-to-1	1	0.00072	-0.01576	0.01501	0.000076	-116.24%
	Duration	0.7856*	0.00023	0.00000	0.00078	0.000000	99.88%
	OLS	0.4328	0.00214	-0.01796	0.01209	0.000030	13.61%
	VAR	0.4382	0.00213	-0.01793	0.01198	0.000031	13.18%
	VECM	0.3801	0.00227	-0.01831	0.01313	0.000029	17.20%
	GARCH	0.5470*	0.00152	-0.01793	0.01159	0.000042	-19.90%

Note: The hedging effectiveness of strategies is evaluated for Euro-Bobl (FGBM) future during 9 December 2013 to 27 April 2016 in the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

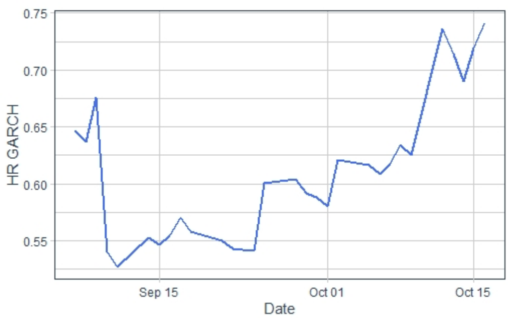
6.2.2 Out-of-sample testing

The hedging strategies are tested further out of the sample for the last 30 days of the chosen periods. For the OLS, VAR, and VECM methods the hedge ratios estimated previously are employed. The duration hedge ratio is calculated for each hedging window a day before entering a hedge. With regard to the time-varying hedge ratios, a series of 1-step-ahead forecasts was obtained while rolling the estimation window of constant size. The forecasted hedge ratios are depicted in Figure 3. While the hedge ratios for FGBL in period 1 and both FGBL and FGBM in period 2 generally fluctuate in a 0.45-0.75 band, the hedge ratios for FGBM in the first period are more volatile, with a massive drop from 1 to less than 0.55 at the beginning of the forecasting period. A decrease is caused by a sudden drop in futures price on 9 October 2009 by 2.27 while the CTD falls by only 0.304 on that day. A similar decline is evident for FGBL where the futures price decreases by 1.64 while the spot price fell only by 0.296.

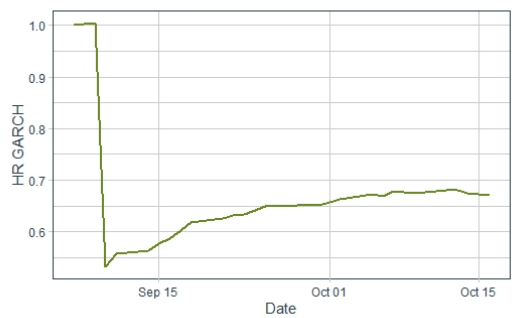
Tables 14 and 15 present the results for the first period for FGBL and FGBM, respectively. For both futures contracts, it is evident that the unhedged position no longer yields the highest mean return. Instead, the greatest mean return is provided by the duration technique and the GARCH model. For a 5-hedge with FGBL, the duration yields 0.97% while the GARCH and the cash only position provide 0.12% and 0.08%, respectively. The hedging strategies mostly provide higher minimum return when compared with the unhedged position. With respect to the percentage variance reduction, the duration method again performs best for both futures contracts. For FGBL, the OLS, VAR, and VECM reduce the variance by around 75% and 89% for 5-day and 10-day hedges, respectively, with VECM performing slightly better in terms of variance reduction. The GARCH model is effective as well, offering 64% and 84% reduction for 5-day and 10-day hedges. With the 20-day hedging horizon, however, the duration hedge ratios aside, the strategies increase the returns' variance, with the GARCH incresing the variance least. For FGBM, after the duration, the VAR provides the greatest variance reduction for a 5-day hedge while the GARCH rather increases the variance. With a 10-day hedge the VAR leads again, while the GARCH reduces the variance as well. Similarly to FGBL, almost all hedging strategies increase the variance for a 20-day hedge, with lowest increase provided by the GARCH.

The results for the second period are given in Tables 16 and 17. Apart from the unhedged portfolio, the highest mean returns are given by the duration and GARCH model for FGBL, while the high mean returns for FGBM are mostly provided by the VECM and other constant hedge ratios. The duration and the one-to-one strategy lower the variance of returns most though the return on one-to-one hedges is mostly low or negative. For FGBL all other hedging strategies perform very well, with the VAR reducing the variance most. Similarly to the in-sample analysis, the GARCH offers lowest reduction though it is still on a relatively high level. For FGBM the GARCH lowers the variance of returns more than OLS, VAR, and VECM which follows the risk-return trade-off pattern.

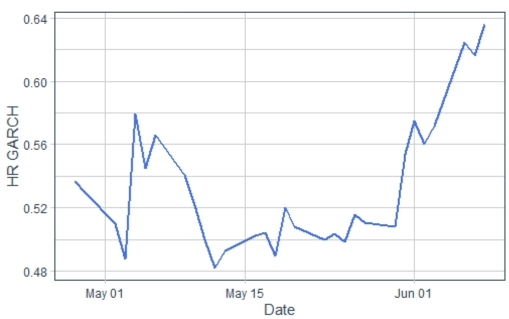
It is important to stress that the out-of-sample results have less statistical power because of low number of rolling windows tested. In many cases the duration method and the GARCH yield highest returns, with the duration also significantly lowering the variance of returns which contradicts the risk-return trade-off. The constant hedge ratios from the VAR and VECM model work relatively well, though there is no consistency in which one should be better. It seems reasonable to say that when using the constant hedge ratios, OLS, VAR, and VECM could be estimated to see which model fits the data most based on the past observations. However, it has been shown that if the model works well in the sample, it is not guaranteed that the performance level will be the same with the out-of-sample data. The GARCH time-varying hedge ratios perform relatively well mostly providing solid reduction in returns' variance. In periods when all time-series hedging strategies increase the variance, the GARCH raises the variance least. Generally, the hedging effectiveness is not influenced by the length of the hedging horizon though there are some differences in period 1.



(a) FGBL - Period 1



(b) FGBM - Period 1



(c) FGBL - Period 2

(d) FGBM - Period 2

Figure 3: GARCH 1-step-ahead forecasts

Table 14: FGBL Out-of-sample testing: Period 1

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00084	-0.01432	0.01255	0.000064	-
	1-to-1	1	0.00090	-0.00257	0.01274	0.000018	72.52%
	Duration	0.7875*	0.00971	0.00960	0.00973	0.000000	99.99%
	OLS	0.9332	0.00090	-0.00336	0.01183	0.000016	74.76%
	VAR	0.9280	0.00090	-0.00342	0.01175	0.000016	74.90%
	VECM	0.9229	0.00090	-0.00348	0.01168	0.000016	75.02%
	GARCH	0.6091*	0.00118	-0.00662	0.00982	0.000022	64.81%
10-day	Cash	-	0.00384	-0.01220	0.01987	0.000120	-
	1-to-1	1	0.00093	-0.00273	0.01157	0.000017	85.47%
	Duration	0.7875*	0.00598	0.00578	0.00603	0.000000	99.99%
	OLS	0.9332	0.00112	-0.00322	0.01019	0.000014	88.50%
	VAR	0.9280	0.00114	-0.00326	0.01008	0.000014	88.69%
	VECM	0.9229	0.00115	-0.00329	0.00998	0.000013	88.86%
	GARCH	0.6091*	0.00250	-0.00570	0.00866	0.000018	84.60%
20-day	Cash	-	0.00901	0.00198	0.01908	0.000022	-
	1-to-1	1	0.00186	-0.00250	0.01153	0.000035	-58.83%
	Duration	0.7875*	0.01079	0.01078	0.01080	0.000000	99.99%
	OLS	0.9332	0.00234	-0.00196	0.01137	0.000031	-40.03%
	VAR	0.9280	0.00238	-0.00192	0.01136	0.000030	-38.64%
	VECM	0.9229	0.00115	-0.00188	0.01134	0.000030	-37.30%
	GARCH	0.6091*	0.00556	-0.00027	0.01256	0.000023	-7.04%
30-day	Cash	-	-0.00321	-0.00321	-0.00321	-	-
	1-to-1	1	0.00888	0.00888	0.00888	-	-
	Duration	0.7875*	0.00624	0.00624	0.00624	-	-
	OLS	0.9332	0.00808	0.00808	0.00808	-	-
	VAR	0.9280	0.00802	0.00802	0.00802	-	-
	VECM	0.9229	0.00796	0.00796	0.00796	-	-
	GARCH	0.6091*	0.00672	0.00672	0.00672	-	-

Note: The hedging effectiveness of strategies is evaluated for Euro-Bund (FGBL) future during 7 September 2009 to 16 October 2009 out of the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 15: FGBM Out-of-sample testing: Period 1

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	-0.00006	-0.00841	0.00534	0.000019	-
	1-to-1	1	0.00097	-0.00150	0.01194	0.000016	13.37%
	Duration	0.8724*	0.01059	0.01049	0.01062	0.000000	99.99%
	OLS	0.9538	0.00092	-0.00182	0.01138	0.000015	20.08%
	VAR	0.9293	0.00090	-0.00199	0.01114	0.000014	23.35%
	VECM	0.9623	0.00093	-0.00176	0.01146	0.000015	18.90%
	GARCH	0.6752*	0.00122	-0.00377	0.01393	0.000023	-21.33%
10-day	Cash	-	0.00112	-0.00817	0.00982	0.000031	-
	1-to-1	1	0.00101	-0.00178	0.01100	0.000017	44.58%
	Duration	0.8724*	0.00891	0.00875	0.00896	0.000000	99.98%
	OLS	0.9538	0.00102	-0.00207	0.01024	0.000015	51.67%
	VAR	0.9293	0.00102	-0.00223	0.00984	0.000014	55.07%
	VECM	0.9623	0.00102	-0.00202	0.01038	0.000016	50.43%
	GARCH	0.6752*	0.00177	-0.00389	0.01075	0.000018	43.92%
20-day	Cash	-	0.00300	-0.00212	0.00770	0.000006	-
	1-to-1	1	0.00195	-0.00197	0.01114	0.000033	-418.10%
	Duration	0.8724*	0.01032	0.01026	0.01034	0.000000	99.98%
	OLS	0.9538	0.00200	-0.00182	0.01084	0.000031	-377.20%
	VAR	0.9293	0.00203	-0.00174	0.01069	0.000029	-356.36%
	VECM	0.9623	0.00199	-0.00185	0.01090	0.000031	-384.57%
	GARCH	0.6752*	0.00370	-0.00189	0.01399	0.000043	-56.15%
30-day	Cash	-	-0.00354	-0.00354	-0.00354	-	-
	1-to-1	1	0.00934	0.00934	0.00934	-	-
	Duration	0.8724*	0.00757	0.00757	0.00757	-	-
	OLS	0.9538	0.00875	0.00875	0.00875	-	-
	VAR	0.9293	0.00843	0.00843	0.00843	-	-
	VECM	0.9623	0.00886	0.00886	0.00886	-	-
	GARCH	0.6752*	0.01005	0.01005	0.01005	-	-

Note: The hedging effectiveness of strategies is evaluated for Euro-Bobl (FGBM) future during 7 September 2009 to 16 October 2009 out of the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 16: FGBL Out-of-sample testing: Period 2

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00322	-0.00384	0.01268	0.000021	-
	1-to-1	1	-0.00015	-0.00112	0.00133	0.000000	98.58%
	Duration	0.4048*	0.00416	0.00415	0.00416	0.000000	99.99%
	OLS	0.7880	0.00056	-0.00076	0.00282	0.000001	95.71%
	VAR	0.7920	0.00055	-0.00074	0.00279	0.000001	95.85%
	VECM	0.7701	0.00062	-0.00083	0.00295	0.000001	95.02%
	GARCH	0.5329*	0.00137	-0.00192	0.00570	0.000005	77.92%
10-day	Cash	-	0.00430	-0.00428	0.01403	0.000035	-
	1-to-1	1	-0.00022	-0.00126	0.00101	0.000000	99.14%
	Duration	0.4048*	0.00820	0.00819	0.00821	0.000000	99.99%
	OLS	0.7880	0.00074	-0.00116	0.00271	0.000001	96.20%
	VAR	0.7920	0.00072	-0.00114	0.00265	0.000001	96.34%
	VECM	0.7701	0.00082	-0.00123	0.00296	0.000002	95.51%
	GARCH	0.5329*	0.00177	-0.00235	0.00657	0.000007	78.87%
20-day	Cash	-	0.00803	0.00411	0.01249	0.000008	-
	1-to-1	1	-0.00042	-0.00139	0.00064	0.000000	94.71%
	Duration	0.4048*	0.00710	0.00709	0.00711	0.000000	99.99%
	OLS	0.7880	0.00137	-0.00018	0.00227	0.000001	91.52%
	VAR	0.7920	0.00133	-0.00020	0.00222	0.000001	91.67%
	VECM	0.77016	0.00152	-0.00008	0.00250	0.000001	90.84%
	GARCH	0.5329*	0.00329	0.00085	0.00565	0.000002	67.44%
30-day	Cash	-	0.02112	0.02112	0.02112	-	-
	1-to-1	1	-0.00049	-0.00049	-0.00049	-	-
	Duration	0.4048*	0.01233	0.01233	0.01233	-	-
	OLS	0.7880	0.00407	0.00407	0.00407	-	-
	VAR	0.7920	0.00398	0.00398	0.00398	-	-
	VECM	0.7701	0.00445	0.00445	0.00445	-	-
	GARCH	0.5329*	0.00920	0.00920	0.00920	-	-

Note: The hedging effectiveness of strategies is evaluated for Euro-Bund (FGBL) future during 28 April 2016 to 8 June 2016 out of the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively. *For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 17: FGBM Out-of-sample testing: Period 2

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00093	-0.00122	0.00470	0.000003	-
	1-to-1	1	0.00002	-0.00058	0.00067	0.000000	97.62%
	Duration	0.7469*	0.00049	0.00049	0.00049	0.000000	99.99%
	OLS	0.4328	0.00054	-0.00076	0.00282	0.000001	64.22%
	VAR	0.4382	0.00053	-0.00075	0.00280	0.000001	64.81%
	VECM	0.3801	0.00059	-0.00081	0.00305	0.000001	58.20%
	GARCH	0.4410*	0.00047	-0.00067	0.00239	0.000001	72.90%
10-day	Cash	-	0.00124	-0.00182	0.00414	0.000004	-
	1-to-1	1	0.00001	-0.00053	0.00037	0.000000	98.62%
	Duration	0.7469*	0.00124	0.00124	0.00125	0.000000	99.99%
	OLS	0.4328	0.00071	-0.00120	0.00242	0.000001	65.06%
	VAR	0.4382	0.00070	-0.00119	0.00240	0.000001	65.65%
	VECM	0.3801	0.00077	-0.00127	0.00262	0.000002	58.96%
	GARCH	0.4410*	0.00062	-0.00109	0.00206	0.000001	73.00%
20-day	Cash	-	0.00228	0.00076	0.00434	0.000001	-
	1-to-1	1	0.00001	-0.00032	0.00044	0.000000	95.27%
	Duration	0.7469*	0.00090	0.00089	0.00090	0.000000	99.99%
	OLS	0.4328	0.00130	0.00030	0.00242	0.000000	62.47%
	VAR	0.4382	0.00129	0.00029	0.00240	0.000000	63.05%
	VECM	0.3801	0.00142	0.00035	0.00265	0.000001	56.60%
	GARCH	0.4410*	0.00114	0.00023	0.00201	0.000000	75.80%
30-day	Cash	-	0.00569	0.00569	0.00569	-	-
	1-to-1	1	0.00020	0.00020	0.00020	-	-
	Duration	0.7469*	0.00158	0.00158	0.00158	-	-
	OLS	0.4328	0.00331	0.00331	0.00331	-	-
	VAR	0.4382	0.00328	0.00328	0.00328	-	-
	VECM	0.3801	0.00360	0.00360	0.00360	-	-
	GARCH	0.4410*	0.00295	0.00295	0.00295	-	-

Note: The hedging effectiveness of strategies is evaluated for Euro-Bobl (FGBM) future during 28 April 2016 to 8 June 2016 out of the sample. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively. *For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

6.3 Robustness testing

To confirm the previous findings and obtain more consistency in the results the same analysis is conducted with the joined datasets of the two subsamples. An immediate advantage is an increase in the sample size for hedge ratio estimation which could have an impact on the models' outcome.

However, it is important to consider that there is a time gap between the subsamples over the years 2009 to 2013. As is evident from the previous Figure 1 and the summary statistics in Table 2, both futures and CTD prices rose over the years with the biggest jump in FGBL. The price increase is attributable mainly to sinking interest rates with Bund yield decreasing from around 4% in April 2007 to around 1.7% in December 2013. More liquid Bund futures market could also be subject to the flight-to-safety while also being affected by the T7 technology upgrade in 2013.

Nonetheless, given the main goal of the analysis is to re-assess the effectiveness of the hedging strategies rather than to study the dynamics between the prices, the data is artificially shifted to bring the series onto the same level. The aim is to avoid misleading results, that could potentially be caused by a jump in prices between the subsets, and to enable the analysis. To obtain a continuous set of prices, each price series in the second subsample is shifted by a constant so that the first observation of the second subsample matches the last observation of the same series in the first subsample.

The results for the joint dataset are available in Appendix B. The futures and spot series are again found to be non-stationary and integrated of order one, with ADF and KPSS test results provided in Table B1. Furthermore, the residuals from the OLS regression with level variables are not stationary therefore it is confirmed the futures and CTD prices are not cointegrated. The hedge ratio is therefore estimated by the OLS with differenced logarithms. The results in Table B4 show statistically significant coefficients that are estimated to be 0.78 and 0.75 for FGBL and FGBM, respectively. Based on the results of Box-Pierce and Ljung-Box tests in Table B5 that indicate autocorrelation in the regression residuals, the series are further modelled with the VAR and VECM models or second order, see Tables B6, B8, and the variance-covariance matrix in Table B10. The error correction term is again found to be statistically significant though for FGBM it is greater than one, signalling the prices do not converge. Consistent with the subsample results the OLS, VAR, and VECM hedge ratios are of similar value. Before the GARCH estimation the series are again examined by autocorrelation and ARCH-LM tests. The results in Table B11 show that while the null of no conditional heteroskedasticity is not rejected for all series, the autocorrelation tests and the ACF and PACF of squared residuals in Figures B1 and B2 suggest the series exhibit signs of dependencies. Table B12 providing the results for the GARCH model shows a very similar outcome to the previous analyses. The long-run average variance is found to be insignificant, while the GARCH effect is significant and mostly large in value indicating a higher volatility persistence.

The hedging performance in the sample is provided in Tables 18 and 19 for FGBL and FGBM, respectively. Similarly to previous results the one-to-one hedge yields relatively low returns while also being outperformed by other strategies in percentage variance reduction. The duration method consistently reduces the variance most but the obtained mean returns are very low or negative. The highest mean return is delivered by the GARCH again. While reducing the variance of returns least it still provides the reduction of over 40% for FGBL as the other methodologies. Interestingly, for FGBM the classical trade-off between risk and return seem to be absent as GARCH provides the highest return from all the hedging strategies while also reducing the variance most (next to the leading duration strategy). The same pattern can be observed for the VAR model, however, the differences between OLS, VAR, and VECM are small and there is again no consistency in which model outperforms the other.

Table 18: Robustness: FGBL In-sample testing

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00133	-0.03090	0.11953	0.000120	-
	1-to-1	1	0.00046	-0.02270	0.11902	0.000063	46.98%
	Duration	0.6822*	-0.00040	-0.00081	-0.00018	0.000000	99.97%
	OLS	0.7794	0.00065	-0.02000	0.11911	0.000063	47.68%
	VAR	0.7758	0.00066	-0.02001	0.11911	0.000063	47.64%
	VECM	0.7604	0.00067	-0.02002	0.11911	0.000063	47.45%
	GARCH	0.6419*	0.00078	-0.02491	0.11912	0.000066	44.78%
10-day	Cash	-	0.00273	-0.04306	0.11653	0.000220	-
	1-to-1	1	0.00092	-0.02200	0.11893	0.000123	43.97%
	Duration	0.6822*	-0.00254	-0.00458	-0.00143	0.000001	99.65%
	OLS	0.7794	0.00132	-0.02133	0.11840	0.000119	45.73%
	VAR	0.7758	0.00133	-0.02136	0.11839	0.000119	45.70%
	VECM	0.7604	0.00135	-0.02148	0.11836	0.000120	45.58%
	GARCH	0.6419*	0.00158	-0.02743	0.11770	0.000124	43.43%
20-day	Cash	-	0.00559	-0.05226	0.12852	0.000422	-
	1-to-1	1	0.00185	-0.02225	0.11910	0.000244	42.22%
	Duration	0.6822*	-0.00166	-0.00296	-0.00097	0.000000	99.93%
	OLS	0.7794	0.00268	-0.02114	0.12002	0.000235	44.17%
	VAR	0.7758	0.00269	-0.02115	0.12005	0.000236	44.15%
	VECM	0.7604	0.00275	-0.02121	0.12022	0.000236	44.05%
	GARCH	0.6419*	0.00324	-0.02664	0.12361	0.000246	41.70%
30-day	Cash	-	0.00847	-0.06808	0.12765	0.000646	-
	1-to-1	1	0.00281	-0.02529	0.12911	0.000363	43.73%
	Duration	0.6822*	-0.00411	-0.00701	-0.00260	0.000001	99.78%
	OLS	0.7794	0.00406	-0.02073	0.12668	0.000354	45.21%
	VAR	0.7758	0.00408	-0.02073	0.12664	0.000354	45.18%
	VECM	0.7604	0.00417	-0.02074	0.12647	0.000355	45.05%
	GARCH	0.6419*	0.00490	-0.02559	0.12534	0.000375	41.99%

Note: The hedging effectiveness of strategies is evaluated for Euro-Bund (FGBL) future in the sample with the joint dataset. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 19: Robustness: FGBM In-sample testing

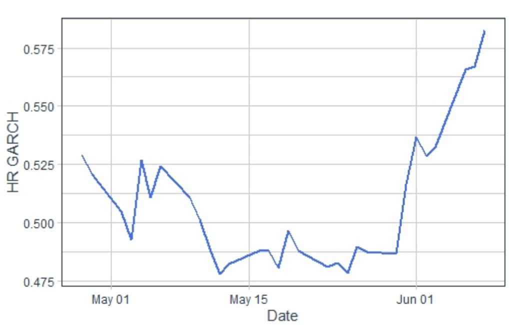
Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00086	-0.01640	0.08835	0.000051	-
	1-to-1	1	0.00041	-0.01483	0.08967	0.000039	24.24%
	Duration	0.8349*	0.00003	-0.00006	0.00008	0.000000	99.99%
	OLS	0.7538	0.00052	-0.01321	0.08935	0.000036	28.55%
	VAR	0.7465	0.00053	-0.01317	0.08934	0.000036	28.57%
	VECM	0.7612	0.00052	-0.01326	0.08936	0.000036	28.52%
	GARCH	0.6852*	0.00053	-0.01529	0.08910	0.000035	30.68%
10-day	Cash	-	0.00174	-0.01821	0.08575	0.000095	-
	1-to-1	1	0.00083	-0.01514	0.08896	0.000076	20.04%
	Duration	0.8349*	-0.00077	-0.00157	-0.00034	0.000000	99.93%
	OLS	0.7538	0.00106	-0.01382	0.08817	0.000070	25.61%
	VAR	0.7465	0.00106	-0.01379	0.08815	0.000070	25.67%
	VECM	0.7612	0.00105	-0.01385	0.08819	0.000070	25.54%
	GARCH	0.6852*	0.00108	-0.01618	0.08757	0.000067	28.71%
20-day	Cash	-	0.00355	-0.02921	0.09092	0.000184	-
	1-to-1	1	0.00168	-0.01514	0.08881	0.000148	19.41%
	Duration	0.8349*	-0.00090	-0.00147	-0.00059	0.000000	99.98%
	OLS	0.7538	0.00214	-0.01354	0.08927	0.000137	25.38%
	VAR	0.7465	0.00215	-0.01353	0.08928	0.000137	25.45%
	VECM	0.7612	0.00213	-0.01355	0.08926	0.000138	25.30%
	GARCH	0.6852*	0.00218	-0.01557	0.08976	0.000131	28.64%
30-day	Cash	-	0.00537	-0.03741	0.09249	0.000285	-
	1-to-1	1	0.00254	-0.01647	0.09948	0.000222	22.05%
	Duration	0.8349*	-0.00191	-0.00319	-0.00121	0.000000	99.94%
	OLS	0.7538	0.00324	-0.01540	0.09604	0.000207	27.39%
	VAR	0.7465	0.00326	-0.01546	0.09594	0.000207	27.44%
	VECM	0.7612	0.00322	-0.01535	0.09614	0.000207	27.33%
	GARCH	0.6852*	0.00330	-0.01779	0.09524	0.000199	30.20%

Note: The hedging effectiveness of strategies is evaluated for Euro-Bobl (FGBM) future in the sample with the joint dataset. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

For the out-of-sample testing, time-varying hedge ratios comprise the series of one-step-ahead forecasts while the estimation window for the forecasts is now larger than the one used for the subsamples. The forecasted hedge ratios are displayed in Figure 4. The span of the hedge ratios is now much smaller when compared to the subsample analyses with FGBL ranging from 0.475 to 0.6 and FGBM remaining close to 0.48.

The out-of-sample test results for the last 30 days of the sample are given in Tables 20 and 21. Following the previous out-of-sample analyses the highest returns are achieved with not only the unhedged position but also with the duration and the GARCH models while the duration method still leads in the percentage variance reduction. As opposed to the subsample results, the variance is not increased by any of the strategies, rather the performance is more stable and the percentage variance reduction is generally greater.

Overall, the robustness results are very similar to the outcomes of the individual subsample analyses, nonetheless, the performance of the models is more consistent and settled. The one-to-one hedge is generally outperformed by other strategies. It has been confirmed the duration hedge leads to the greatest variance reduction that however might also squeeze the return. The GARCH model delivers an attractive return while reducing the risk relatively well.



(a) FGBL

(b) FGBM

Figure 4: Robustness: GARCH 1-step-ahead forecasts

Table 20: Robustness: FGBL Out-of-sample testing

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00322	-0.00384	0.01268	0.000021	-
	1-to-1	1	-0.00058	-0.00254	0.00085	0.000001	95.63%
	Duration	0.5012*	0.00303	0.00302	0.00304	0.000000	99.99%
	OLS	0.7794	0.00026	-0.00073	0.00218	0.000000	98.14%
	VAR	0.7758	0.00027	-0.00072	0.00220	0.000000	98.07%
	VECM	0.7604	0.00033	-0.00068	0.00233	0.000000	97.73%
	GARCH	0.5082*	0.00124	-0.00172	0.00501	0.000004	82.35%
10-day	Cash	-	0.00430	-0.00428	0.01403	0.000035	-
	1-to-1	1	-0.00080	-0.00290	0.00115	0.000001	96.23%
	Duration	0.5012*	0.00589	0.00586	0.00590	0.000000	99.99%
	OLS	0.7794	0.00032	-0.00101	0.00145	0.000000	98.65%
	VAR	0.7758	0.00034	-0.00101	0.00149	0.000000	98.58%
	VECM	0.7604	0.00042	-0.00100	0.00170	0.000001	98.24%
	GARCH	0.5082*	0.00162	-0.00213	0.00583	0.000006	83.10%
20-day	Cash	-	0.00803	0.00411	0.01249	0.000008	-
	1-to-1	1	-0.00151	-0.00281	-0.00010	0.000001	92.20%
	Duration	0.5012*	0.00506	0.00504	0.00508	0.000000	99.99%
	OLS	0.7794	0.00059	-0.00068	0.00133	0.000000	94.02%
	VAR	0.7758	0.00063	-0.00066	0.00136	0.000000	93.95%
	VECM	0.7604	0.00077	-0.00056	0.00154	0.000000	93.60%
	GARCH	0.5082*	0.00301	0.00075	0.00504	0.000002	74.98%
30-day	Cash	-	0.02112	0.02112	0.02112	-	-
	1-to-1	1	-0.00324	-0.00324	-0.00324	-	-
	Duration	0.5012*	0.00882	0.00882	0.00882	-	-
	OLS	0.7794	0.00210	0.00210	0.00210	-	-
	VAR	0.7758	0.00219	0.00219	0.00219	-	-
	VECM	0.7604	0.00256	0.00256	0.00256	-	-
	GARCH	0.5082*	0.00834	0.00834	0.00834	-	-

Note: The hedging effectiveness of strategies is evaluated for Euro-Bund (FGBL) future out of the sample with the joint dataset. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

Table 21: Robustness: FGBM Out-of-sample testing

Horizon	Method	HR	Mean return	Min return	Max return	Variance	Variance reduction
5-day	Cash	-	0.00093	-0.00122	0.00470	0.000003	-
	1-to-1	1	-0.00006	-0.00078	0.00053	0.000000	97.70%
	Duration	0.7594*	0.00037	0.00037	0.00037	0.000000	99.99%
	OLS	0.7538	0.00019	-0.00038	0.00116	0.000000	93.04%
	VAR	0.7465	0.00019	-0.00039	0.00119	0.000000	92.71%
	VECM	0.7612	0.00018	-0.00038	0.00112	0.000000	93.37%
	GARCH	0.4849*	0.00045	-0.00066	0.00242	0.000001	73.52%
10-day	Cash	-	0.00124	-0.00182	0.00414	0.000004	-
	1-to-1	1	-0.00009	-0.00056	0.00037	0.000000	98.64%
	Duration	0.7594*	0.00097	0.00096	0.00097	0.000000	99.99%
	OLS	0.7538	0.00024	-0.00065	0.00099	0.000000	94.10%
	VAR	0.7465	0.00025	-0.00066	0.00102	0.000000	93.77%
	VECM	0.7612	0.00023	-0.00063	0.00096	0.000000	94.42%
	GARCH	0.4849*	0.00060	-0.00106	0.00207	0.000001	74.41%
20-day	Cash	-	0.00228	0.00076	0.00434	0.000001	-
	1-to-1	1	-0.00017	-0.00045	0.00031	0.000000	95.43%
	Duration	0.7594*	0.00068	0.00067	0.00068	0.000000	99.99%
	OLS	0.7538	0.00043	-0.00011	0.00083	0.000000	90.67%
	VAR	0.7465	0.00045	-0.00010	0.00085	0.000000	90.35%
	VECM	0.7612	0.00042	-0.00012	0.00080	0.000000	91.00%
	GARCH	0.4849*	0.00110	0.00020	0.00202	0.000000	71.86%
30-day	Cash	-	0.00569	0.00569	0.00569	-	-
	1-to-1	1	-0.00024	-0.00024	-0.00024	-	-
	Duration	0.7594*	0.00118	0.00118	0.00118	-	-
	OLS	0.7538	0.00122	0.00122	0.00122	-	-
	VAR	0.7465	0.00126	0.00126	0.00126	-	-
	VECM	0.7612	0.00117	0.00117	0.00117	-	-
	GARCH	0.4849*	0.00282	0.00282	0.00282	-	-

Note: The hedging effectiveness of strategies is evaluated for Euro-Bobl (FGBM) future out of the sample with the joint dataset. Percentage variance reduction is calculated according to the equation $PVR = \frac{Var(U) - Var(H)}{Var(U)}$, where $Var(U)$ and $Var(H)$ are the variances of the unhedged and hedged portfolios, respectively.
*For non-constant hedge ratios, the mean of hedge ratios within a sample is provided.

7 Conclusion

The purpose of the thesis was to evaluate the effectiveness of various hedging strategies and to inspect whether enhanced methods, especially the one considering time-varying hedge ratios, contribute to lower portfolio risk and offer advantages over simple constant hedges. The analysis was focused on interest rate futures hedging whereby Euro-Bund and Euro-Bobl futures traded at Eurex were used to hedge the corresponding CTD bonds. Hedge ratios were estimated with OLS, VAR, VECM, GARCH, and the duration-based methodology. Their hedging effectiveness was subsequently measured in terms of portfolio variance reduction achieved with a particular hedging strategy while also considering risk-return trade-off. The analysis was carried out for two subperiods while also conducting a robustness check with the joint dataset.

Although there was an apparent correlation between futures and bonds, the cointegration between the series has not been confirmed. This is potentially due to the nature of the bond market and the economic situation of the time. Generally, the hedge ratios were estimated to be higher during the global financial crisis (period 1) relative to the ones predicted for the time after the European sovereign debt crisis (period 2), signalling a weakened relationship between CTD bonds and their futures. The reason behind could be explained by the environment of low interest rates governing period 2, and relatively smaller variance of prices during that time. In almost all cases the hedge ratio was estimated to be below one which follows the aforementioned idea of Ederington (1979) that, contrary to the traditional theory, investors may find it optimal to hedge only a proportion of their entire portfolio. The VAR and VECM, correcting for serial correlation and convergence of prices, generally produced hedge ratios similar to the OLS. Based on the presence of ARCH effects, a time-varying hedge ratio that accounts for conditional information entering the markets was estimated with the GARCH model. This hedge ratio was found to be substantially volatile with a wide range of values.

It was shown that in, almost all cases, the hedging strategies work well in terms of variance reduction and the futures contracts are efficient at limiting potential risks. However, generally there is an apparent trade-off whereby the more the variance is cut the smaller the returns are, with the unhedged position yielding the highest mean return. This is visible especially with the duration strategy that on many occasions decreased the variance by 99% while the returns were low or negative in the sample. Constant hedge ratios produced by the OLS, VAR, and VECM, performed similarly providing generally high variance reduction while there was no consistency in which one is obviously superior. From all the strategies, GARCH limited the variance least, nonetheless, mostly it provided the reduction of at least 65% while also delivering one of the highest returns along with the unhedged position. On the other hand, the one-to-one strategy provided, on some occasions, high variance reduction, but it was often outperformed by other methods, in both return and variance. Generally, the hedging efficiency was not influenced by the length of hedging horizon. Similarly, the relative performance of models was consistent between the two periods tested, however, the overall hedging effectiveness was smaller after the European sovereign debt crisis given the reduced cohesion of the futures and CTDs influenced by lower variance of price series.

None of the hedging strategies performed better than the other in all aspects, though some do have appealing characteristics. Eventually, it depends on the risk-return trade-off and the investor himself. Based on the results, a risk averse investor would employ the duration method securing his bond portfolio from volatile returns, while a less risk averse investor would opt for a time-varying hedge ratio providing some room for a decent return.

The work suffers from some limitations. The periods chosen bear the effects of the crises which could eventually have an impact on models' outcome. The analysis would be worth repeating once the low yields return to their natural levels to confirm consistency. Furthermore, the analysis does not consider transaction costs which could make the dynamic strategy costly in the case of frequent portfolio rebalancing. The investor would then have to consider whether the costs are more than compensated by sufficient profit from the strategy. Care should therefore be taken when interpreting the results.

The research could be extended further. Despite the fact that all bonds are subject to interest rate risk, it would be beneficial to examine how the strategies perform when interest rate futures are used to hedge riskier bonds. Moreover, given the efficiency of the duration method, which was specifically developed for fixed-income assets, it could be of benefit to further investigate whether any specific characteristics of a bond could be exploited to produce greater returns while maintaining the same level of risk protection.

References

Adler, M. & Detemple, J. (1988). Hedging with futures in an intertemporal portfolio context. *The Journal of Futures Markets (1986-1998)*, *8*(3), 249.

Alexander, P. C. (2008). *Market risk analysis: Pricing, hedging and trading financial instruments*. John Wiley & Sons.

Alizadeh, A. & Nomikos, N. (2004). A markov regime switching approach for hedging stock indices. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *24*(7), 649–674.

Anderson, R. W. & Danthine, J.-P. (1980). Hedging and joint production: Theory and illustrations. *The Journal of Finance*, *35*(2), 487–498.

Baillie, R. T. & DeGennaro, R. P. (1990). Stock returns and volatility. *Journal of financial and Quantitative Analysis*, *25*(2), 203–214.

Baillie, R. T. & Myers, R. J. (1991). Bivariate garch estimation of the optimal commodity futures hedge. *Journal of Applied Econometrics*, *6*(2), 109–124.

Banerjee, A., Dolado, J. J., Galbraith, J. W. & Hendry, D. (1993). Co-integration, error correction, and the econometric analysis of non-stationary data. *OUP Catalogue*.

Barnhart, S. W. & Szakmary, A. C. (1991). Testing the unbiased forward rate hypothesis: Evidence on unit roots, co-integration, and stochastic coefficients. *Journal of Financial and Quantitative Analysis*, *26*(2), 245–267.

Barrios, S., Iversen, P., Lewandowska, M. & Setzer, R. (2009). *Determinants of intra-euro area government bond spreads during the financial crisis*. Directorate General Economic and Financial Affairs (DG ECFIN).

Benninga, S., Eldor, R. & Zilcha, I. (1984). The optimal hedge ratio in unbiased futures markets. *Journal of futures markets*, *4*(2), 155–159.

Bera, A. K., Bubnys, E. & Park, T. H. (1993). Arch effects and efficient estimation of hedge ratios for stock index futures. *Advances in Futures and Options Research*, *6*, 313–328.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3), 307–327.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of economics and statistics*, *69*(3), 542–547.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model. *Review of Economics and statistics*, *72*(3), 498–505.

Bollerslev, T., Engle, R. F. & Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. *Journal of political Economy*, *96*(1), 116–131.

Box, G. E. & Pierce, D. A. (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. *Journal of the American statistical Association*, *65*(332), 1509–1526.

Briys, E., Crouhy, M. & Schlesinger, H. (1990). Optimal hedging under intertemporally dependent preferences. *The Journal of Finance*, *45*(4), 1315–1324.

Brooks, C. (2014). *Introductory econometrics for finance*. Cambridge university press.

Brooks, C., Černỳ, A. & Miffre, J. (2012). Optimal hedging with higher moments. *Journal of Futures Markets*, *32*(10), 909–944.

Brown, S. L. (1985). A reformulation of the portfolio model of hedging. *American Journal of Agricultural Economics*, *67*(3), 508–512.

Caporin, M., Jimenez-Martin, J.-A. & Gonzalez-Serrano, L. (2014). Currency hedging strategies in strategic benchmarks and the global and euro sovereign financial crises. *Journal of International Financial Markets, Institutions and Money*, *31*, 159–177.

Carter, C. A. & Loyns, R. (1985). Hedging feedlot cattle: A canadian perspective. *American Journal of Agricultural Economics*, *67*(1), 32–39.

Castelino, M. G. (1990). Minimum-variance hedging with futures revisited. *Journal of Portfolio Management*, *16*(3), 74.

Castelino, M. G. (1992). Hedge effectiveness: Basis risk and minimum-variance hedging. *The Journal of Futures Markets (1986-1998)*, *43*(1), 187.

Cecchetti, S. G., Cumby, R. E. & Figlewski, S. (1988). Estimation of the optimal futures hedge. *The Review of Economics and Statistics*, 623–630.

Čerović, S., Pepić, M., Čerović, S. & Čerović, N. (2014). Duration and convexity of bonds. *Singidunum Journal of Applied Sciences*, *11*(1), 53–66.

Chakraborty, A. & Barkoulas, J. T. (1999). Dynamic futures hedging in currency markets. *The European Journal of Finance*, *5*(4), 299–314.

Chen, S.-S., Lee, C.-F. & Shrestha, K. (2013). Futures hedge ratios: A review. *Encyclopedia of Finance*, 871–890.

Chen, Y.-T., Ho, K.-Y. & Tzeng, L. Y. (2014). Riskiness-minimizing spot-futures hedge ratio. *Journal of Banking & Finance*, *40*, 154–164.

Chou, W., Denis, K. F. & Lee, C.-F. (1996). Hedging with the nikkei index futures: The convential model versus the error correction model. *The Quarterly Review of Economics and Finance*, *36*(4), 495–505.

Czekierda, B. & Zhang, W. (2010). *Dynamic hedge ratios on currency futures* (Master's thesis, Goeteborgs universitet). 30 pages.

Daigler, R. T. (1998). *Comparing hedge ratio methodologies for fixed-income investments*. Florida International University, College of Business, Department of Finance.

Davidson, R. & MacKinnon. (1993). Estimation and inference in econometrics.

Dickey, D. A. & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*, *74*(366a), 427–431.

Ederington, L. H. (1979). The hedging performance of the new futures markets. *The Journal of Finance*, *34*(1), 157–170.

Ejsing, J. & Sihvonen, J. (2009). Liquidity premia in german government bonds.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987–1007.

Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, *20*(3), 339–350.

Engle, R. F. & Granger, C. W. (1987). Co-integration and error correction: Representation, estimation, and testing. *Econometrica*, *55*(2), 251–276.

Eurex. (2007). Interest rate derivatives - fixed income trading strategies. Retrieved December 15, 2019, from https://www.eurexchange.com/resource/blob/298900/fb9a739a9da7ea294164c90678435c13/data/brochure_fixed_income_trading_strategies_en.pdf

Eurex. (2019). Benchmark fixed income derivatives. Retrieved December 15, 2019, from https://www.eurexchange.com/resource/blob/38430/c75c387c98cb2f27019048a%205d521bb61/data/factsheet_eurex_benchmark_fixed_income_derivatives.pdf

Fabozzi, F. J. (2007). *Fixed income analysis*. 2nd ed. New Jersey: John Wiley & Sons.

Fan, J. H., Roca, E. & Akimov, A. (2010). *Hedging with futures contract: Estimation and performance evaluation of optimal hedge ratios in the european union emissions trading scheme*. Griffith University, Department of Accounting, Finance and Economics.

Ferguson, R. & Leistikow, D. (1999). Futures hedge profit measurement, error-correction model vs. regression approach hedge ratios, and data error effects. *Financial Management*, 118–125.

Figlewski, S. (1984). Hedging performance and basis risk in stock index futures. *The Journal of Finance*, *39*(3), 657–669.

Gagnon, L. & Lypny, G. (1997). The benefits of dynamically hedging the toronto 35 stock index. *Canadian Journal of Administrative Sciences/Revue Canadienne des Sciences de l'Administration*, *14*(1), 69–78.

Gay, G. D., Kolb, R. W. & Chiang, R. (1983). Interest rate hedging: An empirical test of alternative strategies. *Journal of Financial Research*, *6*(3), 187–197.

Ghosh, A. (1993a). Cointegration and error correction models: Intertemporal causality between index and futures prices. *Journal of futures markets*, *13*(2), 193–198.

Ghosh, A. (1993b). Hedging with stock index futures: Estimation and forecasting with error correction model. *Journal of futures markets*, *13*(7), 743–752.

Ghosh, A. (1995). The hedging effectiveness of ecu futures contracts: Forecasting evidence from an error correction model. *Financial Review*, *30*(3), 567–581.

Ghosh, A. & Clayton, R. (1996). Hedging with international stock index futures: An intertemporal error correction model. *Journal of Financial Research*, *19*(4), 477–491.

Granger, C. W. (1981). Some properties of time series data and their use in econometric model specification. *Journal of econometrics*, *16*(1), 121–130.

Hatemi-J, A. & El-Khatib, Y. (2012). Stochastic optimal hedge ratio: Theory and evidence. *Applied Economics Letters*, *19*(8), 699–703.

Heaney, R. (1998). A test of the cost-ofcarry relationship using the london metal exchange lead contract. *The Journal of Futures Markets (1986-1998)*, *18*(2), 177.

Herbst, A. F., Kare, D. D. & Caples, S. C. (1989). Hedging effectiveness and minimum risk hedge ratios in the presence of autocorrelation: Foreign currency futures. *The Journal of Futures Markets (1986-1998)*, *9*(3), 185.

Herbst, A. F., Kare, D. D. & Marshall, J. F. (1993). A time varying, convergence adjusted hedge ratio model. *Advances in Futures and Options Research*, *6*, 137–155.

Hill, J. & Schneeweis, T. (1982). The hedging effectiveness of foreign currency futures. *Journal of Financial Research*, *5*(1), 95–104.

Holmes, P. (1996). Stock index futures hedging: Hedge ratio estimation, duration effects, expiration effects and hedge ratio stability. *Journal of Business Finance & Accounting*, *23*(1), 63–77.

Hsu, C.-C., Tseng, C.-P. & Wang, Y.-H. (2008). Dynamic hedging with futures: A copula-based garch model. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *28*(11), 1095–1116.

Hull, J. C. (2012). *Options, futures, and other derivatives.* 8th ed. New Jersey: PrenticeHall.

Johnson, L. L. (1960). The theory of hedging and speculation in commodity futures. *The Review of Economic Studies*, *27*(3), 139–151.

Junkus, J. C. & Lee, C.-F. (1984). Use of three stock index futures in hedging decisions. *BEBR faculty working paper; no. 1041.*

Kenourgios, D., Samitas, A. & Drosos, P. (2008). Hedge ratio estimation and hedging effectiveness: The case of the s&p 500 stock index futures contract. *International Journal of Risk assessment and management*, *9*(1-2), 121–134.

Kluza, S. & Sławiński, A. (2004). Factors affecting t-bond prices: From investors perspective. *22nd NBP Conference on Monetary policy in the environment of structural changes.*

Kočenda, E. & Černý, A. (2015). *Elements of time series econometrics: An applied approach.* Charles University in Prague, Karolinum Press.

Kolb, R. W. & Overdahl, J. A. (2003). *Financial derivatives.* John Wiley & Sons.

Kolb, R. W. & Overdahl, J. A. (2010). *Financial derivatives: Pricing and risk management.* John Wiley & Sons.

Krehbiel, T. & Adkins, L. C. (1993). Cointegration tests of the unbiased expectations hypothesis in metals markets. *The Journal of Futures Markets (1986-1998)*, *13*(7), 753.

Kroner, K. F. & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. *Journal of financial and quantitative analysis*, *28*(4), 535–551.

Ku, Y.-H. H., Chen, H.-C. & Chen, K.-h. (2007). On the application of the dynamic conditional correlation model in estimating optimal time-varying hedge ratios. *Applied Economics Letters*, *14*(7), 503–509.

Kwiatkowski, D., Phillips, P. C., Schmidt, P. & Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of econometrics*, *54*(1-3), 159–178.

Lai, K. S. & Lai, M. (1991). A cointegration test for market efficiency. *Journal of Futures Markets*, *11*(5), 567–575.

Landes, W. J., Stoffels, J. D. & Seifert, J. A. (1985). An empirical test of a duration-based hedge: The case of corporate bonds. *The Journal of Futures Markets (pre-1986)*, *5*(2), 173.

Lee, H.-T. (2009). A copula-based regime-switching garch model for optimal futures hedging. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *29*(10), 946–972.

Lee, H.-T. (2010). Regime switching correlation hedging. *Journal of Banking & Finance*, *34*(11), 2728–2741.

Lee, H.-T. & Yoder, J. K. (2007). A bivariate markov regime switching garch approach to estimate time varying minimum variance hedge ratios. *Applied Economics*, *39*(10), 1253–1265.

Leistikow, D. (1993). Impacts of shifts in uncertainty on spot and futures price change serial correlation and standardized covariation measures. *The Journal of Futures Markets (1986-1998)*, *13*(8), 873.

Li, M.-Y. L. (2010). Dynamic hedge ratio for stock index futures: Application of threshold vecm. *Applied Economics*, *42*(11), 1403–1417.

Lien, D. (1996). The effect of the cointegration relationship on futures hedging: A note. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *16*(7), 773–780.

Lien, D. (2009). A note on the hedging effectiveness of garch models. *International Review of Economics & Finance*, *18*(1), 110–112.

Lien, D. & Luo, X. (1994). Multiperiod hedging in the presence of conditional heteroskedasticity. *Journal of Futures Markets*, *14*(8), 927–955.

Lien, D. & Shrestha, K. (2008). Hedging effectiveness comparisons: A note. *International Review of Economics & Finance*, *17*(3), 391–396.

Lien, D. & Tse, Y. K. (1999). Fractional cointegration and futures hedging. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *19*(4), 457–474.

Lien, D., Tse, Y. K. & Tsui, A. K. (2002). Evaluating the hedging performance of the constant-correlation garch model. *Applied Financial Economics*, *12*(11), 791–798.

Lindahl, M. (1992). Minimum variance hedge ratios for stock index futures: Duration and expiration effects. *The Journal of Futures Markets (1986-1998)*, *12*(1), 33.

Ljung, G. M. & Box, G. E. (1978). On a measure of lack of fit in time series models. *Biometrika*, *65*(2), 297–303.

Lo, A. W. & MacKinlay, A. C. (1988). Stock market prices do not follow random walks: Evidence from a simple specification test. *The review of financial studies*, *1*(1), 41–66.

Lombana Betancourt, A. E. & Al Azzawi, L. (2013). *Estimation of Time-Varying Hedge Ratios for Coffee* (Master's thesis, Lund University, Sweden). 42 pages.

Lypny, G. & Powalla, M. (1998). The hedging effectiveness of dax futures. *The European Journal of Finance*, *4*(4), 345–355.

Malliaris, A. G. & Urrutia, J. (1991). Tests of random walk of hedge ratios and measures of hedging effectiveness for stock indexes and foreign currencies. *The Journal of Futures Markets (1986-1998)*, *11*(1), 55.

Markellos, R. N. & Psychoyios, D. (2013). Interest rate volatility and risk management: Evidence from cboe treasury options. *Available at SSRN 1585744*.

Moosa, I. (2003). The sensitivity of the optimal hedge ratio to model specification. *Finance Letters*, *1*(1), 15–20.

Myers, R. J. (1991). Estimating time-varying optimal hedge ratios on futures markets. *Journal of Futures Markets*, *11*(1), 39–53.

Myers, R. J. & Thompson, S. R. (1989). Generalized optimal hedge ratio estimation. *American Journal of Agricultural Economics*, *71*(4), 858–868.

Nguyen, P., Kim, J. H. & Henry, D. (2017). *Estimation of optimal hedge ratio: A wild bootstrap approach*. La Trobe University, Department of Economics and Finance.

Park, H. Y. & Bera, A. K. (1987). Interest-rate volatility, basis risk and heteroscedasticity in hedging mortgages. *Real Estate Economics*, *15*(2), 79–97.

Park, S. Y. & Jei, S. Y. (2010). Estimation and hedging effectiveness of time-varying hedge ratio: Flexible bivariate garch approaches. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *30*(1), 71–99.

Park, T. H. & Switzer, L. N. (1995a). Bivariate garch estimation of the optimal hedge ratios for stock index futures: A note. *Journal of futures markets*, *15*(1), 61–67.

Park, T. H. & Switzer, L. N. (1995b). Time-varying distributions and the optimal hedge ratios for stock index futures. *Applied Financial Economics*, *5*(3), 131–137.

Pepić, M. (2014). Managing interest rate risk with interest rate futures. *Ekonomika preduzeća*, *62*(3-4), 201–209.

Power, G. J., Vedenov, D. V., Anderson, D. P. & Klose, S. (2013). Market volatility and the dynamic hedging of multi-commodity price risk. *Applied Economics*, *45*(27), 3891–3903.

Sephton, P. S. (1993). Optimal hedge ratios at the winnipeg commodity exchange. *Canadian Journal of Economics*, 175–193.

Statista. (2019). Largest derivatives exchanges worldwide in 2018, by number of contracts traded. Retrieved August 9, 2019, from https://www.statista.com/statistics/272832/largest-international-futures-exchanges-by-number-of-contracts-traded/

Stein, J. L. (1961). The simultaneous determination of spot and futures prices. *The American Economic Review*, *51*(5), 1012–1025.

Su, Y.-K. & Wu, C.-C. (2014). A new range-based regime-switching dynamic conditional correlation model for minimum-variance hedging. *Journal of Mathematical Finance*, *4*(03), 207.

Taevs, A. L. & Jacob, D. P. (1986). Futures and alternative hedge ratio methodologies. *The Journal of Portfolio Management*, *12*(3), 60–70.

Thomas, S. & Brooks, R. (2001). Garch based hedge ratios for australian share price index futures: Does asymmetry matter. *Accounting, Accountability and Performance*, *7*(1), 61–76.

Van Der Heijden, M., Beetsma, R. & Romp, W. (2018). 'whatever it takes' and the role of eurozone news. *Applied Economics Letters*, *25*(16), 1166–1169.

Viswanath, P. (1993). Efficient use of information, convergence adjustments, and regression estimates of hedge ratios. *The Journal of Futures Markets (1986-1998)*, *13*(1), 43.

Wahab, M. & Lashgari, M. (1993). Price dynamics and error correction in stock index and stock index futures markets: A cointegration approach. *Journal of Futures Markets*, *13*(7), 711–742.

White, H. (2000). A reality check for data snooping. *Econometrica*, *68*(5), 1097–1126.

Wilkinson, K. J., Rose, L. C. & Young, M. R. (1999). Comparing the effectiveness of traditional and time varying hedge ratios using new zealand and australian debt futures contracts. *Financial Review*, *34*(3), 79–94.

Witt, H. J., Schroeder, T. C. & Hayenga, M. L. (1987). Comparison of analytical approaches for estimating hedge ratios for agricultural commodities. *Journal of Futures Markets*, *7*(2), 135–146.

Working, H. (1953). Futures trading and hedging. *The American Economic Review*, *43*(3), 314–343.

Yang, W. J. & Allen, D. E. (2005). Multivariate garch hedge ratios and hedging effectiveness in australian futures markets. *Accounting & Finance*, *45*(2), 301–321.

Ye, T. & Chen, Z. (2006). *The hedging effectiveness of currency futures* (Master's thesis, Faculty of Business Administration-Simon Fraser University). 40 pages.

Young, M., Hogan, W. & Batten, J. (2004). The effectiveness of interest-rate futures contracts for hedging japanese bonds of different credit quality and duration. *International Review of Financial Analysis*, *13*(1), 13–25.

Appendix A

Table A1: Autocorrelation tests

FGBL	OLS levels	OLS percentage changes
Period 1:	Box-Pierce Test	
	χ^2=3446.8***, df=6	χ^2=76.737***, df=6
	Ljung-Box Test	
	χ^2=3476.3***, df=6	χ^2=77.138***, df=6
Period 2:	Box-Pierce Test	
	χ^2= 3602.8***, df=6	χ^2= 106.94***, df=6
	Ljung-Box Test	
	χ^2= 3634.6***, df=6	χ^2= 107.62***, df=6

FGBM	OLS levels	OLS percentage changes
Period 1:	Box-Pierce Test	
	χ^2= 3472.4***, df=6	χ^2=256.78***, df=6
	Ljung-Box Test	
	χ^2=3502.1***, df=6	χ^2=258.19***, df=6
Period 2:	Box-Pierce Test	
	χ^2= 3378*** , df=6	χ^2=8.4098 , df=6
	Ljung-Box Test	
	χ^2=3407*** , df=6	χ^2=8.4626 , df=6

Note: Autocorrelation tests for the residuals of the OLS regressions $\ln S_t = \alpha + \beta \ln F_t + \epsilon_t$ and $\Delta \ln S_t = a + b\Delta \ln F_t + u_t$, are carried out for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures during 2 April 2007 to 4 September 2009 (Period 1, in the sample) and 9 December 2013 to 27 April 2016 (Period 2, in the sample). *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table A3: VAR model: Period 1

Dependent variable: diff.log.CTD1

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0002	0.0002	0.96	0.3350
diff.log.CTD1.-1	0.3940	0.0937	4.20	0.0000
diff.log.FGBL1.-1	-0.2800	0.0960	-2.92	0.0037
diff.log.CTD1.-2	-0.0548	0.0936	-0.59	0.5587
diff.log.FGBL1.-2	-0.0180	0.0956	-0.19	0.8507

Dependent variable: diff.log.FGBL1

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0002	0.67	0.5030
diff.log.CTD1.-1	0.1167	0.0915	1.28	0.2025
diff.log.FGBL1.-1	-0.0469	0.0937	-0.50	0.6170
diff.log.CTD1.-2	-0.1370	0.0913	-1.50	0.1340
diff.log.FGBL1.-2	0.0816	0.0933	0.87	0.3822

Dependent variable: diff.log.CTD1

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	0.78	0.4337
diff.log.CTD1.-1	0.5746	0.0734	7.82	0.0000
diff.log.FGBM1.-1	-0.5362	0.0811	-6.61	0.0000
diff.log.CTD1.-2	0.1578	0.0789	2.00	0.0460
diff.log.FGBM1.-2	-0.1132	0.0852	-1.33	0.1847
diff.log.CTD1.-3	-0.1029	0.0735	-1.40	0.1622
diff.log.FGBM1.-3	0.0708	0.0818	0.87	0.3868

Dependent variable: diff.log.FGBM1

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	1.22	0.2234
diff.log.CTD1.-1	0.1418	0.0665	2.13	0.0334
diff.log.FGBM1.-1	-0.1335	0.0735	-1.82	0.0698
diff.log.CTD1.-2	-0.0641	0.0715	-0.90	0.3700
diff.log.FGBM1.-2	0.0439	0.0772	0.57	0.5700
diff.log.CTD1.-3	0.0422	0.0666	0.63	0.5268
diff.log.FGBM1.-3	-0.0670	0.0741	-0.90	0.3658

Note: VAR model is estimated according to the equation 4.6 for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures and the corresponding CTD during 2 April 2007 to 4 September 2009 (Period 1, in the sample). diff.log- indicates a differenced logarithm of a variable while -1, -2, and -3 denote the first, the second, and the third lag of a respective variable.

Table A5: VAR model: Period 2

Dependent variable: diff.log.CTD2				
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0002	0.0001	1.08	0.2792
diff.log.CTD2.-1	0.2893	0.0635	4.56	0.0000
diff.log.FGBL2.-1	-0.3033	0.0637	-4.76	0.0000
diff.log.CTD2.-2	0.0956	0.0633	1.51	0.1315
diff.log.FGBL2.-2	-0.1514	0.0650	-2.33	0.0203
Dependent variable: diff.log.FGBL2				
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0002	0.0001	1.76	0.0794
diff.log.CTD2.-1	0.1152	0.0630	1.83	0.0677
diff.log.FGBL2.-1	-0.0927	0.0632	-1.47	0.1430
diff.log.CTD2.-2	-0.1601	0.0628	-2.55	0.0110
diff.log.FGBL2.-2	0.0349	0.0645	0.54	0.5882

Dependent variable: diff.log.CTD2				
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	-0.0000	0.0001	-0.34	0.7377
diff.log.CTD2.-1	-0.0887	0.0523	-1.69	0.0907
diff.log.FGBM2.-1	-0.0073	0.0355	-0.20	0.8378
diff.log.CTD2.-2	-0.0516	0.0521	-0.99	0.3226
diff.log.FGBM2.-2	0.0011	0.0354	0.03	0.9750
Dependent variable: diff.log.FGBM2				
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	0.82	0.4100
diff.log.CTD2.-1	-0.0314	0.0764	-0.41	0.6809
diff.log.FGBM2.-1	0.0215	0.0518	0.42	0.6777
diff.log.CTD2.-2	-0.2760	0.0761	-3.63	0.0003
diff.log.FGBM2.-2	0.0855	0.0517	1.65	0.0985

Note: VAR model is estimated according to the equation 4.6 for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures and the corresponding CTD during 9 December 2013 to 27 April 2016 (Period 2, in the sample). diff.log- indicates a differenced logarithm of a variable while -1 and -2 denote the first and the second lag of a respective variable, respectively.

Table A7: VECM model: Period 1

	Dependent variable: diff.log.CTD1			
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0002	0.63	0.5270
ECT	-0.0387	0.0029	-13.41	0.0000
diff.log.CTD1.-1	-0.3513	0.0842	-4.17	0.0000
diff.log.FGBL1.-1	0.4257	0.0970	4.39	0.0000
diff.log.CTD1.-2	-0.2344	0.0842	-2.78	0.0055
diff.log.FGBL1.-2	0.2356	0.0878	2.68	0.0075
	Dependent variable: diff.log.FGBL1			
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0002	0.70	0.4817
ECT	-0.0405	0.0028	-14.66	0.0000
diff.log.CTD1.-1	0.1127	0.0806	1.40	0.1627
diff.log.FGBL1.-1	-0.0531	0.0928	-0.57	0.5673
diff.log.CTD1.-2	-0.0833	0.0806	-1.03	0.3014
diff.log.FGBL1.-2	0.0837	0.0840	1.00	0.3194

	Dependent variable: diff.log.CTD1			
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	0.72	0.4716
ECT	0.2833	0.0403	7.04	0.0000
diff.log.CTD1.-1	-0.4740	0.0762	-6.22	0.0000
diff.log.FGBM1.-1	0.2028	0.1198	1.69	0.0909
diff.log.CTD1.-2	-0.2432	0.0784	-3.10	0.0020
diff.log.FGBM1.-2	0.0870	0.1047	0.83	0.4064
diff.log.CTD1.-3	-0.2631	0.0726	-3.62	0.0003
diff.log.FGBM1.-3	0.1284	0.0847	1.52	0.1299
	Dependent variable: diff.log.FGBM1			
	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	1.20	0.2287
ECT	0.4161	0.0351	11.87	0.0000
diff.log.CTD1.-1	-0.1692	0.0663	-2.55	0.0110
diff.log.FGBM1.-1	0.1082	0.1043	1.04	0.3001
diff.log.CTD1.-2	-0.2103	0.0683	-3.08	0.0022
diff.log.FGBM1.-2	0.1589	0.0912	1.74	0.0821
diff.log.CTD1.-3	-0.1500	0.0633	-2.37	0.0180
diff.log.FGBM1.-3	0.0933	0.0738	1.26	0.2064

Note: VECM model is estimated according to the equation 4.15 for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures and the corresponding CTD during 2 April 2007 to 4 September 2009 (Period 1, in the sample). diff.log- indicates a differenced logarithm of a variable while -1, -2, and -3 denote the first, the second, and the third lag of a respective variable.

Table A9: VECM model: Period 2

Dependent variable: diff.log.CTD2

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0002	0.0001	1.49	0.1376
ECT	-0.1859	0.0137	-13.61	0.0000
diff.log.CTD2.-1	-0.4006	0.0599	-6.69	0.0000
diff.log.FGBL2.-1	0.4419	0.0732	6.04	0.0000
diff.log.CTD2.-2	-0.1797	0.0600	-3.00	0.0028
diff.log.FGBL2.-2	0.1762	0.0629	2.80	0.0052

Dependent variable: diff.log.FGBL2

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0002	0.0001	1.66	0.0964
ECT	-0.2033	0.0133	-15.23	0.0000
diff.log.CTD2.-1	0.2571	0.0586	4.39	0.0000
diff.log.FGBL2.-1	-0.1000	0.0715	-1.40	0.1627
diff.log.CTD2.-2	0.0305	0.0586	0.52	0.6029
diff.log.FGBL2.-2	-0.0085	0.0615	-0.14	0.8898

Dependent variable: diff.log.CTD2

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0000	0.0001	0.21	0.8332
ECT	-0.7789	0.0548	-14.22	0.0000
diff.log.CTD2.-1	-0.1863	0.0583	-3.20	0.0015
diff.log.FGBM2.-1	0.1768	0.0330	5.36	0.0000
diff.log.CTD2.-2	-0.0999	0.0480	-2.08	0.0378
diff.log.FGBM2.-2	0.0803	0.0312	2.58	0.0102

Dependent variable: diff.log.FGBM2

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0000	0.0001	0.19	0.8475
ECT	-1.0829	0.0808	-13.41	0.0000
diff.log.CTD2.-1	0.8349	0.0859	9.72	0.0000
diff.log.FGBM2.-1	-0.4424	0.0487	-9.09	0.0000
diff.log.CTD2.-2	0.2959	0.0708	4.18	0.0000
diff.log.FGBM2.-2	-0.1738	0.0460	-3.78	0.0002

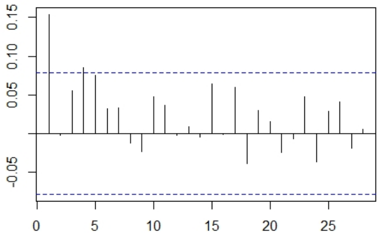
Note: VECM model is estimated according to the equation 4.15 for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures and the corresponding CTD during 9 December 2013 to 27 April 2016 (Period 2, in the sample). diff.log- indicates a differenced logarithm of a variable while -1 and -2 denote the first and the second lag of a respective variable.

Table A11: Autocorrelation and heteroskedasticity tests

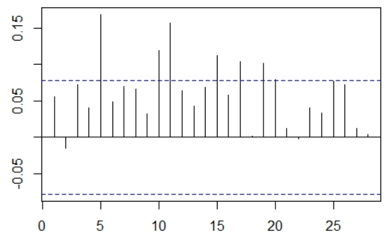
FGBL	diff.log.CTD	diff.log.FGBL
Period 1:	Box-Pierce Test	
	χ^2=25.494***, df=6	χ^2=25.946***, df=6
	Ljung-Box Test	
	χ^2=25.672***, df=6	χ^2=26.213***, df=6
	ARCH-Lagrange Multiplier Test	
	χ^2=7.3784, df=6	χ^2=24.182***, df=6
Period 2:	Box-Pierce Test	
	χ^2=39.036***, df=6	χ^2= 6.5466, df=6
	Ljung-Box Test	
	χ^2= 39.411***, df=6	χ^2= 6.6056, df=6
	ARCH-Lagrange Multiplier Test	
	χ^2=12.874**, df=6	χ^2=0.15835, df=6

FGBM	diff.log.CTD	diff.log.FGBM
Period 1:	Box-Pierce Test	
	χ^2=8.6633, df=6	χ^2=41.325***, df=6
	Ljung-Box Test	
	χ^2=8.7299, df=6	χ^2=41.777***, df=6
	ARCH-Lagrange Multiplier Test	
	χ^2=0.10629, df=6	χ^2=34.109***, df=6
Period 2:	Box-Pierce Test	
	χ^2= 55.73*** , df=6	χ^2= 0.69947, df=6
	Ljung-Box Test	
	χ^2= 56.097***, df=6	χ^2= 0.70615, df=6
	ARCH-Lagrange Multiplier Test	
	χ^2=146.6***, df=6	χ^2=0.23005, df=6

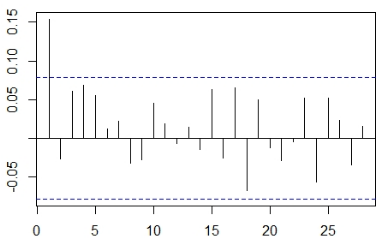
Note: Autocorrelation and ARCH-LM tests for the residuals of the two equations of the VAR model are carried out for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures during 2 April 2007 to 4 September 2009 (Period 1, in the sample) and 9 December 2013 to 27 April 2016 (Period 2, in the sample). *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.



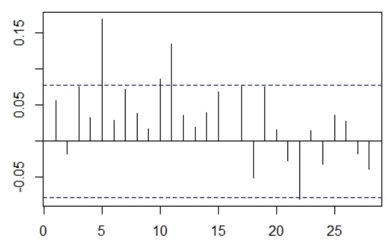
(a) ACF - diff.log.CTD1



(b) ACF - diff.log.FGBL1

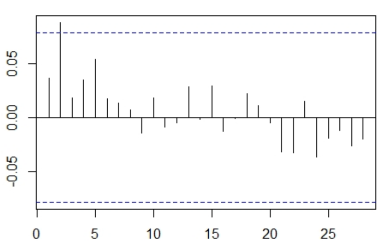


(c) PACF - diff.log.CTD1

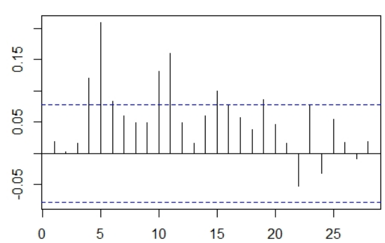


(d) PACF - diff.log.FGBL1

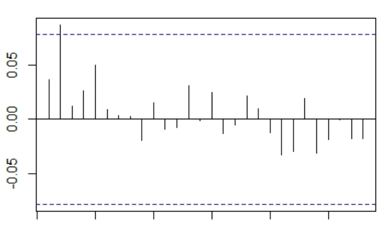
Figure A1: FGBL ACF and PACF of squared residuals: Period 1



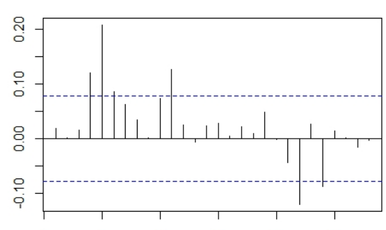
(a) ACF - diff.log.CTD1



(b) ACF - diff.log.FGBM1

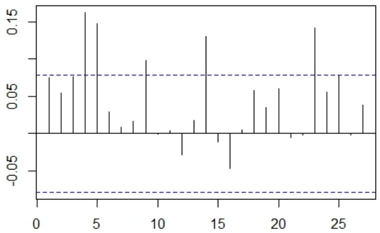


(c) PACF - diff.log.CTD1

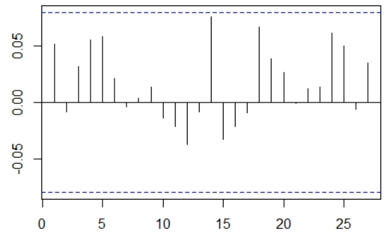


(d) PACF - diff.log.FGBM1

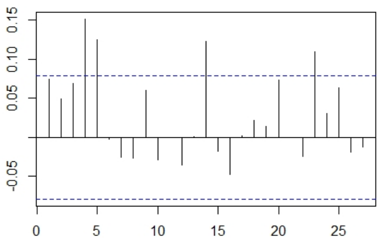
Figure A2: FGBM ACF and PACF of squared residuals: Period 1



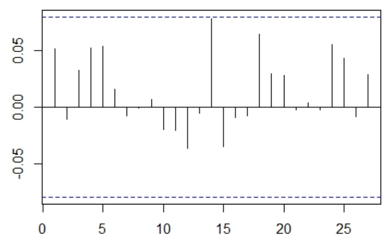
(a) ACF - diff.log.CTD2



(b) ACF - diff.log.FGBL2

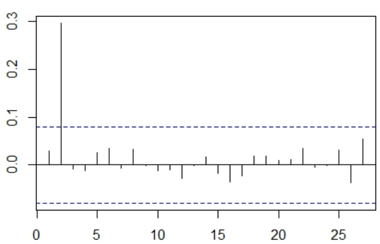


(c) PACF - diff.log.CTD2

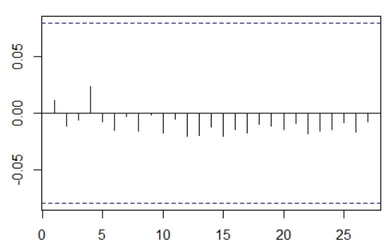


(d) PACF - diff.log.FGBL2

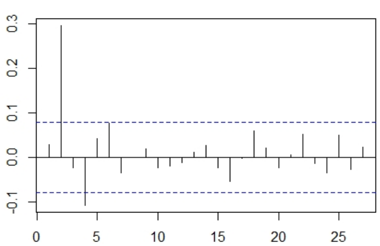
Figure A3: FGBL ACF and PACF of squared residuals: Period 2



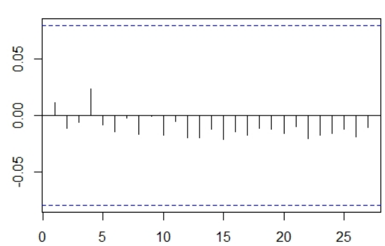
(a) ACF - diff.log.CTD2



(b) ACF - diff.log.FGBM2



(c) PACF - diff.log.CTD2



(d) PACF - diff.log.FGBM2

Figure A4: FGBM ACF and PACF of squared residuals: Period 2

Table A13: GARCH model: Period 1

	Estimate	Std. Error	t value	Pr(>|t|)
diff.log.CTD1				
omega	0.0000	0.0000	0.48	0.6343
alpha	0.0777	0.0319	2.44	0.0148
beta	0.8875	0.0373	23.80	0.0000
diff.log.FGBL1				
omega	0.0000	0.0000	0.22	0.8266
alpha	0.0414	0.0104	3.98	0.0000
beta	0.9518	0.0114	83.18	0.0000
Joint.C1	0.9900	0.0268	36.91	0.0000

	Estimate	Std. Error	t value	Pr(>|t|)
diff.log.CTD1				
omega	0.0000	0.0000	0.03	0.9776
alpha	0.0186	0.0260	0.71	0.4757
beta	0.9804	0.0124	79.33	0.0000
diff.log.FGBM1				
omega	0.0000	0.0000	0.13	0.9004
alpha	0.0581	0.0144	4.02	0.0000
beta	0.9366	0.0146	64.04	0.0000
Joint.C1	0.9900	0.0779	12.71	0.0000

Note: The GARCH residual structure of the VAR model is estimated according to the equation 4.17 for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures and the corresponding CTD during 2 April 2007 to 4 September 2009 (Period 1, in the sample). Omega, alpha, and beta denote a constant, ARCH effect, and GARCH effect, respectively. Joint.C1 indicates joint significance of the variables.

Table A15: GARCH model: Period 2

	Estimate	Std. Error	t value	Pr(>|t|)
diff.log.CTD2				
omega	0.0000	0.0000	0.59	0.5585
alpha	0.1543	0.0586	2.63	0.0084
beta	0.7550	0.0573	13.17	0.0000
diff.log.FGBL2				
omega	0.0000	0.0000	0.05	0.9615
alpha	0.0167	0.0124	1.34	0.1798
beta	0.9823	0.0078	125.22	0.0000
Joint.C1	0.9900	0.1106	8.95	0.0000

	Estimate	Std. Error	t value	Pr(>|t|)
diff.log.CTD2				
omega	0.0000	0.0000	0.00	0.9974
alpha	0.0251	0.1806	0.14	0.8895
beta	0.9632	0.1252	7.69	0.0000
diff.log.FGBM2				
omega	0.0000	0.0000	0.07	0.9432
alpha	0.0013	0.0005	2.72	0.0066
beta	0.9902	0.0012	853.00	0.0000
Joint.C1	0.9900	0.1996	4.96	0.0000

Note: The GARCH residual structure of the VAR model is estimated according to the equation 4.17 for Euro-Bund (FGBL) and Euro-Bobl (FGBM) futures and the corresponding CTD during 9 December 2013 to 27 April 2016 (Period 2, in the sample). Omega, alpha, and beta denote a constant, ARCH effect, and GARCH effect, respectively. Joint.C1 indicates joint significance of the variables.

Appendix B

Table B1: Robustness: ADF and KPSS tests

		ADF	KPSS
lnFGBL	level	-2.9469	14.976***
	Δ	-11.736***	0.0892
lnCTD	level	-1.1691	13.68***
	Δ	-11.022***	0.1761
$\hat{\epsilon}_t$	level	-0.7968	2.624***
lnFGBM	level	-2.5107	14.533***
	Δ	-11.532***	0.0454
lnCTD	level	-2.2348	9.322***
	Δ	-10.418***	0.0803
$\hat{\epsilon}_t$	level	-2.7284	1.7323***

Note: Tests are conducted for the joint dataset of Period 1 and Period 2. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively. $\hat{\epsilon}_t$ are residuals from the OLS equation $\ln S_t = \alpha + \beta \ln F_t + \epsilon_t$.

Table B2: Robustness: OLS levels

	Dependent variable:
	log(CTD$_{\text{FGBL}}$)
Constant	0.3548*
	(0.1862)
log(FGBL)	0.8973***
	(0.0385)
Observations	1,288
R^2	0.799
Adjusted R^2	0.799
Residual Std. Error	0.037 (df = 1286)
F Statistic	5,103.980*** (df = 1; 1286)

	Dependent variable:
	log(CTD$_{\text{FGBM}}$)
Constant	1.3275***
	(0.5734)
log(FGBM)	0.7027***
	(0.1214)
Observations	1,288
R^2	0.735
Adjusted R^2	0.734
Residual Std. Error	0.019 (df = 1286)
F Statistic	3,561.426*** (df = 1; 1286)

Note: The results are obtained from the OLS estimation of $\ln S_t = \alpha + \beta \ln F_t + \epsilon_t$ for the price series from the joint dataset. Figures in parentheses are HAC standard errors.*, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table B4: Robustness: OLS percentage changes

	Dependent variable: diff(log(CTD_{FGBL}))
Constant	0.0000
	(0.0000)
diff(log(FGBL))	0.7794***
	(0.0360)
Observations	1,287
R^2	0.698
Adjusted R^2	0.697
Residual Std. Error	0.002 (df = 1285)
F Statistic	2,963.355*** (df = 1; 1285)

	Dependent variable: diff(log(CTD_{FGBM}))
Constant	0.0000
	(0.0001)
diff(log(FGBM))	0.7538***
	(0.0579)
Observations	1,287
R^2	0.548
Adjusted R^2	0.548
Residual Std. Error	0.002 (df = 1285)
F Statistic	1,560.495*** (df = 1; 1285)

Note: The results are obtained from the OLS estimation of $\Delta \ln S_t = a + b\Delta \ln F_t + u_t$ for the price series from the joint dataset. Figures in parentheses are HAC standard errors.*, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table B5: Robustness: Autocorrelation tests

	OLS levels	OLS percentage changes
FGBL	Box-Pierce Test	
	χ^2= 8651.1***, df=7	χ^2= 156.23***, df=7
	Ljung-Box Test	
	χ^2= 8691.2***, df=7	χ^2= 156.65***, df=7
FGBM	Box-Pierce Test	
	χ^2= 8365***, df=7	χ^2=281.02***, df=7
	Ljung-Box Test	
	χ^2= 8403.5***, df=7	χ^2=281.79***, df=7

Note: Autocorrelation tests for the residuals of the OLS regressions $\ln S_t = \alpha + \beta \ln F_t + \epsilon_t$ and $\Delta \ln S_t = a + b\Delta \ln F_t + u_t$, are carried out for the price series from the joint dataset. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table B6: Robustness: VAR model

Dependent variable: diff.log.CTD

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	1.36	0.1740
diff.log.CTD.-1	0.3526	0.0516	6.83	0.0000
diff.log.FGBL.-1	-0.2606	0.0473	-5.51	0.0000
diff.log.CTD.-2	0.0233	0.0517	0.45	0.6524
diff.log.FGBL.-2	-0.0855	0.0477	-1.79	0.0734

Dependent variable: diff.log.FGBL

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0002	0.0001	1.60	0.1099
diff.log.CTD.-1	0.1528	0.0562	2.72	0.0066
diff.log.FGBL.-1	-0.0860	0.0515	-1.67	0.0950
diff.log.CTD.-2	-0.1501	0.0563	-2.67	0.0077
diff.log.FGBL.-2	0.0437	0.0520	0.84	0.4007

Dependent variable: diff.log.CTD

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0000	0.0001	0.64	0.5223
diff.log.CTD.-1	0.3607	0.0424	8.50	0.0000
diff.log.FGBM.-1	-0.2831	0.0416	-6.81	0.0000
diff.log.CTD.-2	0.1420	0.0425	3.34	0.0009
diff.log.FGBM.-2	-0.0899	0.0421	-2.13	0.0331

Dependent variable: diff.log.FGBM

	Estimate	Std. Error	t value	Pr(>|t|)
Constant	0.0001	0.0001	1.36	0.1751
diff.log.CTD.-1	0.1067	0.0434	2.46	0.0142
diff.log.FGBM.-1	-0.0766	0.0426	-1.80	0.0723
diff.log.CTD.-2	-0.0676	0.0435	-1.55	0.1204
diff.log.FGBM.-2	0.0242	0.0431	0.56	0.5739

Note: VAR model is estimated according to the equation 4.6 for the price series from the joint dataset. diff.log- indicates a differenced logarithm of a variable while -1 and -2 denote the first and the second lag of a respective variable.

Table B8: Robustness: VECM model

	Estimate	Std. Error	t value	Pr(>|t|)
Dependent variable: diff.log.CTD				
Constant	0.0002	0.0001	1.45	0.1478
ECT	-0.1258	0.0065	-19.41	0.0000
diff.log.CTD.-1	-0.3599	0.0471	-7.64	0.0000
diff.log.FGBL.-1	0.3834	0.0506	7.57	0.0000
diff.log.CTD.-2	-0.1906	0.0473	-4.03	0.0001
diff.log.FGBL.-2	0.1711	0.0448	3.82	0.0001
Dependent variable: diff.log.FGBL				
Constant	0.0002	0.0001	1.60	0.1109
ECT	-0.1485	0.0069	-21.39	0.0000
diff.log.CTD.-1	0.2303	0.0504	4.57	0.0000
diff.log.FGBL.-1	-0.1056	0.0542	-1.95	0.0516
diff.log.CTD.-2	0.0092	0.0507	0.18	0.8564
diff.log.FGBL.-2	-0.0038	0.0479	-0.08	0.9375

	Estimate	Std. Error	t value	Pr(>|t|)
Dependent variable: diff.log.CTD				
Constant	0.0001	0.0001	0.95	0.3406
ECT	0.1002	0.0072	13.95	0.0000
diff.log.CTD.-1	-0.5194	0.0415	-12.51	0.0000
diff.log.FGBM.-1	0.3399	0.0530	6.41	0.0000
diff.log.CTD.-2	-0.1729	0.0413	-4.18	0.0000
diff.log.FGBM.-2	0.1135	0.0427	2.66	0.0079
Dependent variable: diff.log.FGBM				
Constant	0.0001	0.0001	1.45	0.1472
ECT	0.1484	0.0070	21.19	0.0000
diff.log.CTD.-1	-0.0101	0.0405	-0.25	0.8028
diff.log.FGBM.-1	0.0444	0.0517	0.86	0.3908
diff.log.CTD.-2	-0.0528	0.0403	-1.31	0.1902
diff.log.FGBM.-2	0.0516	0.0416	1.24	0.2152

Note: VECM model is estimated according to the equation 4.15 for the price series from the joint dataset. diff.log- indicates a differenced logarithm of a variable while -1 and -2 denote the first and the second lag of a respective variable, respectively.

Table B10: Robustness: VAR & VECM variance-covariance matrices

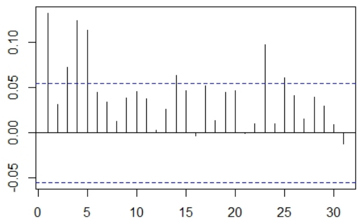
	VAR		VECM	
FGBL	σ_{ss}	σ_{ff}	σ_{ss}	σ_{ff}
σ_{ss}	1.3480e-05	1.2382e-05	1.3941e-05	1.2151e-05
σ_{ss}	1.2382e-05	1.5961e-05	1.2151e-05	1.5981e-05
σ_{sf}/σ_{ff}	0.7758		0.7604	
FGBM	σ_{ss}	σ_{ff}	σ_{ss}	σ_{ff}
σ_{ss}	5.5937e-06	4.3745e-06	6.1655e-06	4.4592e-06
σ_{ss}	4.3745e-06	5.8599e-06	4.4592e-06	5.8581e-06
σ_{sf}/σ_{ff}	0.7465		0.7612	

Note: σ_{ff} and σ_{ss} denote the variance of ϵ_{ft} and ϵ_{st}, respectively, and σ_{sf} denotes the covariance of ϵ_{st} and ϵ_{ft} from the VAR equation 4.6 and the VECM equation 4.15 for FGBL and the corresponding CTD from the joint dataset.

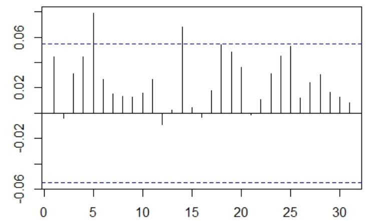
Table B11: Robustness: Autocorrelation and heteroskedasticity tests

	diff.log.CTD	diff.log.FGBL
FGBL	Box-Pierce Test	
	χ^2= 70.233***, df=7	χ^2= 15.438**, df=7
	Ljung-Box Test	
	χ^2= 70.523***, df=7	χ^2= 15.511**, df=7
	ARCH-Lagrange Multiplier Test	
	χ^2=8.3288, df=7	χ^2=0.30634, df=7
FGBM	Box-Pierce Test	
	χ^2= 75.197***, df=7	χ^2= 7.4365, df=7
	Ljung-Box Test	
	χ^2= 75.468***, df=7	χ^2= 7.473, df=7
	ARCH-Lagrange Multiplier Test	
	χ^2= 0.79713, df=7	χ^2= 0.27187, df=7

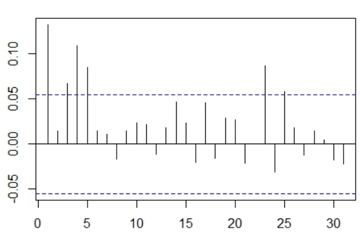
Note: Autocorrelation and ARCH-LM tests for the residuals of the two equations of the VAR model are carried out for the price series from the joint dataset. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.



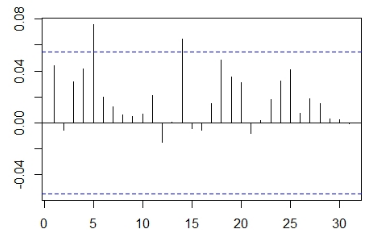
(a) ACF - diff.log.CTD



(b) ACF - diff.log.FGBL

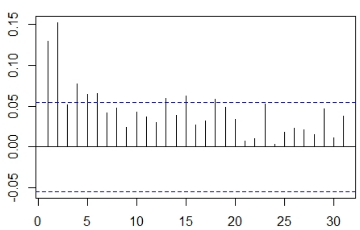


(c) PACF - diff.log.CTD

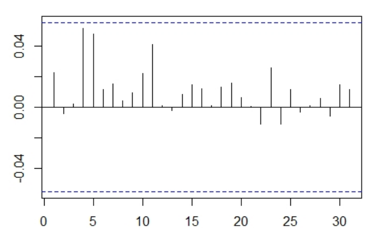


(d) PACF - diff.log.FGBL

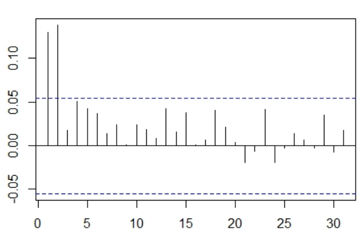
Figure B1: Robustness: FGBL ACF and PACF of squared residuals



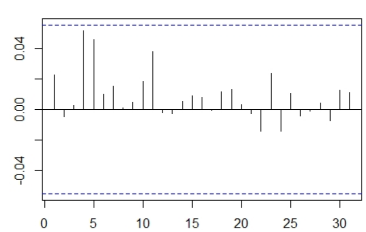
(a) ACF - diff.log.CTD



(b) ACF - diff.log.FGBM



(c) PACF - diff.log.CTD



(d) PACF - diff.log.FGBM

Figure B2: Robustness: FGBM ACF and PACF of squared residuals

Table B12: Robustness: GARCH model

		diff.log.CTD		
	Estimate	Std. Error	t value	Pr(>|t|)
omega	0.0000	0.0000	0.51	0.6131
alpha	0.1112	0.0367	3.04	0.0024
beta	0.8530	0.0383	22.28	0.0000
		diff.log.FGBL		
	Estimate	Std. Error	t value	Pr(>|t|)
omega	0.0000	0.0000	0.12	0.9013
alpha	0.0287	0.0173	1.65	0.0986
beta	0.9624	0.0156	61.70	0.0000
Joint.C1	0.9900	0.0477	20.76	0.0000

		diff.log.CTD		
	Estimate	Std. Error	t value	Pr(>|t|)
omega	0.0000	0.0000	0.03	0.9800
alpha	0.0211	0.0026	8.22	0.0000
beta	0.9779	0.0005	1901.5	0.0000
		diff.log.FGBM		
	Estimate	Std. Error	t value	Pr(>|t|)
omega	0.0000	0.0000	0.00	0.9999
alpha	0.0172	0.0021	7.94	0.0000
beta	0.9817	0.0007	1359.7	0.0000
Joint.C1	0.9900	0.0569	17.4	0.0000

Note: The GARCH residual structure of the VAR model is estimated according to the equation 4.17 for the price series from the joint dataset. Omega, alpha, and beta denote a constant, ARCH effect, and GARCH effect, respectively. Joint.C1 indicates joint significance of the variables.