

Report on
*The tree property and the continuum
function,*
the PhD Thesis of Šárka Stejskalová

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Overview

König's Lemma states that every finitely branching countably infinite tree has a countably infinite branch. This is a fundamental result, and it is natural to ask whether it may be generalised to higher cardinals. We thus say that a cardinal κ has the *tree property* if every tree of height κ with levels of size strictly less than κ (that is, every κ -tree) has a branch of length κ . Aronszajn showed that the tree property fails for $\kappa = \aleph_1$, but the question of whether it can hold at higher cardinals has a rich history, and much remains unknown.

Related to the tree property is the *weak tree property*. A κ -tree with no κ -branch, witnessing the failure of the tree property at κ , is called an *Aronszajn tree*. If $\kappa = \mu^+$ for some μ , a κ -Aronszajn tree that can be decomposed as the union of μ many antichains is called a *special κ -Aronszajn tree*. The statement that there is a special κ -Aronszajn tree is strictly stronger than the statement that there is a κ -Aronszajn tree; dually, the weak tree property states that there is no special κ -Aronszajn tree.

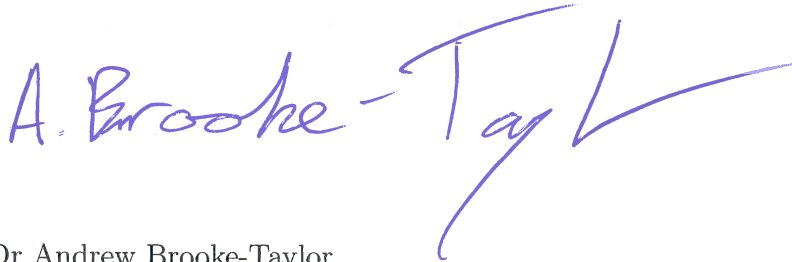
For the tree property to hold at a double successor cardinal κ^{++} , we must have $2^\kappa \geq \kappa^{++}$. It is thus natural to ask whether the tree property or weak tree property imposes any other constraints on the continuum function. This thesis adds considerable weight to the possibility that the answer to this question is “no.” For many of the configurations of the tree property or weak

tree property that are known to be consistent (relative to large cardinal assumptions), models are constructed with given the configurations, such that moreover the pattern of the continuum function is essentially arbitrary in the corresponding range of cardinals. This is an important step for our understanding of the tree property.

The new models that are exhibited in the thesis are generally built by modifying constructions from the literature for the specified tree property configurations, showing that the corresponding arguments for the tree property and weak tree property can be carried out for these modified constructions, while they simultaneously allowing the continuum function to be set to be as desired. The existing arguments that are built upon are already quite involved, so the arguments in the thesis extending them are highly detailed, complex constructions.

Conclusions

This work meets the standard requirements of a doctoral thesis. I recommend the thesis for public defence, and I propose giving it the grade of “pass.”



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