

Matlab Code for an AR(2) Process

```
clear all;

F = [0.6 0.35; 1 0];
Q = [1 0; 0 0];
h = sym('h%d%d', [2 1]);
h(1,1) = 1;
X = sym('x%d%d', [2 2]);
k = 1;

U = X - F*(X - ((1 - 2^(-2*k))/(h.*X*h))*X*(h*h.))*F.' - Q;

E = zeros(2, 2);
E11 = E;
E11(1,1) = 1;
E21 = E;
E21(2, 1) = 1;
E12 = E;
E12(1, 2) = 1;
E22 = E;
E22(2, 2) = 1;
E1 = E11(:,1);
E2 = E21(:,1);

Z11 = E11 - F*E11*F.' - ((1 - 2^(-
2*k))/(h.*X*h))*h.*E11*h*F*X*(h*h.)*X*F.' + ((1 - 2^(-
2*k))/(h.*X*h))*F*(E11*(h*h.)*X + X*(h*h.)*E11)*F.';
Z21 = E21 - F*E21*F.' - ((1 - 2^(-
2*k))/(h.*X*h))*h.*E21*h*F*X*(h*h.)*X*F.' + ((1 - 2^(-
2*k))/(h.*X*h))*F*(E21*(h*h.)*X + X*(h*h.)*E21)*F.';
Z12 = E12 - F*E12*F.' - ((1 - 2^(-
2*k))/(h.*X*h))*h.*E12*h*F*X*(h*h.)*X*F.' + ((1 - 2^(-
2*k))/(h.*X*h))*F*(E12*(h*h.)*X + X*(h*h.)*E12)*F.';
Z22 = E22 - F*E22*F.' - ((1 - 2^(-
2*k))/(h.*X*h))*h.*E22*h*F*X*(h*h.)*X*F.' + ((1 - 2^(-
2*k))/(h.*X*h))*F*(E22*(h*h.)*X + X*(h*h.)*E22)*F.';

Z1 = - ((1 - 2^(-2*k))/(h.*X*h)^2)*(E1.*X*h + h.*X*E1)*F*X*(h*h.)*X*F.'
+ ((1 - 2^(-2*k))/(h.*X*h))*F*(X*E1*h.*X + X*h*E1.*X)*F.';
Z2 = - ((1 - 2^(-2*k))/(h.*X*h)^2)*(E2.*X*h + h.*X*E2)*F*X*(h*h.)*X*F.'
+ ((1 - 2^(-2*k))/(h.*X*h))*F*(X*E2*h.*X + X*h*E2.*X)*F.';

Xh2 = sym('xh2%d%d', [2 2]);
Xh2(2, 2) = 0;

M2 = Z11*Xh2(1,1) + Z12*Xh2(1,2) + Z21*Xh2(2,1) + Z22*Xh2(2,2) + Z2;

W = [reshape(U, [4,1]);reshape(M2, [4,1])];

W(1);
W(2);
W(3);
W(4);
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W(5);
W(6);
W(7);
W(8);

values = [0.1;1;0.1;1;1;1;1;1];

options = optimoptions('fsolve');
options.MaxIterations = 10000;
options.MaxFunctionEvaluations = 20000;

sol = fsolve(@feqnAR2v2, values, options);

sol(1)
sol(2)
sol(3)
sol(4)
sol(5)

o2 = ([1 sol(1)]*[sol(2) sol(3); sol(4) sol(5)]*[1; sol(1)])/(2^(2*k)-1)

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function F=feqnAR2v2(z)
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h21=z(1);
x11=z(2);
x12=z(3);
x21=z(4);
x22=z(5);
xh211=z(6);
xh212=z(7);
xh221=z(8);

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F(1)=(16*x11)/25 - (21*x12)/100 - (21*x21)/100 - (49*x22)/400 +
(9*x21*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/25 +
(21*x22*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/100 +
(21*x21*((3*h21*x21)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21^2*x22)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/100 +
(49*x22*((3*h21*x21)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21^2*x22)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/400 +
(9*x11*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/25 +
(21*x12*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/100 +
(21*x11*((3*x21)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x22)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/100 +
(49*x12*((3*x21)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x22)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/400 - 1;
F(2)=x21 - (7*x12)/20 - (3*x11)/5 + (3*x21*((3*h21*x11)/(4*(x11 + h21*x21 +
h21*(x12 + h21*x22))) + (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 +
h21*x22)))))/5 + (7*x22*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 +
h21*x22))) + (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/20 +
(3*x11*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/5 +
(7*x12*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/20;

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$$\begin{aligned}
& h21*x21 + h21*(x12 + h21*x22))^2) + (21*x21*(x12 + x21 + \\
& 2*h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)))/20 - \\
& xh211*((3*x11*((9*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) + h21*((9*x12)/(20*(x11 \\
& + h21*x21 + h21*(x12 + h21*x22))) + (21*x22)/(80*(x11 + h21*x21 + h21*(x12 \\
& + h21*x22))))))/5 + (7*x12*((9*x11)/(20*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))) + (21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& h21*((9*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + (21*x22)/(80*(x11 \\
& + h21*x21 + h21*(x12 + h21*x22))))))/20 - (27*(2*x11 + h21*x12 + \\
& h21*x21))/(100*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (3*x21*(h21*((9*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))))) + \\
& h21^2*((9*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/5 + \\
& (7*x22*(h21*((9*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))))) + \\
& h21^2*((9*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/20 - (63*(x12 + \\
& h21*x22))/(400*(x11 + h21*x21 + h21*(x12 + h21*x22))) - (63*(x21 + \\
& h21*x22))/(400*(x11 + h21*x21 + h21*(x12 + h21*x22))) - 16/25 + (63*(x12^2 \\
& + x22*(x11 + h21*x12) + h21*x12*x22))/(400*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))) - (3*x21*(h21*((9*x11*(x12 + x21 + 2*h21*x22))/(20*(x11 + \\
& h21*x21 + h21*(x12 + h21*x22))^2) + (21*x21*(x12 + x21 + \\
& 2*h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)) + \\
& h21^2*((9*x12*(x12 + x21 + 2*h21*x22))/(20*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))^2) + (21*x22*(x12 + x21 + 2*h21*x22))/(80*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))^2)))/5 - (7*x22*(h21*((9*x11*(x12 + x21 + \\
& 2*h21*x22))/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))^2) + (21*x21*(x12 + \\
& x21 + 2*h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)) + \\
& h21^2*((9*x12*(x12 + x21 + 2*h21*x22))/(20*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))^2) + (21*x22*(x12 + x21 + 2*h21*x22))/(80*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))^2)))/20 + (147*(x12*x22 + h21*x22^2 + x22*(x21 + \\
& h21*x22))/(1600*(x11 + h21*x21 + h21*(x12 + h21*x22))); \\
& F(6)=(9*(x11*x12 + x21*(x11 + h21*x12) + h21*x12*x21))/(20*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) - (3*x11*((3*x11*(x12 + x21 + 2*h21*x22))/(4*(x11 + \\
& h21*x21 + h21*(x12 + h21*x22))^2) + (3*h21*x12*(x12 + x21 + \\
& 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)))/5 - \\
& (7*x12*((3*x11*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))^2) + (3*h21*x12*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))^2)))/20 - xh212*((3*x21*((3*h21^2*x11)/(4*(x11 + \\
& h21*x21 + h21*(x12 + h21*x22))) + (3*h21^3*x12)/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))))))/5 + (7*x22*((3*h21^2*x11)/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + (3*h21^3*x12)/(4*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22)))))/20 - (21*(x22*h21^2 + 2*x12*h21 + x11))/(80*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + (3*x11*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))) + (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/5 + \\
& (7*x12*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/20 - \\
& (9*(x21*h21^2 + x11*h21))/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& 7/20 + (21*(x12^2 + x22*(x11 + h21*x12) + h21*x12*x22))/(80*(x11 + h21*x21 \\
& + h21*(x12 + h21*x22))) - xh211*((3*x21*((3*h21*x11)/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))))))/5 - (9*(2*x11 + h21*x12 + h21*x21))/(20*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + (7*x22*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))) + (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/20 - \\
& (21*(x12 + h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (3*x11*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/5 + \\
& (7*x12*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))))))/20 + 3/5) - \\
& (3*x21*((3*h21*x11*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 +
\end{aligned}$$

$$\begin{aligned}
& h21*x22))^2) + (3*h21^2*x12*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))^2))/5 - (7*x22*((3*h21*x11*(x12 + x21 + \\
& 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))^2) + (3*h21^2*x12*(x12 \\
& + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)))/20 - \\
& xh221*((3*x21*((3*h21^2*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (3*h21^3*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/5 + \\
& (7*x22*((3*h21^2*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (3*h21^3*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/20 + \\
& (3*x11*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/5 + \\
& (7*x12*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))))/20 - \\
& (9*(x12*h21^2 + x11*h21))/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) - 1); \\
F(7) = & (9*(x11*x12 + x21*(x11 + h21*x12) + h21*x12*x21))/(20*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + (21*(x11*x22 + x21*(x21 + h21*x22) + \\
& h21*x21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) - \\
& xh212*(x21*(h21*((9*h21*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*h21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& h21^2*((9*h21*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*h21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& x11*(h21*((9*h21*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*h21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (9*h21*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*h21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) - (9*(x21*h21^2 + \\
& x11*h21))/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) - 1) - \\
& xh211*(x11*((9*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + h21*((9*x12)/(20*(x11 \\
& + h21*x21 + h21*(x12 + h21*x22)))) + (21*x22)/(80*(x11 + h21*x21 + h21*(x12 \\
& + h21*x22)))) - (9*(2*x11 + h21*x12 + h21*x21))/(20*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + x21*(h21*((9*x11)/(20*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22)))) + (21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& h21^2*((9*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) - (21*(x21 + \\
& h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) + 3/5) - \\
& x11*(h21*((9*x12*(x12 + x21 + 2*h21*x22))/(20*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))^2) + (21*x22*(x12 + x21 + 2*h21*x22))/(80*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))^2)) + (9*x11*(x12 + x21 + 2*h21*x22))/(20*(x11 + \\
& h21*x21 + h21*(x12 + h21*x22))^2) + (21*x21*(x12 + x21 + \\
& 2*h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)) - \\
& xh221*(x21*(h21*((9*h21*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*h21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& h21^2*((9*h21*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*h21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) - (21*(x22*h21^2 \\
& + 2*x21*h21 + x11))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& x11*(h21*((9*h21*x12)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (21*h21*x22)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (9*h21*x11)/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + \\
& (21*h21*x21)/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))) - (9*(x12*h21^2 + \\
& x11*h21))/(20*(x11 + h21*x21 + h21*(x12 + h21*x22))) + 7/20) - \\
& x21*(h21*((9*x11*(x12 + x21 + 2*h21*x22))/(20*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22))^2) + (21*x21*(x12 + x21 + 2*h21*x22))/(80*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))^2)) + h21^2*((9*x12*(x12 + x21 + 2*h21*x22))/(20*(x11 \\
& + h21*x21 + h21*(x12 + h21*x22))^2) + (21*x22*(x12 + x21 + \\
& 2*h21*x22))/(80*(x11 + h21*x21 + h21*(x12 + h21*x22))^2)); \\
F(8) = & (3*(x11*x12 + x21*(x11 + h21*x12) + h21*x12*x21))/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) - xh221*(x21*((3*h21^2*x11)/(4*(x11 + h21*x21 + \\
& h21*(x12 + h21*x22))) + (3*h21^3*x12)/(4*(x11 + h21*x21 + h21*(x12 + \\
& h21*x22)))) + x11*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + \\
& (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) - (3*(x12*h21^2 + \\
& x11*h21))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) - \\
& xh212*(x21*((3*h21^2*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) +
\end{aligned}$$

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(3*h21^3*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
x11*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) - (3*(x21*h21^2 +
x11*h21))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) - x11*((3*x11*(x12 +
x21 + 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))^2) +
(3*h21*x12*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 +
h21*x22))^2) - xh211*(x21*((3*h21*x11)/(4*(x11 + h21*x21 + h21*(x12 +
h21*x22)))) + (3*h21^2*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) -
(3*(2*x11 + h21*x12 + h21*x21))/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
x11*((3*x11)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22))) +
(3*h21*x12)/(4*(x11 + h21*x21 + h21*(x12 + h21*x22)))) + 1) -
x21*((3*h21*x11*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 + h21*(x12 +
h21*x22))^2) + (3*h21^2*x12*(x12 + x21 + 2*h21*x22))/(4*(x11 + h21*x21 +
h21*(x12 + h21*x22))^2));

```

end